# H

National Qualifications 2019

**X847/76/11**

# Mathematics Paper 1 (Non-calculator)

THURSDAY, 2 MAY

9:00 AM – 10:30 AM

**Total marks — 70**

Attempt ALL questions.

**You may NOT use a calculator.**

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.



*X8477611*

A/PB

## FORMULAE LIST

### Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

### Scalar product

$\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where $\theta$ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

### Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

### Table of standard derivatives

| $f(x)$ | $f'(x)$ |
|---|---|
| $\sin ax$ | $a\cos ax$ |
| $\cos ax$ | $-a\sin ax$ |

### Table of standard integrals

| $f(x)$ | $\int f(x)dx$ |
|---|---|
| $\sin ax$ | $-\frac{1}{a}\cos ax + c$ |
| $\cos ax$ | $\frac{1}{a}\sin ax + c$ |

MARKS

**Attempt ALL questions**

**Total marks — 70**

**1.** Find the $x$-coordinates of the stationary points on the curve with equation $y = \frac{1}{2}x^4 - 2x^3 + 6$. **4**

**2.** The equation $x^2 + (k-5)x + 1 = 0$ has equal roots.

Determine the possible values of $k$. **3**

**3.** Circle $C_1$ has equation $x^2 + y^2 - 6x - 2y - 26 = 0$.

Circle $C_2$ has centre (4,–2).

The radius of $C_2$ is equal to the radius of $C_1$.

Find the equation of circle $C_2$. **2**

**4.** A sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c,$$

where the first three terms of the sequence are 6, 9 and 11.

(a) Find the values of $m$ and $c$. **3**

(b) Hence, calculate the fourth term of the sequence. **1**

**5.** (a) Show that the points A(1,5,–3), B(4,–1,0) and C(8,–9,4) are collinear. **3**

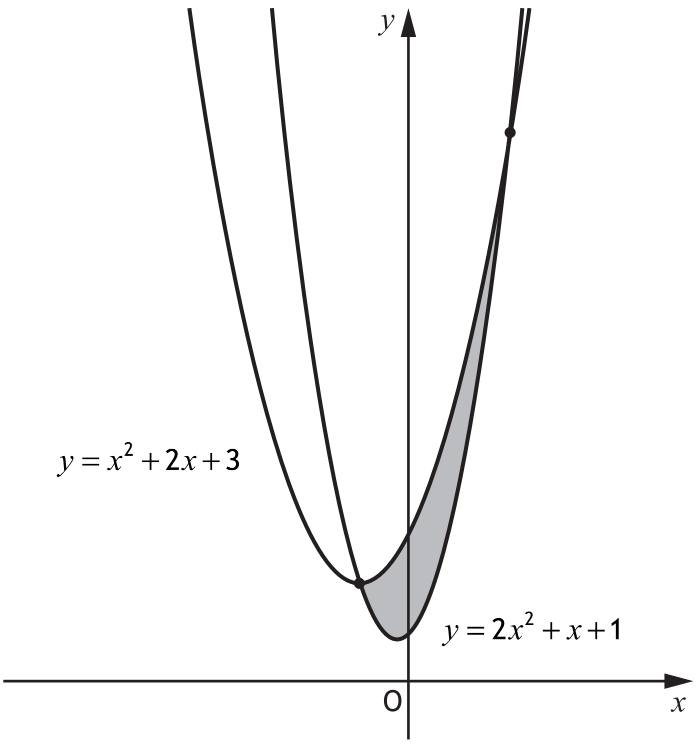
(b) State the ratio in which B divides AC. **1**

**6.** Given that $y = \dfrac{1}{(1-3x)^5}$, $x \neq \dfrac{1}{3}$, find $\dfrac{dy}{dx}$. **3**

**7.** The line, $L$, makes an angle of 30° with the positive direction of the $x$-axis.

Find the equation of the line perpendicular to $L$, passing through (0,–4). **4**

**8.** The graphs of $y = x^2 + 2x + 3$ and $y = 2x^2 + x + 1$ are shown below.




The graphs intersect at the points where $x = -1$ and $x = 2$.

(a) Express the shaded area, enclosed between the curves, as an integral. **1**

(b) Evaluate the shaded area. **3**

MARKS

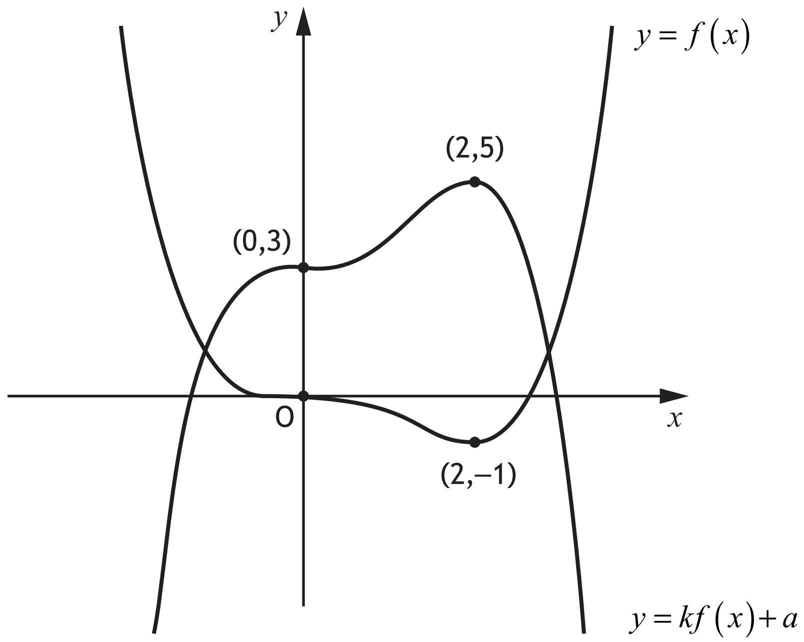
**9.** Vectors $\mathbf{u}$ and $\mathbf{v}$ have components $\begin{pmatrix} p \\ -2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 2p+16 \\ -3 \\ 6 \end{pmatrix}$, $p \in \mathbb{R}$.

(a) (i) Find an expression for $\mathbf{u}.\mathbf{v}$. **1**

(ii) Determine the values of $p$ for which $\mathbf{u}$ and $\mathbf{v}$ are perpendicular. **3**

(b) Determine the value of $p$ for which $\mathbf{u}$ and $\mathbf{v}$ are parallel. **2**

**10.** The diagram shows the graphs with equations $y = f(x)$ and $y = kf(x) + a$.




(a) State the value of $a$. **1**

(b) Find the value of $k$. **1**

**11.** Evaluate $\int_0^{\frac{\pi}{9}} \cos\left(3x - \frac{\pi}{6}\right) dx$. **4**

**MARKS**

**12.** Functions $f$ and $g$ are defined by

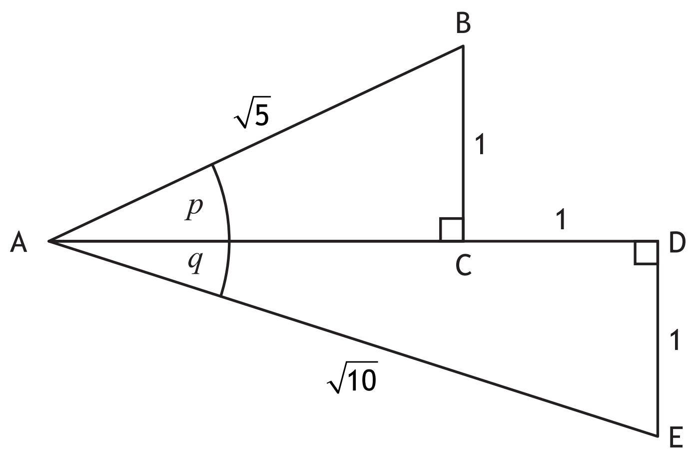
- $f(x) = \frac{1}{\sqrt{x}}$, where $x > 0$
- $g(x) = 5 - x$, where $x \in \mathbb{R}$.

(a) Determine an expression for $f(g(x))$. **2**

(b) State the range of values of $x$ for which $f(g(x))$ is undefined. **1**

**13.** Triangles ABC and ADE are both right angled.

Angles $p$ and $q$ are as shown in the diagram.




(a) Determine the value of

(i) $\cos p$ **1**

(ii) $\cos q$. **1**

(b) Hence determine the value of $\sin(p+q)$. **3**

**14.** (a) Evaluate $\log_{10} 4 + 2\log_{10} 5$. **3**

(b) Solve $\log_2(7x-2) - \log_2 3 = 5, \quad x \geq 1$. **3**

MARKS

**15.** (a) Solve the equation $\sin 2x° + 6\cos x° = 0$ for $0 \leq x < 360$. **4**

(b) Hence solve $\sin 4x° + 6\cos 2x° = 0$ for $0 \leq x < 360$. **1**

**16.** The point P has coordinates $(4,k)$.

C is the centre of the circle with equation $(x-1)^2 + (y+2)^2 = 25$.

(a) Show that the distance between the points P and C is given by $\sqrt{k^2 + 4k + 13}$. **2**

(b) Hence, or otherwise, find the range of values of $k$ such that P lies outside the circle. **4**

**17.** (a) Express $(\sin x - \cos x)^2$ in the form $p + q\sin rx$ where $p$, $q$ and $r$ are integers. **3**

(b) Hence, find $\int (\sin x - \cos x)^2 \, dx$. **2**

**[END OF QUESTION PAPER]**

| For marker's use | | |
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| Question no | Marks/Grades | |
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