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Section	Торіс	Skills		
1	Rounding		I	
1.1	Round to decimal places	Examples: $25.1241 \rightarrow 25.1 (1 \text{ d.p.})$ $34.678 \rightarrow 34.68 (2 \text{ d.p.})$		
1.2	Round to significant figures	Examples: $1276 \rightarrow 1300 \ (2 \text{ s.f.})$ $0.06356 \rightarrow 0.064 \ (2 \text{ s.f.})$ $37,684 \rightarrow 37,700 \ (3 \text{ s.f.})$ $0.005832 \rightarrow 0.00583 \ (3 \text{ s.f.})$		
2	Surds			
2	Surds	A surd is a number expressed in root form that cannot be simplified further. A surd is an irrational number, i.e. a number that cannot be expressed as a fraction, such numbers when expressed in decimal form have an infinite number of decimal places.		
2.2	Basic surd simplification	Learn the Square Numbers: 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169. Identify the largest square number factors that divide into the number being simplified, then take the root of them. Example: $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$		
2.3	Multiplyication of surds	$\sqrt{5} \times \sqrt{15} = \sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$		
2.4	Division of surds	$\frac{\sqrt{48}}{\sqrt{3}} = \sqrt{\frac{48}{3}} = \sqrt{16} = 4$		
2.5	Addition and subtraction of surds	$\sqrt{50} + \sqrt{8} = \sqrt{25} \times \sqrt{2} + \sqrt{4} \times \sqrt{2} = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$		
3	Indices		,	
3	Indices	The term index or indices (plural) is another word for the power or powers of a number. The power of a number is how many times a number or term is multiplied by itself. For example, $3 = 3^1$, so it has a power of 1. This means 3 is only present once. With the number 3^2 , there are two 3's multiplied together, i.e. $3^2 = 3 \times 3 = 9$. NB. $a^0 = 1$		
3.5	The first law of indices	$a^{x} \times a^{y} = a^{x+y}$ Examples: $a^{3} \times a^{2} = a^{3+2} = a^{5}$ $3b^{5} \times 4b^{-2} = 12b^{5+(-2)} = 12b^{3}$		
3.6	The second law of indices	$\frac{a^x}{a^y} = a^{x-y}$ Example: $\frac{6a^7}{2a^5} = 3a^{7-2} = 3a^5$		
3.8	The third law of indices – raising powers to powers	$(a^x)^y = a^{x \times y}$ Example: $(a^3)^4 = a^{3 \times 4} = a^{12}$		
3.9	The fourth law of indices – negative indices	$a^{-x} = \frac{1}{a^x}$ Example: $2a^{-3} = \frac{2}{a^3}$		

Section	Торіс	Skills		
4	Standard Form			
4.1	Change from normal to standard form and vice versa	Standard Form , also known as scientific notation, is a method of writing either very large or very small numbers in a usable and convenient way. The numbers are standardized in that they are always written with the leading number being greater than or equal to 1 and less than 10. Examples: a. $2350 = 2.35 \times 10^3$ b. $0.0000058 = 5.8 \times 10^{-6}$		
4.2-3	Calculations using standard form	$(1.3 \times 10^5) \times (8 \times 10^3) = 10.4 \times 10^8 = 1.04 \times 10^9$		
5	Expanding Brackets			
5.4-10	Multiplication of two brackets	Use FOIL (Firsts Outsides Insides Lasts) or another suitable method $(x + 3)(x - 2) = x^2 + 3x - 2x - 6 = x^2 + x - 6$		
5.11	Multiplication of brackets – two by three	Every term in the first bracket must multiply every term in the second. $(x + 2)(x^2 - 3x - 4) = x^3 - 3x^2 - 4x + 2x^2 - 6x - 8$ $= x^3 - x^2 - 10x - 8$		
6	Factorising			
6.1-3	Common factor	When factorising any expression, the first thing to look for is a common factor . A common factor is a factor that each of the terms share. Example: $4x^2 + 8x = 4x(x + 2)$		
6.4-5	Difference of two squares	A difference of two squares is when one square number is taken away from another. Example: a. $a^2 - 16 = (a + 4)(a - 4)$ b. $4x^2 - 36 = 4(x^2 - 9)$ (NB : common factor first) = 4(x - 3)(x + 3)		
6.6-9	Trinomial	Step 1: Start by considering the First terms in the bracket these will be factors of the first term of the trinomial. Step 2: Move to the Last terms in the brackets. These must be factors of the third term in the trinomial. Step 3: The Outsides and Insides of the brackets must add to give the middle term. Example: $x^2 - x - 6 = (x - 3)(x + 2)$		
6.10	Trinomials – non-unitary coefficient of x^2	This is more difficult. Same process as 6.6-9. The Outsides add Insides give a check of the correct answer: Example: $3x^2 - 13x - 10$ = (3x - 5)(x + 2) Check: $3x \times 2 + (-5) \times x = 6x - 5x = -x$ X = (3x + 2)(x - 5) Check: $3x \times (-5) + 2 \times x = -15x + 2x = -13x \checkmark$ NB: If the answer is wrong, score out and try alterative factors or positions. Keep a note of the factors you have tried.		

Section	Торіс	Skills		
7	Completing the Square			
7.1	Completing the square	$ax^{2} + bx + c = a\left(x + \frac{b}{2a}\right)^{2} + c - \left(\frac{b}{2a}\right)^{2}$ Example: $x^{2} + 8x + 19 = (x + 4)^{2} + 19 - 16$ $= (x + 4)^{2} + 3$		
8	Algebraic Fractions			
8.1-2	Simplifying algebraic fractions by factorising	Step 1: Factorise expression Step 2: Look for common factors. Step 3: Cancel and simplify $\frac{6x^2 - 12x}{x^2 + x - 6} = \frac{6x(x - 2)}{(x + 3)(x - 2)} = \frac{6x}{x + 3}$		
8.3	Multiplying algebraic fractions	Multiply the numerators, then multiply the denominators. NB: It is often better to simply before multiplying. $\frac{6ab}{5c} \times \frac{5ac}{2b} = \frac{3a}{1} \times \frac{a}{1} = \frac{3a \times a}{1} = 3a^2$		
8.4	Dividing algebraic fractions	Invert the second fraction, then multiply $\frac{6x^2}{7y} \div \frac{4x}{3z} = \frac{6x^2}{7y} \times \frac{3z}{4x} = \frac{3x}{7y} \times \frac{3z}{2} = \frac{9xz}{14y}$		
8.5	Addition and subtraction of algebraic fractions	Find a common denominator. This can be done either by working out the lowest common denominator, or by using Smile and Kiss. $\frac{5a}{b} + \frac{3d}{2c} = \frac{10ac}{2bc} \times \frac{3bd}{2bc} = \frac{10ac + 3bd}{2bc}$		
9	Gradients			
9.1-3	The Gradient Formula	Know that gradient is represented by the letter m Step 1: Select two coordinates Step 2: Label them $(x_1, y_1) (x_2, y_2)$ Step 3: Substitute them into gradient formula Example: $(-4, 4), (12, -28)$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-28 - 4}{12 - (-4)} = \frac{-32}{16} = -2$		
10	Circles – Arcs & Sectors			
10.4	The length of an arc of a circle	$\frac{\text{Length of Arc}}{\pi D} = \frac{\text{Angle}}{360} \text{or Length of Arc} = \frac{\text{Angle}}{360} \times \pi D$		
10.6	Finding an angle, radius or diameter	Rearrange the formula and used to find other unknowns: $\frac{\text{Length ofArc}}{\pi \text{D}} = \frac{\text{Area of Sector}}{\pi \text{r}^2} = \frac{\text{Angle}}{360}$		
11	3D Solids - Volume			
11.2	Volume of a cylinder	$V = \pi r^2 h$		
11.3	Volume of a pyramid	$V = \frac{1}{3} \times Area \ of \ base \times height$		

Section	Торіс	Skills		
11.4	Volume of a cone	$V = \frac{1}{3}\pi r^2 h$		
11.5	Volume of a sphere	$V = \frac{4}{3}\pi r^3$		
11.6	Calculating a height or radius using volume formulae	Rearrange the formulae, then substitute in the given values. Example: Cylinder has volume 400cm ³ and radius 6cm, find the height. $V = \pi r^2 h$ $h = \frac{\pi r^2}{V}$ $h = \frac{\pi \times 6^2}{400}$		
11.7	Volume of composite solids	Composite solids are made up of two or more solids. To find the volume of composite solids, find the volume of each solid and add them together.		
12	The Straight Line	1		
12	Gradient	 Represented by <i>m</i> Measure of steepness of slope Positive gradient: the line is <i>increasing</i> Negative gradient: the line is <i>decreasing</i> 		
12	y-intercept	 Represented by <i>c</i> Shows where the line crosses the <i>y</i>-axis Find by making x = 0 		
12.1	The equation of a line from the gradient and <i>y</i> -intercept	Step 1: Find gradient <i>m</i> (section 9.1-3) Step 2: Find y-intercept <i>c</i> Step 3: Substitute into $y = mx + c$ (see above for definitions)		
12.2	Sketch a line from its equation	Step 1: Rearrange equation to the form $y = mx + c$ Step 2: Draw a table of points, determine x and y intercepts, or use gradient to step along from y-intercept. Step 3: Plot points on coordinate axes		
12.3	The equation of a line from two points	Use this when there are only two points (i.e. no <i>y</i> -intercept) Step 1: Find gradient <i>m</i> Step 2: Substitue into $y - b = m(x - a)$ where (a, b) are taken from either one of the points		
12.5	Equations of parallel lines	Parallel lines have the same gradient.		
13	Functions			
13.1	Basic functions	A function in mathematics takes an input value, then applies a rule to it and produces an output value or image. a. If $f(x) = 2x - 1$, evaluate $f(x)$ when $x = 3$. f(x) = 2x - 1 f(3) = 2(3) - 1 f(3) = 6 - 1 f(3) = 5 b. If $g(x) = 4x - 3$. Calculate a when $g(a) = 29$ 4a - 3 = 29 4a = 32 a = 8		

Section	Торіс	Skills		
14	Solving Equations 1 – Lin	ear Equations		
14.6	Equations with brackets	When solving equations with brackets, first expand the brackets and simplify, then solve the equations. Example: $2(x + 4) = 10$ 2x + 8 = 10 2x = 2 x = 1		
14.7	Forming equations	The perimeter of the rectangle is 40 <i>cm</i> . Calculate the value of <i>x</i> . $(x + 2)$ cm 2(x + 2) + 2(2x - 3) = 40 $6x - 2 = 40$ $6x = 42$ $x = 7cm$		
14.9	Equations with fractions	To solve equations involving more than one fraction, multiply everything by the lowest common denominator of the fractions. Example: Solve $\frac{1}{5}(b-2) = \frac{3}{2}$ $(\times 10)$ $(\times 10)$ $\frac{10}{5}(b-2) = \frac{30}{2}$ 2(b-2) = 15 2b-4 = 15 2b = 19 $b = \frac{19}{2}$		
14.8,10	Solving inequations	Inequations are solved <i>in the same way as equations</i> , with two exceptions. Firstly, if multiplying or dividing by a negative value, the inequality sign is reversed. Secondly, when the unknown is on the right of the inequality, the sign must be reversed to move the unknown to the left. a. $-c \ge 5$ b. $4 \ge d$ $c \le -5$ $d \le 4$		
15	Solving Equations 2 – Sin	nultaneous Linear Equations		
15.1	Simultanoues equations by substitution	Simultaneous equations can be solved by substitution if one of the equations has an unknown with a coefficient equal to one, or if the unknown is of equal value in both equations. Example: Solve the system of equations $y = x + 2$ and y = 3x - 6. Substitute $y = x + 2$ into $y = 3x - 6$ x + 2 = 3x - 6 8 = 2x 2x = 8 x = 4 Substitute $x = 4$ into $y = x + 2$ y = 4 + 2 y = 6 Solution: $x = 4$ and $y = 6$.		

Section	Торіс	Skills		
15.2	Simultaneous equations by elimination	Step 1: Scale equations to make one unknown equal with opposite sign.Step 2: Add Equations to eliminate equal term and solve.Step 3: Substitute number to find second term.Example: $2x + 4y = 2$ (A) $x + 4y = 5$ (B)(B) $\times -1$ $-x - 4y = -5$ (C) $x = -3$ (A)+(C) $x = -3$ Substitute $x = -3$ into (A). 		
15.3	Forming equations to solve simultaneously	Form equations from a variety of contexts to solve for unknowns.		
16	Changing the Subject of	a Formula	I	
16.3-6	Changing the subject of a formula	The subject of a formula is the value that is on its own on one side of the formula (usually the left). Change the subject in each of the following questions to x . $y = \sqrt{\frac{a+2j}{b}}$ $y^2 = \frac{a+2j}{b}$ $by^2 = a + 2j$ $a = by^2 - 2j$		
17	Quadratic Functions & G	raphs	I	
17	Quadratic Graphs	The graph of a quadratic function is called a parabola . Parabolas are either 'n' or 'u' shaped, depending on whether the coefficient of the x^2 term is positive , the graph has a minimum turning point . This produces a 'u' shape graph (as in the blue graph on the right). When the coefficient of the x^2 term is negative , the graph has a maximum turning point . This produces an 'n' shape graph (as in the orange graph on the right).		
17.4-5	Solving quadratic equations by factorising	To solve a quadratic function is to find the <i>x</i> -coordinates at which the graph of the quadratic function is equal to zero, i.e. where the graph intersects the <i>x</i> -axis. a. $x^2 + 2x = 0$ x(x + 2) = 0 x = 0 or $x + 2 = 0x = -2b. x^2 - 16 = 0(x + 4)(x - 4) = 0x = -4$ or $x = 4$		

Section	Торіс	Skills		
17.7	Solving quadratics using the Quadratic Formula	This method is used when a quadratic cannot be factorised. These questions usually ask for decimal place accuracy. Example: Solve the quadratic equation $x^2 + 4x - 2 = 0$ to 1 decimal place. Since $ax^2 + bx + c$, then $a = 1, b = 4, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-2)}}{2(1)}$ $x = \frac{-(4) \pm \sqrt{24}}{2}$ $x = -2 - \sqrt{6}$ or $x = -2 + \sqrt{6}$ x = -4.4 (to 1 d.p.) $x = 0.4$ (to 1 d.p.)		
17.8	Sketching quadratics from factorised form	Step 1: Identify the roots and shape from values of k , m and n from the form $y = k(x - m)(x - n)$. Step 2: Find y-intercept by making $x = 0$. Step 3: Sketch graph, noting turning point, y-intercept and axis of symmetry. NB: For National 5, $k = \pm 1$.		
17.9	Forming quadratic equations	Example: The rectangle and the square have the same area. By forming an equation, find the dimensions of the rectangle. $(x-4)cm \qquad xcm \qquad $		

Section	Торіс	Skills		
18	Properties of Shape		I	
18.1	Angles in polygons	To calculate an interior angle of a regular polygon, we can use the formula $180 - (360 \div n)$ where n is the number of sides. To calculate an exterior angle , either find the supplement of the interior angle, or simply $360 \div n$.		
18.2	Angles in circles			
19	Pythagoras' Theorem		I	
19.2	The Converse of Pythagoras	The converse of Pythagoras' Theorem is used to determine whether a triangle is right-angled or not. (The Cosine Rule may also be used, see section 22.) Example: Determine whether the triangle in the diagram is right-angled. Longest Side Other sides $1.3^2 = 1.69$ $1.2^2 + 0.5^2 = 1.44 + 0.25 = 1.69$ $1.3^2 = 1.2^2 + 0.5^2$ \therefore by the converse of Pythagoras' Theorem, the triangle is right-angled. NB: If the triangle is not right-angled, there is no need to state 'by the converse of Pythagoras'.		

Section	Торіс	Skills		
19.3	Pythagoras' Theorem in circles	Pythagoras' Theorem is used within circles where right-angled triangles can be formed. Most commonly this occurs when a radius and chord intersect at right-angles (this is called a perpendicular bisector). Example: The circle, centre <i>O</i> , has a radius of 10 <i>cm</i> . The chord in the diagram is 12 <i>cm</i> long. Calculate the value of <i>x</i> . Identify the right-angled triangle then use Pythagoras' Theorem. $xcm \int_{6cm}^{10cm} x^2 = 10^2 - 6^2$ $x = \sqrt{64}$ $x = 8cm$		
19.4	The distance between two points	To find the distance between two points or coordinates, Pythagoras' Theorem is used. This often appears in the form of the Distance Formula: $Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, but it may be simpler and often more helpful to sketch a right-angled triangle and work out the vertical and horizontal difference, just like when calculating the gradient.		
19.5	Pythagoras' Theorem in three dimensions	Example: The diagram on the right represents a cuboid. Calculate the length of the diagonal <i>AB</i> . $AB^{2} = 9^{2} + 12^{2} + 8^{2}$ $AB^{2} = 81 + 144 + 64$ $AB^{2} = 289$ $AB = \sqrt{289}$ $AB = 17cm$		
21	Trigonometric Functions			
21.3	Solving trigonometric equations	Solve the equations $\sin x^\circ = \frac{1}{2}$, for $0 \le x \le 360$. $\sin x^\circ = \frac{1}{2}$ $x^\circ = \sin^{-1}\left(\frac{1}{2}\right)$ $x^\circ = 30^\circ, 180^\circ - 30^\circ$ $x^\circ = 30^\circ, 150^\circ$ $x^\circ = 30^\circ, 150^\circ$ $x^\circ = 30^\circ, 150^\circ$		

Section	Торіс	Skills		
21.4	Using trigonometric identities	In National 5 Mathematics there are two trigonometric identities to be remembered and used: and $\tan x = \frac{\sin x}{\cos x}$ $\sin^2 x + \cos^2 x = 1$ These identities are equalities that are useful in simplifying trigonometric expressions. Example: Show that $\tan x \cos x = \sin x$. $LHS = \tan x \cos x$ $= \frac{\sin x}{\cos x} \times \cos x$ $= \sin x$ = RHS		
22	Triangle Trigonometry			
22	Triangle Trigonometry			
22.1	Finding the area of a triangle	$A = \frac{1}{2}ab\sin C$		
22.2	Using the Sine Rule to find a side	As a general rule, if there is more than one angle in the triangle, either known or to be calculated, the Sine Rule should be used. Example: Calculate the value of x. $ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} $ $ \frac{a}{\sin A} = \frac{c}{\sin C} $ $ \frac{a}{\sin A} = \frac{19}{\sin 40} $ $ \frac{x}{\sin 35} = \frac{19}{\sin 40} $ $ x = \frac{19 \sin 35}{\sin 40} $ $ x = 17.0cm (1 \text{ d.p.}) $		

Section	Торіс	Skills		
22.3	Using the Sine Rule to find an angle	Example: Calculate size of angle x. $ \begin{array}{c} a\\ c\\ 22m\\ b\\ a\\ B\\ c\\ c\\ 22m\\ c\\ c\\$		
22.4	Using the Cosine Rule to find a side	The Cosine Rule is used when there is only one angle involved in the calculation. Example: Calculate the length of side x . $a^2 = b^2 + c^2 - 2bc \cos A$ $x^2 = 15^2 + 19^2 - 2 \times 15 \times 19 \cos 38$ $x = \sqrt{15^2 + 19^2} - 2 \times 15 \times 19 \cos 38$ x = 11.7 cm (1 d.p.)		
21.5	Using the Cosine Rule to find an angle	Example: Calculate the length of side x. A $25cm$ c x° $31cm$ B $cos A = \frac{b^{2} + c^{2} - a^{2}}{2bc}$ $cos x = \frac{25^{2} + 31^{2} - 28^{2}}{2 \times 25 \times 31}$ $x = cos^{-1} \left(\frac{25^{2} + 31^{2} - 28^{2}}{2 \times 25 \times 31}\right)$ $x = 58.8^{\circ} (1 \text{ d.p.})$		
24	Percentages			
24.4	Appreciation including compound interest	The term appreciation usually refers to the value of something increasing (appreciation) over time. This is usually described as a percentage. Example: Bethany invests £4000 in a savings account with an interest rate of 4%. How much will her investment be worth after 3 years? Calculate the multiplier as a decimal: 100% + 4% = 104% = 1.04 Use the multiplier three times: $4000 \times 1.04 \times 1.04 \times 1.04 = 4000 \times 1.04^3 = £4499.46$		

Section	Торіс	Skills		
24.5	Using reverse percentages	Using reverse percentages is often called working backwards or reversing the change or finding an initial value. Example: A restaurant bill cost £170.50 after a 10% service charge was added on. How much was the bill before the service charge was added? 110% = 170.5 $1\% = 170.5 \div 110 = 1.55$ $100\% = 1.55 \times 100 = £155$		
25	Fractions			
25.4	Addition of mixed numbers	$2\frac{1}{3} + 3\frac{1}{2} = 5\frac{2}{6} + \frac{3}{6} = 5\frac{5}{6}$		
25.5	Subtraction of mixed numbers	$4\frac{2}{3} - 1\frac{1}{4} = 3\frac{8}{12} - \frac{3}{12} = 5\frac{5}{12}$		
25.7	Multiplication of mixed numbers	$3\frac{1}{2} \times 2\frac{1}{5} = \frac{7}{2} \times \frac{11}{5} = \frac{77}{10} = 7\frac{7}{10}$		
25.8	Division of mixed numbers	$\frac{3}{4} \div 1\frac{2}{5} = \frac{3}{4} \div \frac{7}{5} = \frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$		
26	Statistics			
26.2	Quartiles and interquartile range	Example: Find the intervarile range of the following data: 12 18 9 19 13 25 11 15 9 11 12 13 15 18 19 25 $Q_1 = \frac{11+12}{2} = 11.5$ $IQR = Q_3 - Q_1$ $Q_2 = \frac{13+15}{2} = 14$ $IQR = 18.5 - 11.5$ $Q_1 = \frac{18+19}{2} = 18.5$ $IQR = 7$		
26.3	Mean & standard deviation	Example: Calculate the mean and standard deviation of the following data.: 22 38 19 29 13 25 When using either formula, calculate the mean \overline{x} , then draw a table to calculate the values. Finally substitute into the formula. $\overline{x} = 24.3$ $\frac{x}{22} -2.3 5.29}{38} 13.7 187.69}{19} -5.3 28.09}{29} 4.7 22.09}$ $s = \sqrt{\frac{\Sigma(x - \overline{x})^2}{n - 1}}$ $s = \sqrt{\frac{371.34}{5}}$ $s = 8.6 (1 \text{ d.p.})$		

26.4	Comparing data	In National 5 Mathematics there are two things to compare when comparing two or more sets of data; the average and the		
		spread of the data. The average the mean and the spread is the standard deviation.		