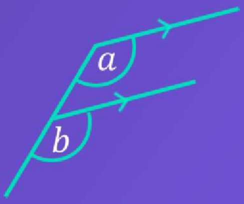
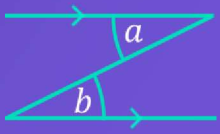
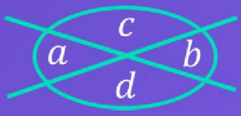
Chapter 1: Fundamental Skills

On exam formulae list? ✓/✗

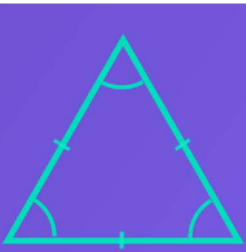
Percentages and Compounding

Percentage Change	$\% \, Change = \frac{New \, quantity}{Old \, quantity} - 1$	To work out the percentage change (either an increase or a decrease) between two quantities, take the new quantity and divide by the original quantity, then subtract 1.	✗
Compound Appreciation	$Value_n = Original \, value \times (1 + app \, \%)^n$	When something appreciates, it grows in value. The formula calculates the value ($Value^n$) we obtain if we take some starting value ($Original \, value$) and let it grow at a certain appreciation rate ($app \, \%$) over a defined number of periods (n).	✗
Compound Depreciation	$Value_n = Original \, value \times (1 - dep \, \%)^n$	When something depreciates, it decreases in value. The formula calculates the value ($Value^n$) we obtain if we take some starting value ($Original \, value$) and let it decrease at a certain depreciation rate ($dep \, \%$) over a defined number of periods (n).	✗

Related Angles

Supplementary Angles		The angles along any straight line add to 180°. These are called supplementary angles. For the example shown: $a + b = 180°$	✗
Corresponding Angles		When two parallel lines are intersected by another line, it can create corresponding angles (the "F" shape). Corresponding angles are equal to each other. For the example shown: $a = b$	✗
Alternate Angles		When two parallel lines are intersected by another line, it can also create alternate angles (the "Z" shape). Alternate angles are equal to each other. For the example shown: $a = b$	✗
Vertically Opposite Angles		When two lines intersect, they create vertically opposite angles (the "**X**" shape). Vertically opposite angles are equal to each other. For the example shown: $a = b$ & c$= d$	✗

Interior & Exterior Angles

Interior Angles of a Triangle		The interior angles of any triangle always sum to $180°$. An equilateral triangle – like the diagram shown – has three equal sides and interior angles, so each of the interior angles must be $60°$ since $60° + 60° + 60° = 180°$.	✗

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Interior Angles of a Quadrilateral

The interior angles of any quadrilateral always sum to 360°.

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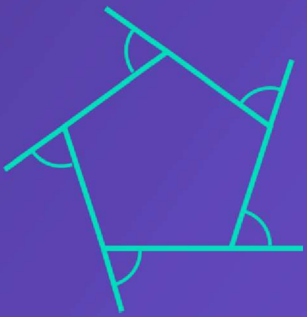
Exterior Angles of a Regular Polygon



The exterior angles of any regular polygon (e.g. squares, rectangles, pentagons, hexagons etc. where all the sides are the same length) always sum to 360°. Taking the regular pentagon shown as an example, each of the five exterior angles must be 72° since $72^\circ + 72 + 72^\circ + 72^\circ + 72^\circ = 360^\circ$.

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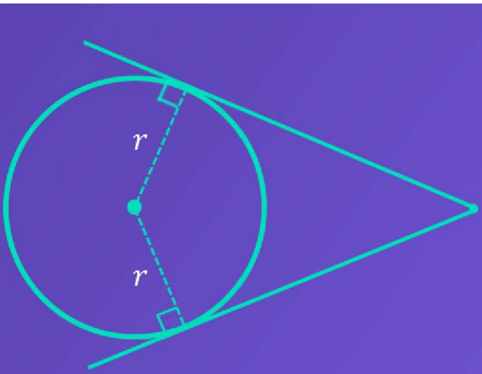
Circle Geometry

Circle Geometry Rule I



A tangent to a curve at a given point is a straight line which "just touches" the curve at that point and no more. In circle geometry, tangents always meet radii at 90° angles.

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Circle Geometry Rule II

Any two chords that start at opposite ends of a circle's diameter will always meet at a 90° angle.

☒

Circle Geometry Rule III

The perpendicular bisector of any chord always passes through the centre of a circle.

☒

☒

Chapter 2: Surds and Indices

On exam formulae list? ✓/✗

Laws of Surds

Law I

$$a\sqrt{m} + b\sqrt{m} = (a+b)\sqrt{m}$$

Law I states that like-surds stack up (recall the key principle in maths that all like-terms add together, or "stack up"). $\sqrt{m}$ just stands for some random surd: it could be $\sqrt{3}$, $\sqrt{5}$, $\sqrt{10}$ or any other surd, it does not matter. "a" and "b" are just multiplying factors, or "amounts" of that surd, which could be any number.

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Law II	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	Law II states that the root of a fraction can be written as a fraction of separate roots.	☒
Law III	$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$	Law III states that products of surds ($\sqrt{a} \times \sqrt{b}$) can be combined into a single surd ($\sqrt{ab}$). This also works the other way around – single surds can be split up into products of two surds.	☒

Laws of Indices

Law I	$x^a \times x^b = x^{a+b}$	Law I states that to multiply index terms you add their powers together.	☒
Law II	$x^a \div x^b = x^{a+b}$	Law II states that to divide index terms you subtract their powers.	☒
Law III	$(x^a)^b = x^{ab}$	Law III states that when an index is raised to a power you multiply the powers.	☒
Law IV	$x^1 = x$	Law IV states that any number raised to the power 1 equals the number itself.	☒
Law V	$x^0 = 1$	Law V states that any number (excluding 0) raised to the power of 0 equals 1.	☒
Law VI	$x^{-a} = \frac{1}{x^a}$	Law VI states that any number raised to a negative power can be written as its reciprocal with a positive power.	☒
Law VII	$x^{\frac{a}{b}} = \sqrt[b]{x^a}$	Law VII states that any number raised to a fractional power can be written as a surd.	☒

Chapter 3: Straight Line

On exam formulae list? ✓/✗

General Equation of a Straight Line

The General Equation of a Straight Line	$y = mc + c$	Any straight line can be described using the General Equation $y = mx + c$ where x and y are variables and m and c are special constants: m is the gradient and c is the y-intercept. For any specific line, the gradient (m) and the y-intercept (c) are fixed.	☒
Gradient Formula	$m = \frac{y_2 - y_1}{x - x_1}$	The gradient defines the slope, or how "steep" a line is. The larger the gradient, the more steeply sloped a line is; the smaller the gradient, the less steeply sloped a line is. The gradient of any line can be calculated using the gradient formula shown where (x_1, y_1) are the x and y components of a first coordinate that lies on the line, and (x_1, y_1) are the x and y components of a second coordinate that also lies on the line.	☒

Determining the Equation of a Specific Line

Alternate Equation for Straight Lines

$y - b = m(x - a)$

A straight line can be expressed in the form $y - b = m(x - a)$ where x and y are variables, m is the gradient and a and b are respectively the x and y coordinates of any point on the line. We exclusively use $y - b = m(x - a)$ to calculate the equation of a line when we do not know the line's y-intercept.

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Chapter 4: Quadratics

On exam formulae list? ✓/✗

General Equation of a Quadratic Function

The General Equation of a Quadratic Function

$y = ax^2 + bx + c$

Any quadratic function can be described using the General Equation $y = ax^2 + bx + c$ where x and y are variables, and a, b and c are constants (often called "coefficients") which change the curvature and position of the quadratic.

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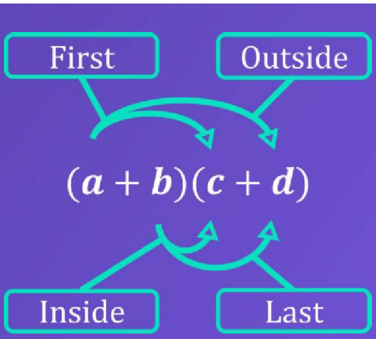
Double Brackets

Expanding Double Brackets



To expand two brackets, multiply each term in the first bracket by each term in the second bracket individually. In cases where each bracket contains two terms, this process is often summarised using the acronym "FOIL" which stands for First, Outside, Inside, Last.

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Roots (x-intercepts)

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Formula is used to calculate the roots (i.e. the x-intercepts) of quadratic functions. In the formula shown left, x represents the roots of the quadratic and a, b and c are the coefficients of the quadratic when written in the form of the General Equation $y = ax^2 + bx + c$. If it is possible to find the roots of the quadratic using factorisation techniques then you should always do so before considering using the Quadratic Formula.

☑

Discriminant

$b^2 - 4ac$

The discriminant is a formula which can be used to determine whether a quadratic has 2, 1 or 0 roots. In the formula shown left a, b and c are the coefficients of the quadratic when written in the form of the General Equation $y = ax^2 + bx + c$. On calculating the discriminant, there are three possible outcomes and each outcome relates to one of the three root scenarios (i.e. 2, 1 or 0 roots):

$b^2 - 4ac < 0$ → No real roots
$b^2 - 4ac = 0$ → 1 real root repeated
$b^2 - 4ac > 0$ → 2 real distinct roots

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Complete the Square Form	$y=(x+p)^2+q$	Completing the Square is a more powerful technique for finding the turning point of a quadratic. Completing the Square works for all quadratics, including those which do not have roots. Completing the Square is a process which takes a quadratic in the form of the General Equation $y=ax^2+bx+c$ and transforms it into a quadratic of the form $y=(x+p)^2+q$. Once in this form, the turning point of the quadratic can be extracted as $(-p,q)$.	✗

Chapter 5: Trigonometry

On exam formulae list? ✓/✗

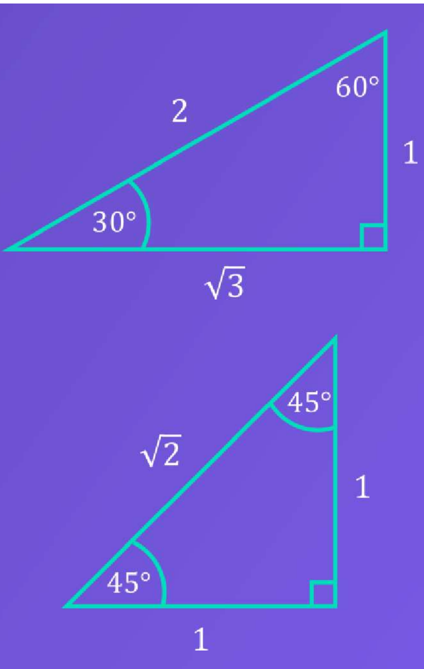
Pythagoras

Pythagoras'Theorem	$a^2=b^2+c^2$	Pythagoras's Theorem states that in right-angled trigonometry, the square of the hypotenuse (i.e. the longest side, usually labelled a and which is always opposite the right angle) is equal to the sum of the squares of the other two sides.	✗
Converse of Pythagoras	$b^2=a^2-c^2$ *or* $c^2=a^2-b^2$	The Converse of Pythagoras is really just a fancy term meaning a rearrangement of Pythagoras' Theorem where one of the shorter sides (i.e. side b or c) is made the subject of the equation. As long as you know two sides of a right-angled triangle, you will always be able to calculate the third side using either Pythagoras' Theorem or the Converse of Pythagoras.	✗

SOH CAH TOA

SOH CAH TOA	 $\sin(x)=\frac{opposite}{hypotenuse}$ $\cos(x)=\frac{adjacent}{hypotenuse}$ $\tan(x)=\frac{opposite}{adjacent}$	The SOH CAH TOA triangles are useful tools to help you decide which trigonometric function (i.e. *sin*, *cos* or *tan*) you should be using in a given situation to calculate missing sides or angles of a right-angled triangle.	✗

Exact Values

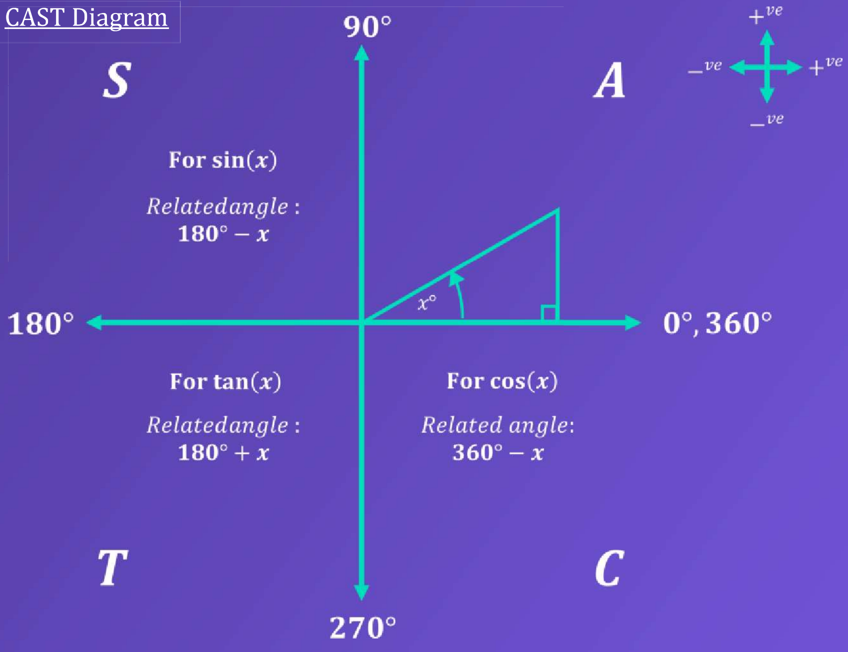
Exact Value Triangles		For your non-calculator exam, you will need to know the exact values of: sin(30), sin(45), sin(60), cos(30), cos(45), cos(60), tan(30), tan(45) and tan(60). Memorising all 9 of these exact values is challenging, so it is easier to remember the two exact value triangles. Using these triangles, all of these exact values can be calculated.	✗

Graphing Trigonometric Functions

The General Equation of a Trigonometric Function	$y = a\,sin(bx + c) + d$	The General Equation of a Trigonometric Function is $y = a\,sin(bx + c) + d$ where a is the amplitude; b is the frequency, or the number of complete waves inside the standard period; c dictates the phase shift (left or right movement); and d determines the vertical shift (up or down movement). The format is exactly the same if dealing with cosine or tangent functions: $y = a\,sin(bx + c) + d$ $y = a\cos(bx + c) + d$ $y = a\tan(bx + c) + d$	☒

CAST Diagram

CAST Diagram



The CAST diagram was developed as a tool to help summarise certain properties of the 3 basic trigonometric functions (i.e. $\sin(x)$, $\cos(x)$ and $\tan(x)$) in one simple diagram. The standard CAST diagram is shown left. All trigonometric functions have related angles, but the rules for calculating the related angles for each function are different due to each function having a different shape and symmetries.

☒

Trigonometric Identities

Trig Identity I	$\frac{\sin x}{\cos x} = \tan x$	Trigonometric identities are used to simplify expressions involving trigonometric functions to make them easier to work with. There are a huge number of identities out there, but at the National 5 level there are only two you are expected to know.	☒
Trig Identity II	$\sin^2 x + \cos^2 x = 1$	As above.	☒

Non-Right-Angled Trigonometry

The Sine Rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	The Sine Rule can be used to calculate the missing sides or angles of a non-right-angled triangle. In the Sine Rule, the lower case letters a, b and c are used to represent the sides of the non-right-angled triangle; the upper case A, B and C are used to represent the corresponding angles (i.e. angle A lies directly opposite side a, and so on). The Sine Rule cannot be used in all situations – to use it, one corresponding side and angle pair must be known, plus one additional piece of information.	☑

The Cosine Rule

Form 1: $a^2 = b^2 + c^2 - 2bc\cos(\mathrm{A})$

Form 2: $\cos(A) = \dfrac{b^2 + c^2 - a^2}{2bc}$

Like the Sine Rule, the Cosine Rule can be used to calculate the missing sides or angles of a non-right-angled triangle, and again the lower case letters a, b and c are used to represent the sides of the non-right-angled triangle whereas the upper case A, B and C are used to represent the corresponding angles (i.e. angle A lies directly opposite side a, and so on).

The Cosine Rule is often stated in two different forms (both shown left). These two forms both describe the same underlying rule – one is simply a rearrangement of the other. The two forms of the Cosine Rule can be used in different situations:

Form 1 of the Cosine Rule can be used to calculate the remaining side of a non-right-angled triangle assuming the two other sides of the triangle, plus the included angle, are known.

Form 2 of the Cosine Rule can be used to calculate any angle inside a non-right-angled triangle assuming all the side lengths of the triangle are known.

✓

Area of a Non-Right-Angled Triangle

$\mathrm{A} = \frac{1}{2}ab\sin C$

The area of any triangle can be calculated using $\mathrm{A} = \frac{1}{2}bh$ where b is the base and h is the height. However, with most non-right-angled triangles you will not know their height. The alternative area of a triangle formula, $\mathrm{A} = \frac{1}{2}ab\sin C$ where a and b represent two sides of the triangle and C represents their included angle, is a more sophisticated formula that allows you to calculate the area of any triangle without knowing its height.

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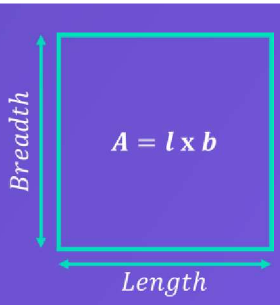
Chapter 6: Area, Volume and Similarity

On exam formulae list? ✓/✗

Area

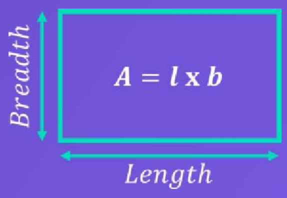
Area of a Square



To calculate the area of a square, multiply its length by its breadth. Note that for a square the length and breadth will have the same value.

✗

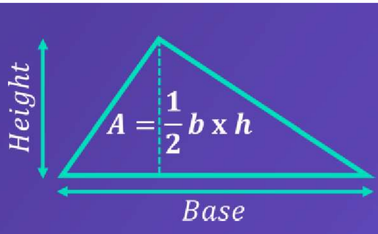
Area of a Rectangle



To calculate the area of a rectangle, multiply its length by its breadth.

✗

Area of a Triangle



To calculate the area of a triangle, multiply its base by its height and divide by two.

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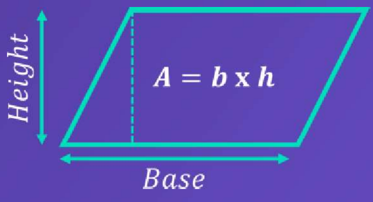
Area of a Parallelogram



To calculate the area of a parallelogram, multiply its base by its height.

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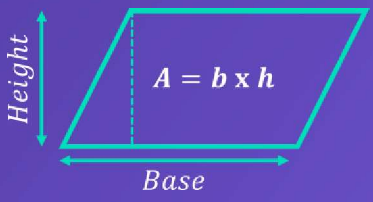
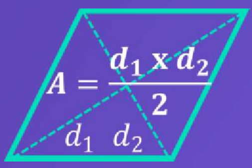
Area of a Parallelogram



To calculate the area of a parallelogram, multiply its base by its height.

☒

Area of a Rhombus



To calculate the area of a rhombus, multiply the diagonal lengths together and divide by two.

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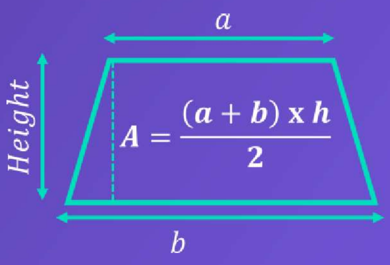
Area of a Trapezium



To calculate the area of a trapezium, add the lengths of the two parallel edges (a and b in the diagram), multiply by the height and divide the result by two.

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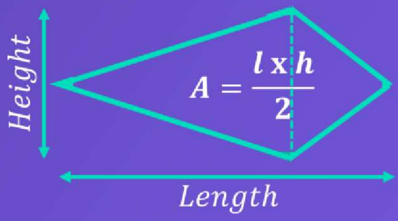
Area of a Kite



To calculate the area of a kite, multiply its length by its height and divide by two.

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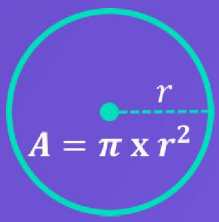
Area of a Circle



To calculate the area of a circle, multiply its radius squared by pi (π).

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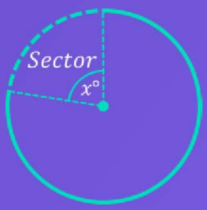
Area of a Sector

$$\text{Sector Area} = \frac{x}{360} \times \pi r^2$$



The area of a sector is determined by the angle subtended by the arc (i.e. $x°$). The sector area formula essentially works out the “small section” or fraction ($\frac{x}{360}$) of the overall area (πr^2) of the circle that the sector represents.

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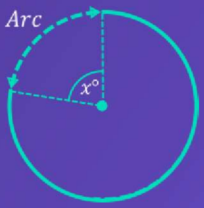
Length of an Arc

$$\text{Arc Length} = \frac{x}{360} \times \pi D$$



Just like the area of a sector, the length of an arc is determined by the angle subtended by the arc (i.e. $x°$). This formula works in essentially the same way as the previous formula for the area of a sector – this formula works out the “small section” or fraction ($\frac{x}{360}$) of the overall circumference (πD) of the circle that the arc represents

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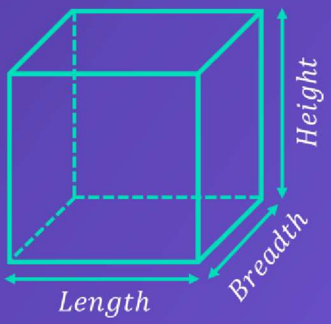
Volume

Volume of a Cube

$$V = l \times b \times h$$

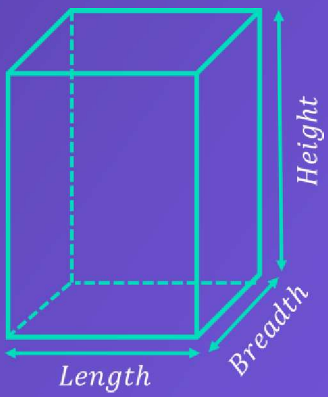


To calculate the volume of a cube, multiply its length, base and height together. Note that for a cube the length, breadth and height will have the same value.

☒

Volume of a Cuboid

$$V = l \times b \times h$$



To calculate the volume of a cuboid, multiply its length, base and height together.

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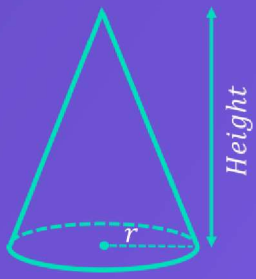
Volume of a Cone

$$V = \frac{1}{3}\pi r^2 \times h$$



To calculate the volume of a cone, multiply its height by its radius squared and by pi then divide by three.

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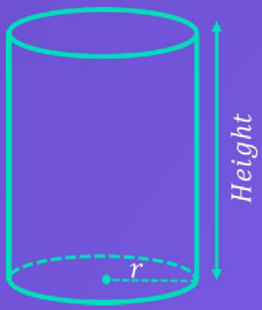
Volume of a Cylinder

$$V = \pi r^2 \times h$$



To calculate the volume of a cylinder, multiply its height by its radius squared and by pi.

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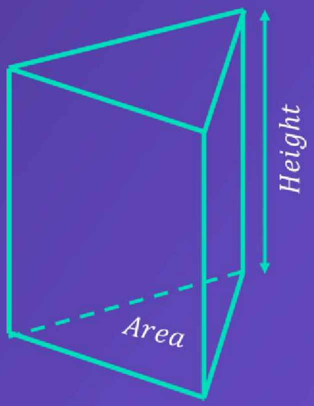
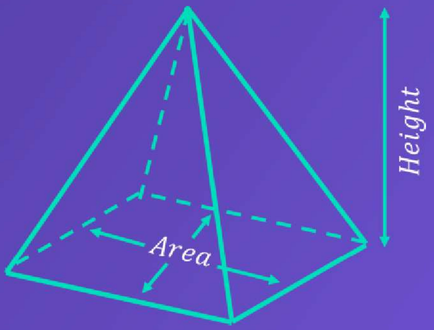
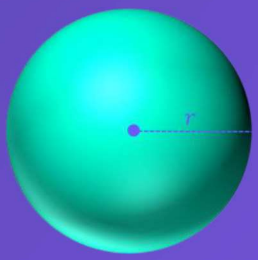
Volume of a Triangular Prism	$V = A \times h$  	To calculate the volume of a triangular prism, multiply the area of one of its triangular faces by its height.	☒
Volume of a Pyramid	$V = \frac{1}{3}A \times h$  	To calculate the volume of a pyramid, multiply the area of its base by its height and divide by three.	☑
Volume of a Sphere	$V = \frac{4}{3}\pi r^3$  	To calculate the volume of a sphere, multiply its radius cubed by pi and by a factor of four thirds.	☑

Similarity

Scale Factor	$\text{Scale factor} = \frac{\text{Large side}}{\text{Corresponding small side}}$	The scale factor is the constant ratio between corresponding sides of similar shapes. To calculate the scale factor between two similar shapes, divide a side from the larger shape by the corresponding side in the smaller shape.	☒
Area Factor	$\text{Area factor} = \frac{\text{Area of large shape}}{\text{Area of small shape}}$ *and* $\text{Area factor} = (\text{Scale factor})^2$	The area factor is the ratio between the areas of similar shapes. Unlike with the scale factor, there is no need to compare "corresponding" areas since all shapes have only one area value. The area factor between two similar shapes is calculated in a very similar manner to the scale factor – simply divide the area of the large shape by the area of the small shape. Note that the area factor is also equal to the scale factor squared. Whilst you can use the actual areas of similar shapes to calculate the area factor directly, if you don't have this information you can instead use the scale factor to calculate the area factors. Everything in similarity is ultimately connected to the scale factor.	☒

Volume Factor	$\text{Volume factor} = \frac{\text{Volume of large shape}}{\text{Volume of small shape}}$ *and* $\text{Volume factor} = (\text{Scale factor})^3$	The volume factor is the ratio between the volumes of similar shapes. Just as with the area factor, all shapes have only one volume value, so there is only one volume ratio to calculate. The volume factor between two similar shapes is calculated in a very similar manner to both the scale and area factor – simply divide the volume of the large shape by the volume of the small shape. Note that the volume factor is also equal to the scale factor cubed. Whilst you can use the actual volumes of similar shapes to calculate the volume factor directly, if you don't have this information you can instead use the scale factor to calculate the volume factors. Everything in similarity is ultimately connected to the scale factor.	☒

Chapter 7: Vectors and 3-D Coordinates

On exam formulae list? ✓/✗

Vectors and Vector Notation

Adding Vectors	$\boldsymbol{a} + \boldsymbol{b} = \binom{x_a}{y_a} + \binom{x_b}{y_b} = \binom{x_a + x_b}{y_a + y_b}$	To add two vectors in component form, add their x-components together and add their y-components together. In the formula left: - x_a is the x-component of the vector $\boldsymbol{a}$; - y_a is the y-component of the vector $\boldsymbol{a}$; - x_b is the x-component of the vector $\boldsymbol{b}$; - y_b is the y-component of the vector $\boldsymbol{b}$.	☒
Magnitude of a 2-D Vector	$\lvert\overrightarrow{AB}\rvert = \sqrt{(x_{ab})^2 + (y_{ab})^2}$	The magnitude of a vector is the distance between the tail and the head of the vector – in other words, its length. In the case of a vector $\overrightarrow{AB}$ (i.e. the vector connecting point A to point B), the mathematical notation for "the magnitude of the vector of the vector $\overrightarrow{AB}$" is $\lvert\overrightarrow{AB}\rvert$. In the formula used to calculate $\lvert\overrightarrow{AB}\rvert$ stated left, x_{ab} is the x-component of the vector $\overrightarrow{AB}$ and y_{ab} is the y-component of the vector $\overrightarrow{AB}$.	☒

3-D Coordinates

Magnitude of a 3–D Vector	$\lvert\overrightarrow{AB}\rvert = \sqrt{(x_{ab})^2 + (y_{ab})^2 + (z_{ab})^2}$	The formula for the magnitude of a vector works in exactly the same way with 3-D vectors as with 2-D vectors. The only difference is that the formula is extended to include the third dimension where z_{ab} represents the z-component of the vector $\overrightarrow{AB}$.	☒

Chapter 8: Statistics

On exam formulae list? ✓/✗

Mean, Median, Mode and Range

Mean	$\text{Mean} = \dfrac{\text{Sum of values in set}}{\text{Number of values in set}}$	The mean, or alternatively, the "arithmetic average", is the sum of the values in a dataset divided by the number of values in the dataset. The purpose of the mean is to calculate a figure which is representative of the dataset as a whole, or representative of the dataset "on average".	☒
Median	$\text{Median} = \text{Middle datapoint}$	The median is the middle number in a dataset when it is ordered numerically. A dataset must be sorted into numerical order before calculating the median. To find the median, simply count in from each side of your dataset in equal steps until you meet in the middle. For datasets with an even number of datapoints, no datapoint lies exactly in the middle – for these datasets, to find the median simply calculate the mean of the middle two results.	☒
Mode	$\text{Mode} = \text{Most frequent result}$	The mode of a dataset is the value which appears most frequently. If no value appears more frequently than the other values in a dataset, then by definition, the dataset must have no mode.	☒
Range	$\text{Range} = \text{Maximum } - \text{minimum}$	The range of a dataset is the difference between the maximum and minimum values in the set. The range defines an interval within which all other results in the dataset can be found.	☒

Boxplots

Interquartile Range	$\text{Interquartile Range} = Q_3 - Q_1$	The interquartile range is equal to the difference between the upper quartile (Q_3) and the lower quartile (Q_1) of a dataset. Upper and lower quartiles normally come up in the context of boxplots. Recall that the upper quartile (Q_3) is the median of the upper half of a dataset when it is ordered numerically, and similarly the lower quartile (Q_1) is the median of the lower half of that same dataset.	☒
Semi−Interquartile Range (SIR)	$\text{SIR} = \dfrac{Q_3 - Q_1}{2}$	The semi-interquartile range is simply the interquartile range (i.e. $Q_3 - Q_1$) divided by two.	☒

Standard Deviation

Standard Deviation	$s = \sqrt{\dfrac{\sum(x-\bar{x})^2}{n-1}}$	The standard deviation of a sample (s) is given by the formula left where x is a datapoint in the sample; $\bar{x}$ (pronounced "x bar") is the mean of the sample; $\sum$ is a standard mathematical symbol which means "sum"; and n is the number of datapoints in the sample (often called the "sample size").	☑

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