Advanced Higher Knowledge to know Prelim

List the 8 trig identities

1) $\sin^2 A + \cos^2 A = 1$

2) $\tan A = \frac{\sin x}{\cos x}$

3) $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

4) $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

5) $1 + \tan^2 A = \sec^2 A$

6) $\cot^2 A + 1 = \operatorname{cosec}^2 A$

7) $\sin 2A = 2\sin A \cos A$

8) $\cos 2A = \cos^2 A - \sin^2 A$

Complete the exact value table

	0	π/6	π/4	π/3	π/2	π	3π/2	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef	0	undef	0

Negative Facts

1) $\sin(-\theta) = -\sin\theta$

2) $\cos(-\theta) = \cos\theta$

3) $\tan(-\theta) = -\tan\theta$

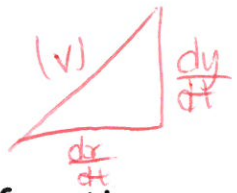
Parametric Equations

$x = f(t) \quad y = f(t)$

Gradient = $m = \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{d^2y}{dx^2} = \frac{d^2y}{dx\,dt} \times \frac{dt}{dx}$

Speed $= \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}$



Volume of Revolution for function around a and b

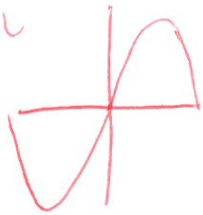
About the x axis: $V = \pi \int_a^b y^2\,dx$

About the y axis: $V = \pi \int_a^b x^2\,dy$

Functions

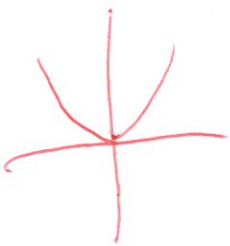
Odd Function: $f(-x) = -f(x)$

180° Rotational symmetry around the origin



Even Function: $f(-x) = f(x)$

Line symmetry about the y axis



Sequences

Arithmetic Term: $U_n = a + (n-1)d$

Geometric term: $U_n = ar^{n-1}$

Sum to infinity: $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Matrices

2 by 2

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\det A = ad - bc$

$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ $A' = A^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

3 by 3

$A = \begin{pmatrix} + & - & + \\ a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

$\det A = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$

Transformation Matrices

Reflection in x axis $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Reflection in y axis $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Scale by factor a $\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$

Complex Numbers

If $z = a + bi$

The modulus is given by $|z| = \sqrt{a^2 + b^2}$

The Argument is given by $\tan\theta = \frac{b}{a}$ $\quad -\pi < \theta < \pi$

The conjugate is given by $\bar{z} = a - bi$

McLaurin Series Useful to Memorise

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots + \frac{x^n}{n!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

Differential Equations

For $\frac{dy}{dx} + P(x) = Q(x)$ the integrating factor $I(x) = e^{\int P(x)\,dx}$

The solution is given by $I(x)\,y = \int I(x)\,Q(x)\,dx$

Complementary Functions

Two real distinct roots $y = Ae^{mx} + Be^{nx}$

Real and Equal $y = (Ax + B)e^{mx}$

Complex and Conjugate $y = e^{px}(A\cos qx + B\sin qx)$

Particular Integrals

if sinax or cosax try $y = P\cos ax + Q\sin ax$

if e^{ax} try $y = Pe^{ax}$

if $y = ax + b$ try $y = Px + Q$

if $y = ax^2 + bx + c$ $y = Px^2 + Qx + R$