N5 - Maths, Relationships (Part 2)

In this booklet:

FURTHER TRIGONOMETRY

By the end of this set of exercises, you should be able to

(a)   recognise the graphs of sine, cosine and tangent functions

(b)   sketch and identify other trigonometric functions

(c)   solve simple trigonometric equations (in degrees)

(d)   define the period of trigonometric functions, either from their graphs or from their equations

(e)   simplify trigonometric expressions using $\sin^2 x + \cos^2 x = 1$
and $\tan x = \dfrac{\sin x}{\cos x}$

A. Sine, cosine and tangent graphs

Exercise 1

You may have drawn the sine, cosine and tangent graphs as part of the introduction to trigonometry in Maths 2 Intermediate 2. If you have retained the graphs, you may miss out questions 1 to 3 of Exercise 1 below.
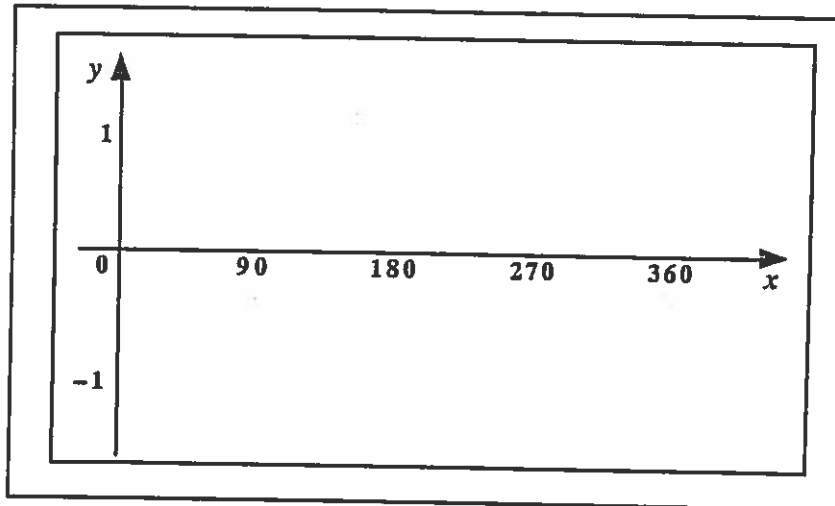
1. The Sine Graph

   (a) Make a copy of this table and use your calculator to help fill it in, giving each answer correct to 2 decimal places.

x	0°	20°	40°	60°	80°	90°	100°	120°	140°	160	180°
$\sin x°$	0·00	0·34	0·64	0·87	0·98	1·00	...	...	...	...	...

x	200°	220°	240°	260°	270°	280°	300°	320°	340°	360
$\sin x°$	...	...	...	...	...	...	...	...	...	...

   (b) Use a piece of 2 mm graph paper to draw a set of axes as illustrated below.

   (c) Plot as accurately as possible the 21 points from your table.

   (d) Join them up smoothly to create the graph of the function $y = \sin x°$.

2. Repeat question 1 (a) to (d) for the function $y = \cos x°$

3. Repeat for the graph of $y = \tan x°$ (a different scale will be required for the vertical axis).

4. Study your three graphs carefully. You should now be able to sketch the sine, cosine (and tangent) graphs fairly quickly (about 30 seconds) indicating the main points where the graphs cut the x and y axes and the general shape of each graph.
   Try them now.

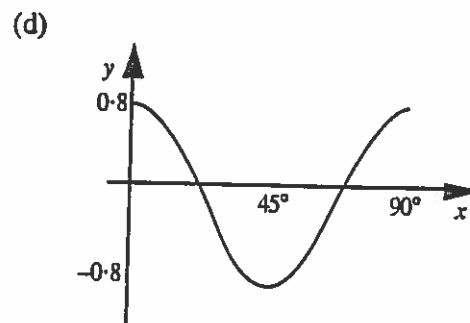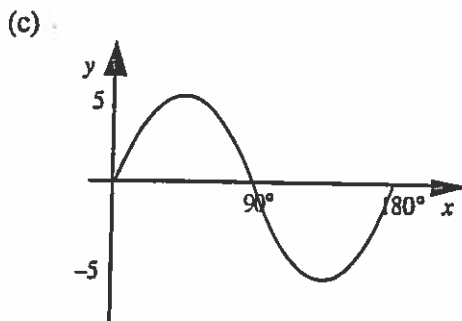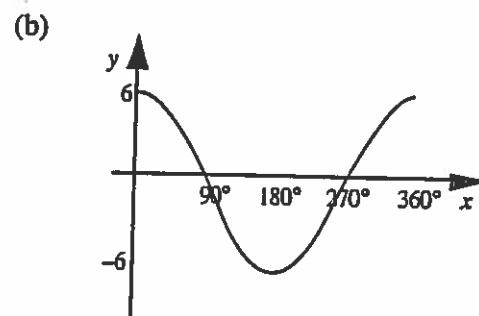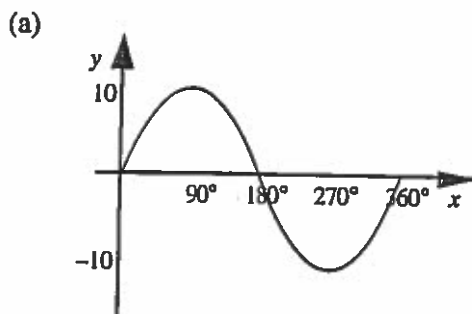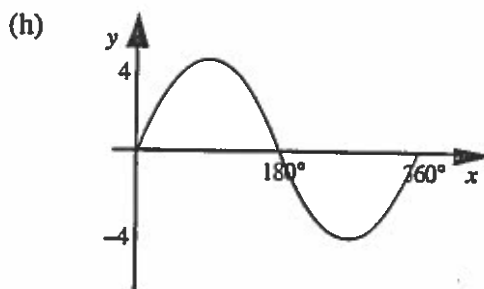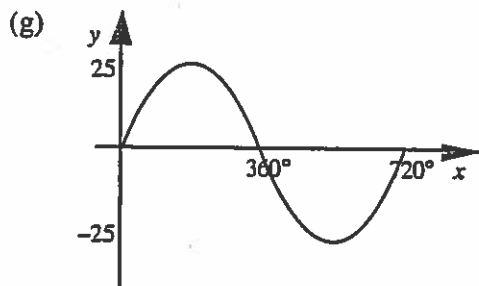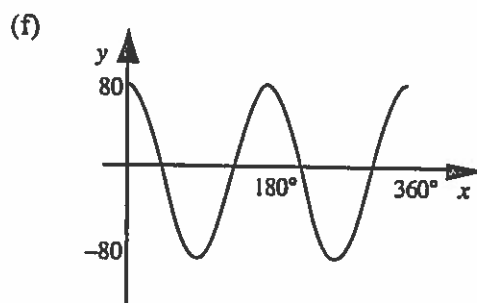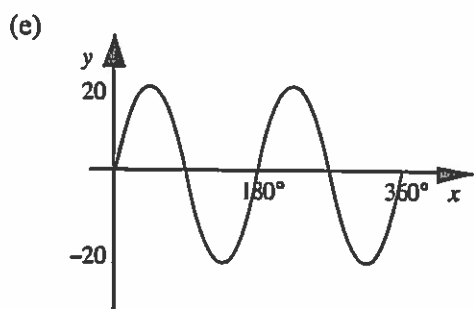Sketches of the three trigonometric graphs, for comparison:
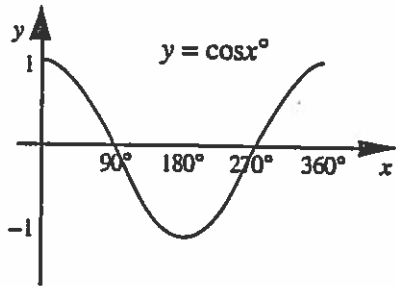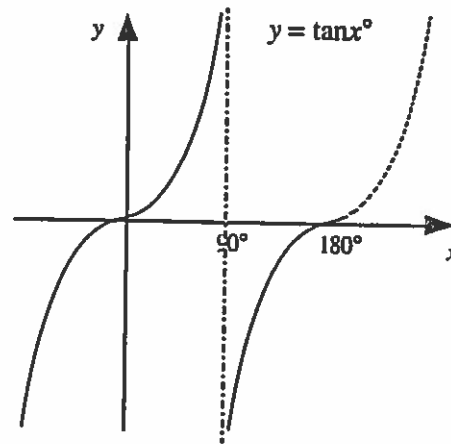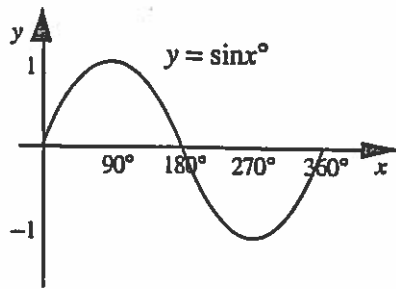
Check your graphs are similar to those shown above.

B. Sketching and identifying trigonometric functions

Exercise 2A

1. Write down the equations of the trigonometric functions represented by the following graphs:

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

(l)
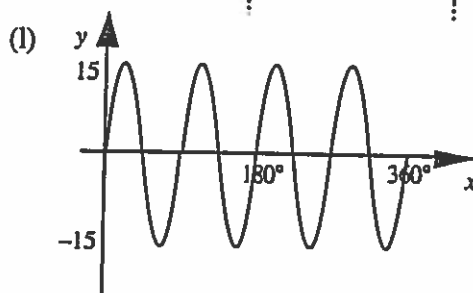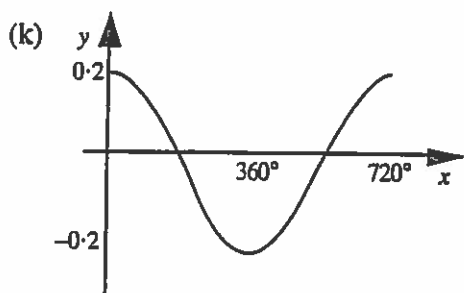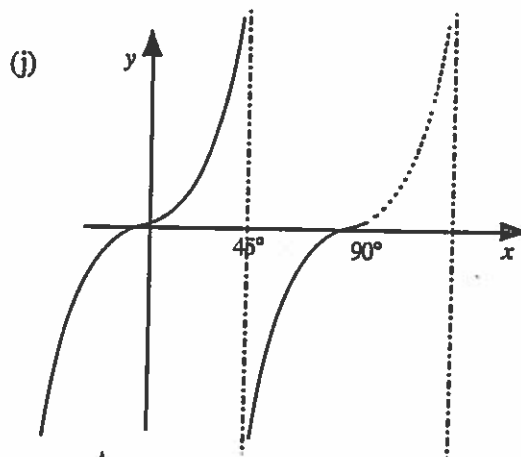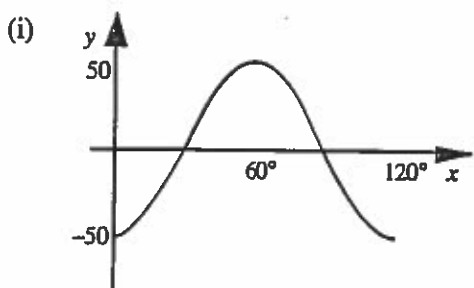
2. Make neat sketches of the following trigonometric functions, clearly indicating

    (i)   the shape of the graph (draw one 'cycle' of it only)

    (ii) the important values on the horizontal axis

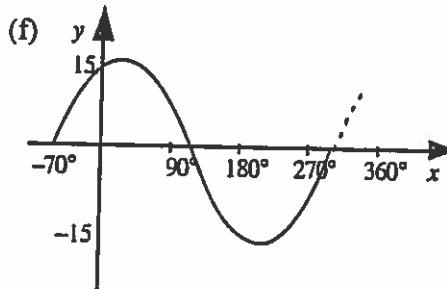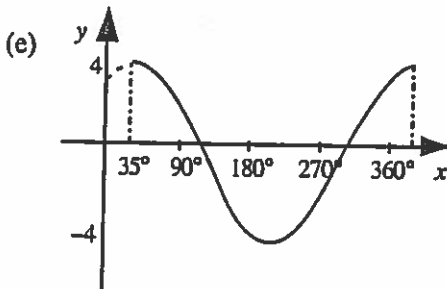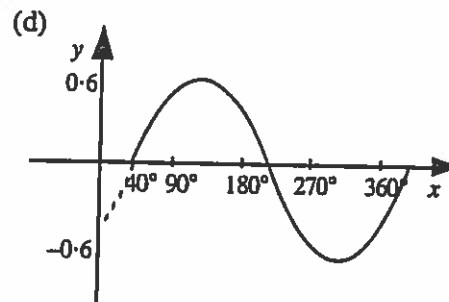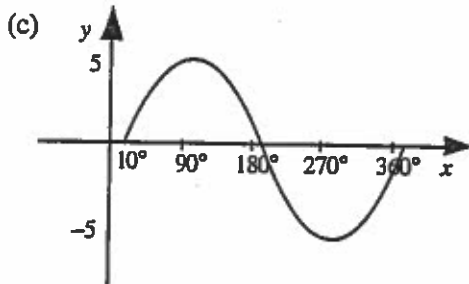   (iii) the maximum and minimum values of the function.

cont'd ...

④

(a) $y = 3\sin x°$ (b) $y = 4\cos x°$ (c) $y = \tan 3x°$

(d) $y = 10\sin 3x°$ (e) $y = 12\cos 2x°$ (f) $y = 0{\cdot}7\sin 4x°$

(g) $y = 1{\cdot}2\cos 4x°$ (h) $y = 30\sin 6x°$ (i) $y = 100\cos 5x°$

(j) $y = -\sin x°$ (k) $y = -6\sin^{1}/_{2}x°$ (l) $y = -20\cos 3x°$

3. Make neat sketches of the following over the given range of values:

(a) $y = 60\sin 2x°$ $0 \le x \le 360$ (b) $y = 2{\cdot}5\cos 3x°$ $0 \le x \le 360$

(c) $y = 40\sin 4x°$ $0 \le x \le 180$ (d) $y = -2\cos 6x°$ $0 \le x \le 180$

(e) $y = -15\sin 8x°$ $0 \le x \le 180$ (f) $y = 1{\cdot}8\cos 30x°$ $0 \le x \le 12$

Exercise 2B

1. Write down the equations of the trigonometric functions in the form $y = k\sin(x - a)°$ or $y = k\cos(x - a)°$ represented by the following graphs:

(a)

(b)

(c)

(d)

(e)

(f)
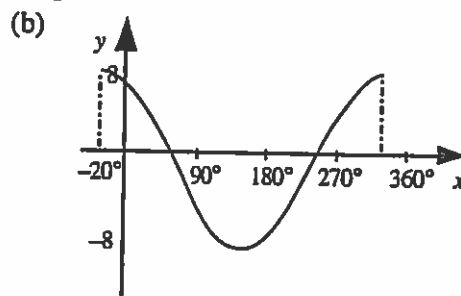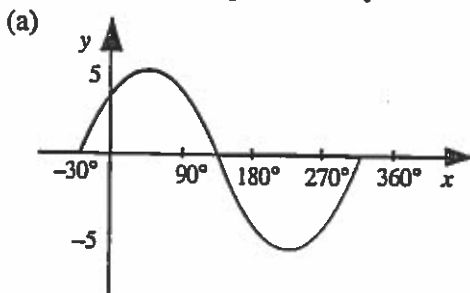
2. Make neat sketches of the following trigonometric functions, showing clearly:

    (i) where each graph cuts the x - axis.   (ii) the maximum and minimum values.

(a) $y = 10\sin(x - 15)°$ (b) $y = 1{\cdot}4\cos(x - 20)°$ (c) $y = 15\sin(x + 10)°$

(d) $y = 2{\cdot}4\cos(x + 30)°$ (e) $y = 300\sin(x - 80)°$ (f) $y = \tan(x - 10)°$

C. Solving trigonometric equations

S (in)	A (ll)
T (an)	C (os)

Exercise 3

1. Find the <u>two</u> solutions for each of the following in the range $0 \le x \le 360$:
   (Give each answer correct to the nearest whole degree).

   (a) $\sin x° = 0.500$ (b) $\cos x° = 0.707$ (c) $\tan x° = 0.869$

   (d) $\cos x° = 0.940$ (e) $\tan x° = 1.280$ (f) $\sin x° = 0.574$

   (g) $\sin x° = 0.990$ (h) $\tan x° = 6.314$ (i) $\cos x° = 0.391$

   (j) $\cos x° = 0.985$ (k) $\sin x° = 0.866$ (l) $\tan x° = 1.732$

2. Rearrange each of the following and solve them in the range $0 \le x \le 360$.
   (Give your answers correct to 1 decimal place).

   (a) $2\cos x° - 1 = 0$ (b) $5\sin x° - 4 = 0$ (c) $10\tan x° - 7 = 0$

   (d) $1 - 3\sin x° = 0$ (e) $5 - 6\cos x° = 0$ (f) $3\tan x° - 5 = 0$

3. Find the <u>two</u> solutions for each of the following in the range $0 \le x \le 360$:
   (Give each answer correct to the nearest whole degree).

   (a) $\sin x° = -0.500$ (b) $\cos x° = -0.707$ (c) $\tan x° = -0.384$

   (d) $\cos x° = -0.292$ (e) $\tan x° = -1.000$ (f) $\sin x° = -0.866$

   (g) $\tan x° = -4$ (h) $\sin x° = -0.174$ (i) $\cos x° = -0.927$

4. Rearrange each of the following and solve them in the range $0 \le x \le 360$.
   (Give your answers correct to 1 decimal place).

   (a) $4\sin x° + 1 = 0$ (b) $5\cos x° + 3 = 0$ (c) $3\tan x° + 1 = 0$

   (d) $7 + 8\cos x° = 0$ (e) $0.4\sin x° + 0.3 = 0$ (f) $5\tan x° + 8 = 0$

5. Solve the following mixture of trigonometric equations in the range $0 \le x \le 360$.
   (Give your answers correct to 1 decimal place).

   (a) $\sin x° = 0.323$ (b) $\cos x° = -0.9$ (c) $\tan x° = 0.678$

   (d) $\cos x° = ^1/_4$ (e) $\sin x° = -0.707$ (f) $\tan x° = -2$

   (g) $\sin x° = ^3/_5$ (h) $\cos x° = -0.111$ (i) $\tan x° = ^5/_8$

   (j) $8\sin x° + 5 = 0$ (k) $6\cos x° + 3 = 0$ (l) $1 - 5\tan x° = 0$

   (m) $20\sin x° - 17 = 0$ (n) $15 - 25\cos x° = 0$ (o) $8\tan x° + 7 = 0$

   (p) $5\sin x° + 3 = 2\sin x° + 5$ (q) $7\cos x° - 1 = \cos x° + 4$

   (r) $10\tan x° + 8 = 3\tan x° + 4$ (s) $6\sin x° + 11 = 3\sin x° + 10$

D. The period of a trigonometric function

Exercise 4

1. Determine the period and the **maximum** and **minimum** values of these trig functions.

(a)

(b)

(c)

(d)

(e)

(f)

(g)
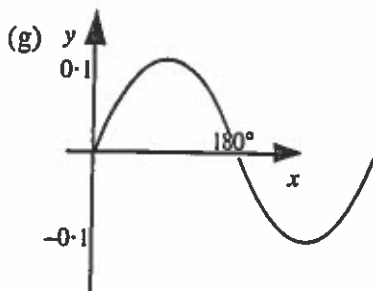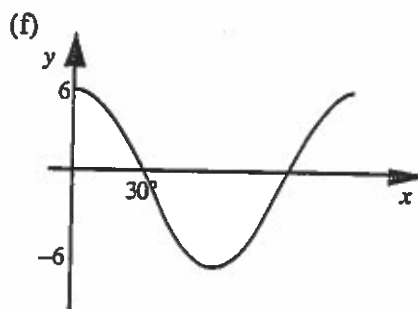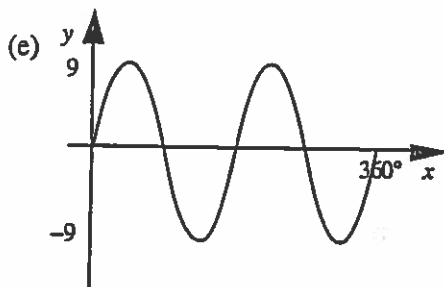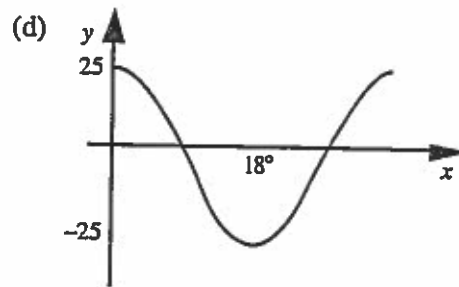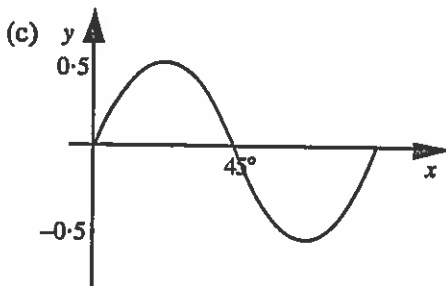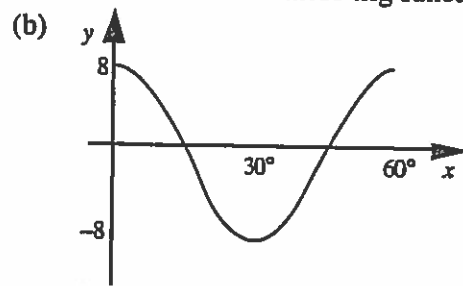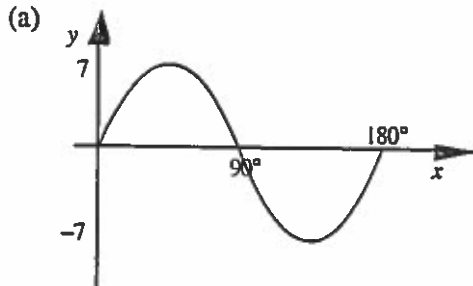
2. Determine the period and the **maximum** and **minimum** values of these trig functions.

(a) $y = 5\sin 2x°$

(b) $y = 3\cos 3x°$

(c) $y = 10\tan 4x°$

(d) $y = 2·2\cos 2x°$

(e) $y = 30\sin 6x°$

(f) $y = -5\cos 30x°$

(g) $y = 50\sin 90x°$

(h) $y = -4\cos 1/2x°$

(i) $y = 18\sin 1/4x°$

(j) $y = 0·9\cos 60x°$

(k) $y = 1/2 \sin 5x°$

(l) $y = 3/4\cos 9x°$

(m) $y = 11\sin 180x°$

(n) $y = 8\sin 1·5x°$

(o) $y = 40\cos 2·5x°$

E. Trigonometric identities

Exercise 5

Remember :- $\sin^2 x + \cos^2 x = 1$; and $\tan x = \dfrac{\sin x}{\cos x}$

1. Simplify the following using the above 2 identities:

   (a) $2\sin^2 x° + 2\cos^2 x°$

   (b) $5\cos^2 x° + 5\sin^2 x°$

   (c) $\dfrac{3\sin x°}{\cos x°}$

   (d) $\dfrac{5\sin x°}{2\cos x°}$

2. Write down a simple expression, identical to:

   (a) $1 - \sin^2 x°$

   (b) $1 - \cos^2 x°$

   (c) $\tan x° \cos x°$

   (d) $\dfrac{\sin x°}{\cos x°}$

3. Simplify:

   (a) $\dfrac{1 - \cos^2 x°}{\sin^2 x°}$

   (b) $\dfrac{1 - \sin^2 x°}{2\cos^2 x°}$

   (c) $\dfrac{\sin^2 x°}{\cos^2 x°}$

   (d) $\dfrac{1 - \sin^2 x°}{\cos x°}$

   (e) $\dfrac{1 - \cos^2 x°}{5\sin x°}$

   (f) $\tan^2 x°(1 - \sin^2 x°)$

4. Prove the following trigonometric identities:

   (a) $3 - 3\sin^2 x° = 3\cos^2 x°$

   (b) $5 - 5\cos^2 x° = 5\sin^2 x°$

   (c) $\sqrt{1 - \cos^2 x°} = \sin x°$

   (d) $\tan x° \sqrt{1 - \sin^2 x°} = \sin x°$

   (e) $\dfrac{1 - \cos^2 x°}{1 - \sin^2 x°} = \tan^2 x°$

   (f) $\dfrac{1 - \sin^2 x°}{1 - \cos^2 x°} = \dfrac{1}{\tan^2 x°}$

Checkup for further trigonometry

1. Make a sketch of the sine, cosine and tangent graphs, indicating all their main features.

2. Write down the equations of the trigonometric functions associated with the following graphs:

(a)

(b)

(c)

(d)
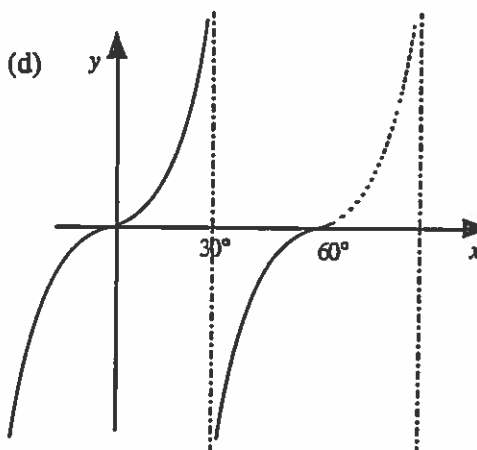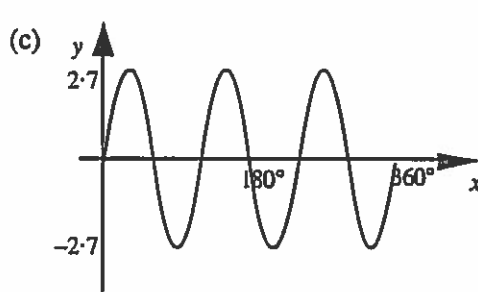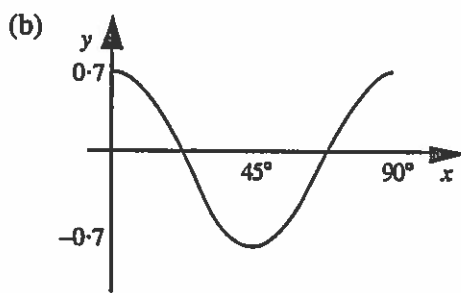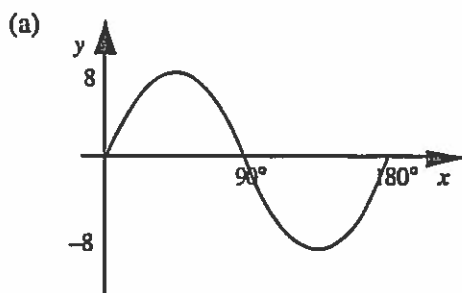
3. Make neat sketches of the following, indicating all the main points and features:
   (a) $y = 20\sin 4x°$ $0 \le x \le 90$ (b) $y = 1·6\cos 2x°$ $0 \le x \le 360$
   (c) $y = -8\sin 8x°$ $0 \le x \le 90$ (d) $y = \tan 2x°$ $0 \le x \le 90$

4. Find the <u>two</u> solutions for each of the following in the range $0 \le x \le 360$:
   (Give your answers correct to 1 decimal place).

   (a) $\sin x° = 0·911$ (b) $\cos x° = 0·444$ (c) $\tan x° = 3$
   (d) $\cos x° = -0·605$ (e) $\tan x° = -0·8$ (f) $\sin x° = -{}^4/_5$
   (g) $2\sin x° - 1 = 0$ (h) $8\cos x° + 6 = 0$ (i) $4\tan x° - 3 = 0$
   (j) $3\cos x° - 2 = 0$ (k) $1 + 4\sin x° = 0$ (l) $5\tan x° = 3\tan x° - 2$

5. What are the periods of the following trigonometric graphs and functions?

(a)

(b)

(c)

(d)
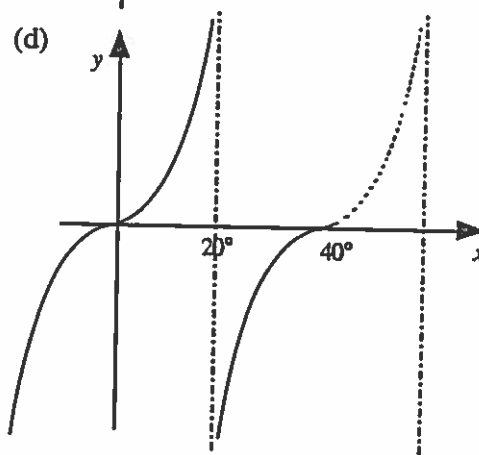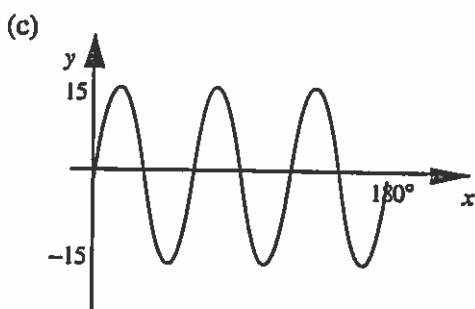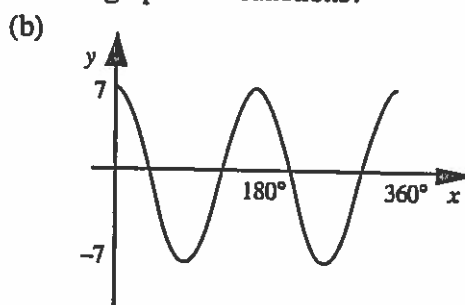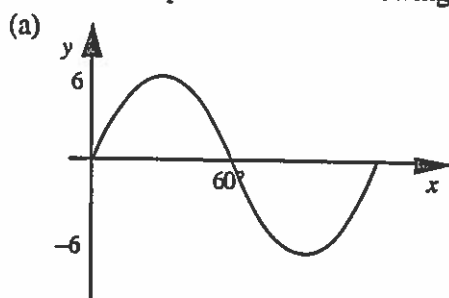
(e) $y = 10\sin 10x°$

(f) $y = 2·3\cos 30x°$

(g) $y = -4\sin 9x°$

(h) $y = 5\tan 4x°$

6. (a) Simplify: (i) $6 - 6\sin^2 x°$ (ii) $\dfrac{\cos x°}{\sin x°}$

(b) Prove these identities :-   (i) $\dfrac{1 - \cos^2 x°}{\sin^2 x°} = 1$ (ii) $(1 - \sin x°)(1 + \sin x°) = \cos^2 x°$

7. Write down the equations of the following trigonometric graphs:

(a)

(b)
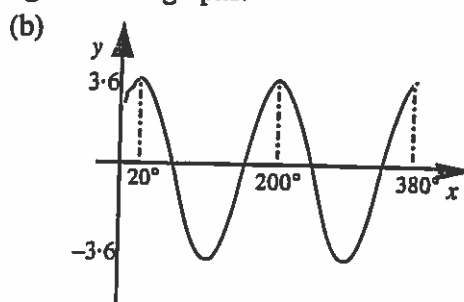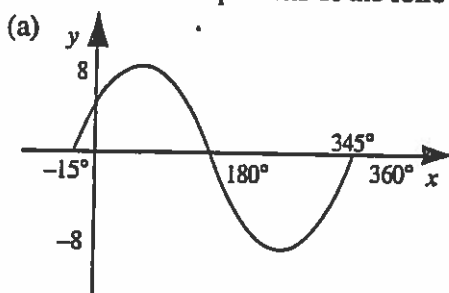
8. Sketch the following trigonometric graphs, indicating their main features:

(a) $y = 18\cos(x + 30)°$ $0 \le x \le 360$ (b) $y = 2\sin(x - 10)°$ $0 \le x \le 360$

Further trigonometry

Exercise 1

1. Check Graphs – see graphs at top of page 15 for comparison.

Exercise 2A

1. (a) $y = 10\sin x°$
    (d) $y = 0\cdot8\cos 4x°$
    (g) $y = 25\sin \frac{1}{2}x°$
    (j) $y = \tan 2x°$

    (b) $y = 6\cos x°$
    (e) $y = 20\sin 2x°$
    (h) $y = -4\sin x°$
    (k) $y = 0\cdot2\cos \frac{1}{2}x°$

    (c) $y = 5\sin 2x°$
    (f) $y = 80\cos 2x°$
    (i) $y = -50\cos 3x°$
    (l) $y = 15\sin 4x°$

2. (a)

    (b)

    (c)

    (d)

    (e)

    (f)

    (g)

    (h)

    (i)

    (j)

    (k)

    (l)

3. (a)

(b)

(c)

(d)

(e)

(f)
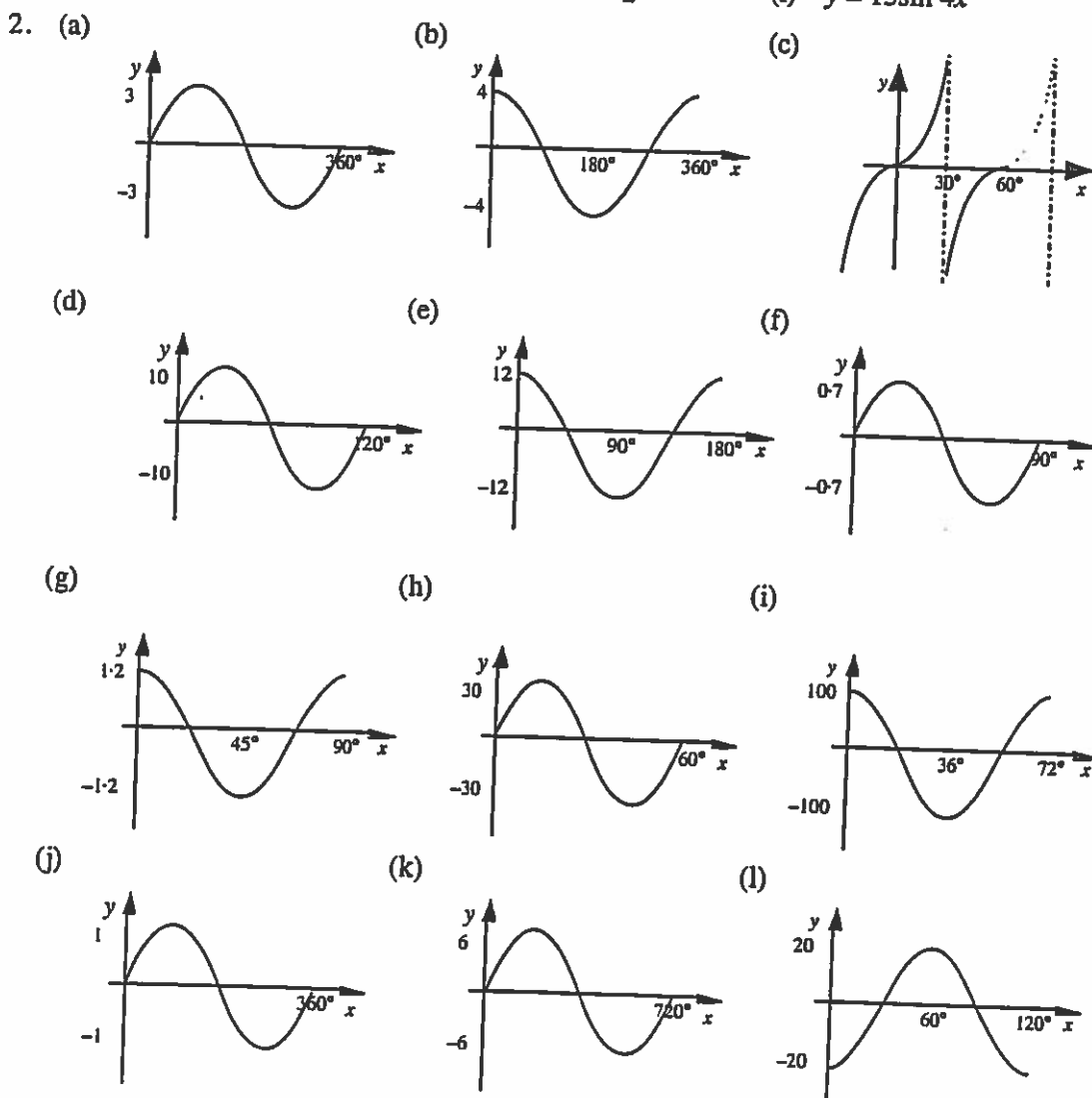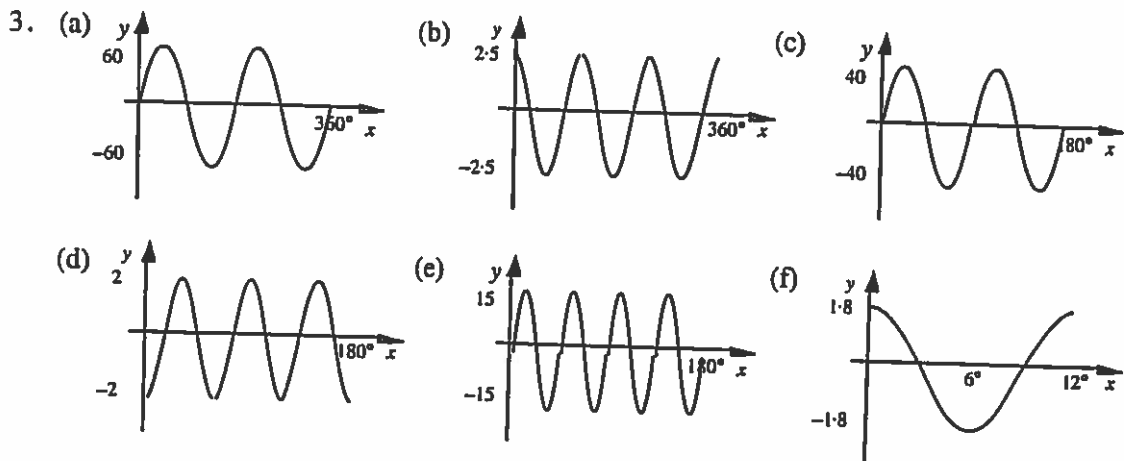
Exercise 2B

1. (a) $y = 5\sin(x + 30)°$
   (c) $y = 5\sin(x - 10)°$
   (e) $y = 4\cos(x - 35)°$

   (b) $y = 8\cos(x + 20)°$
   (d) $y = 0.6\sin(x - 40)°$
   (f) $y = 15\sin(x + 70)°$ or $y = 15\cos(x - 20)°$

2. (a)

(b)

(c)

(d)

(e)

(f)
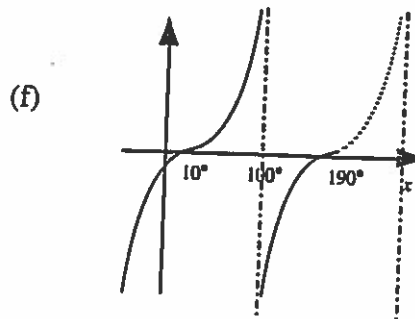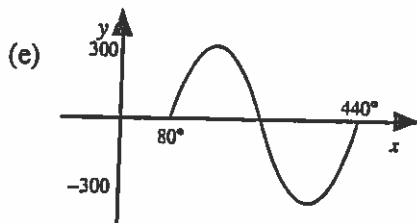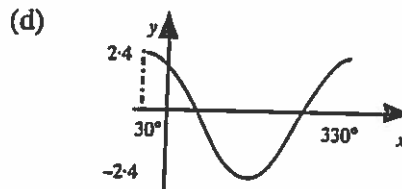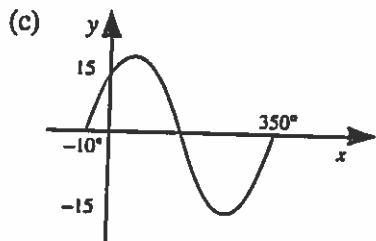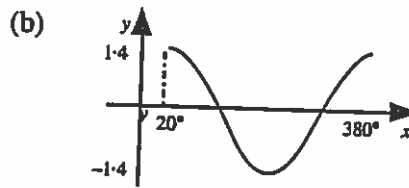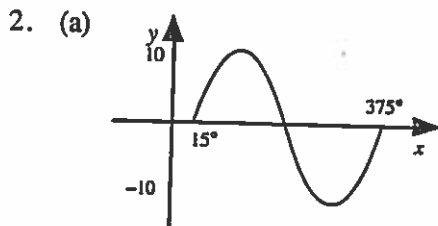
Exercise 3

1. (a) 30, 150   (b) 45, 315   (c) 41, 221
   (d) 20, 340   (e) 52, 232   (f) 35, 145
   (g) 82, 98   (h) 81, 261   (i) 67, 293
   (j) 10, 350   (k) 60, 120   (l) 60, 240.

2. (a) 60, 300   (b) 53·1 or 126·9   (c) 35·0, 215·0
   (d) 19·5, 160·5   (e) 33·6, 326·4   (f) 59·0, 239·0.

3. (a) 210, 330   (b) 135, 225   (c) 159, 339
   (d) 107, 253   (e) 135, 315   (f) 240, 300
   (g) 104, 284   (h) 190, 350   (i) 158, 202.

4. (a) 194·5, 345·5   (b) 126·9, 233·1   (c) 161·6, 341·6
   (d) 151·0, 209·0   (e) 228·6, 311·4   (f) 122·0, 302·0.

5. (a) 18·8, 161·2   (b) 154·2, 205·8   (c) 34·1, 214·1
   (d) 75·5, 284·5   (e) 225, 315   (f) 116·6, 296·6
   (g) 36·9, 143·1   (h) 96·4, 263·6   (i) 32·0, 212·0
   (j) 218·7, 321·3   (k) 120, 240   (l) 11·3, 191·3
   (m) 58·2, 121·8   (n) 53·1, 306·9   (o) 138·8, 318·8
   (p) 41·8, 138·2   (q) 33·6, 326·4   (r) 150·3, 330·3
   (s) 199·5, 340·5°.

Exercise 4

1.

Graph	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Period	180	60	90	36	180	120	360
Max/Min	±7	±8	±0·5	±25	±9	±6	±0·1

2.

Question	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Period	180	120	45	180	60	12	4	720
Max/Min	±5	±3	±10	±2·2	±30	±5	±50	±4

Question	(i)	(j)	(k)	(l)	(m)	(n)	(o)
Period	1440	6	72	40	2	240	144
Max/Min	±18	±0·9	±0·5	±0·75	±11	±8	±40

Exercise 5

1. (a) 2   (b) 5   (c) $3\tan x$ (d) $\frac{5}{2}\tan x$

2. (a) $\cos^2 x^\circ$ (b) $\sin^2 x^\circ$ (c) $\sin x^\circ$ (d) $\tan x^\circ$

3. (a) 1   (b) $\frac{1}{2}$ (c) $\tan^2 x^\circ$ (d) $\cos x^\circ$ (e) $\frac{1}{5}\sin x^\circ$ (f) $\sin^2 x^\circ$

4. All proofs.

Checkup for further trigonometry

1. See sketches on page 15.

2. (a) $y = 8\sin 2x^\circ$ (b) $y = 0.7\cos 4x^\circ$ (c) $y = -2.7\sin 3x^\circ$ (d) $y = \tan 3x^\circ$

3. (a)

(b)

(c)

(d)
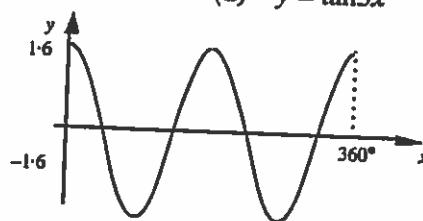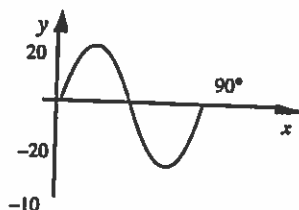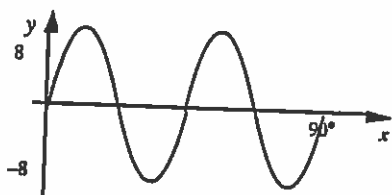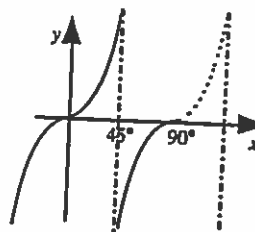
4. (a)   65·6, 114·4   (b)   63·6, 296·4   (c)   71·6, 251·6
    (d)   127·2, 232·8   (e)   141·3, 321·3   (f)   233·1, 306·9
    (g)   30, 150   (h)   138·6, 221·4   (i)   36·9, 216·9
    (j)   48·2, 311·8   (k)   194·5, 345·5   (l)   135, 315.

5. (a) 120°   (b) 180°   (c) 60°   (d) 40°
   (e) 36°   (f) 12°   (g) 40°   (h) 90°.

6. (a) (i) $6\cos^2 x$ (ii) $\dfrac{1}{\tan x}$ (b) Proof.

7. (a) $y = 8\sin(x + 15)^\circ$ (b) $y = 3.6\cos(x - 20)^\circ$

8. (a)
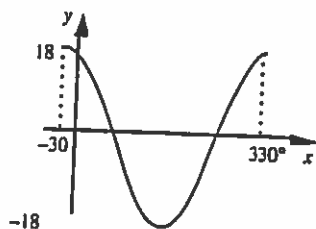
(b)

PYTHAGORAS' THEOREM

EXERCISE 7.
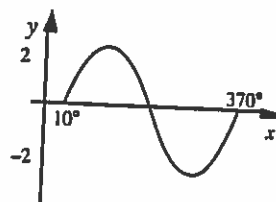
① A RECTANGLE MEASURES 15cm LONG BY 8cm BROAD. CALCULATE THE LENGTH OF ONE OF THE DIAGONALS OF THIS RECTANGLE.

② A LADDER IS 18m LONG. HOW FAR UP THE WALL WILL THE LADDER REACH IF THE FOOT OF THE LADDER IS 3.5m FROM THE FOOT OF THE WALL?

③ A SHIP SAILS 17km EAST AND THEN SAILS 8km NORTH. HOW FAR IS IT FROM ITS STARTING POINT?

④ A KITE IS FLYING ON 100m OF STRING. TWO BOYS WANT TO FIND OUT HOW HIGH THE KITE IS AT THE MOMENT THAT THE STRING IS FULLY OUT.
ONE BOY STANDS IMMEDIATELY UNDER THE KITE. IF THE DISTANCE BETWEEN THEM IS 55m, WHAT IS THE HEIGHT OF THE KITE?

⑤ THE DIAGRAM OPPOSITE SHOWS THE END VIEW OF A LEAN-TO SHED.

CALCULATE THE LENGTH OF THE SLOPING EDGE OF THE ROOF.
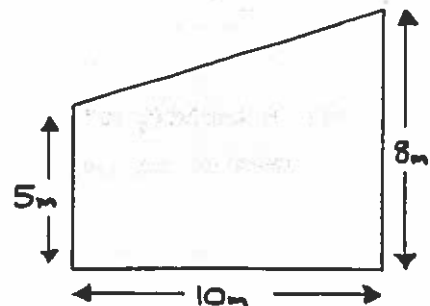
⑥ THE TOPS OF TWO MASTS ON A SHIP ARE JOINED BY A WIRE 25m LONG. IF THE MASTS ARE 35m AND 55m HIGH, HOW FAR APART ARE THEY?

⑦ THE DIAGRAM OPPOSITE SHOWS THE CROSS SECTION OF THE ROOF OF A HOUSE.

CALCULATE THE HEIGHT OF THE CENTRE POST.
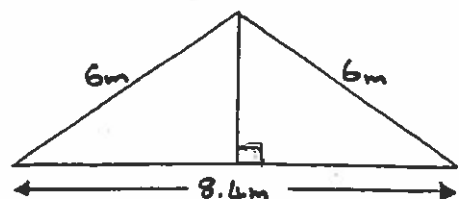
⑧ THE DIAGRAM OPPOSITE SHOWS A SECTION OF HORIZONTAL CYLINDRICAL PIPE OF RADIUS 50cm. WATER IS LYING IN THE PIPE AND IS 20cm DEEP AT THE MIDDLE OF THE SECTION. CALCULATE THE WIDTH OF THE WATER SURFACE.
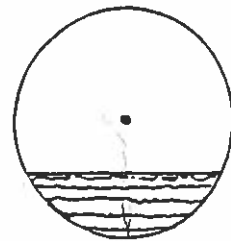
SOLVING PROBLEMS INVOLVING 3D

(1) CALCULATE THE LENGTH OF THE SLOPING SIDE.

← 10cm →

12cm

← 20cm →

(2)

← 30cm →

25cm

← 16cm →
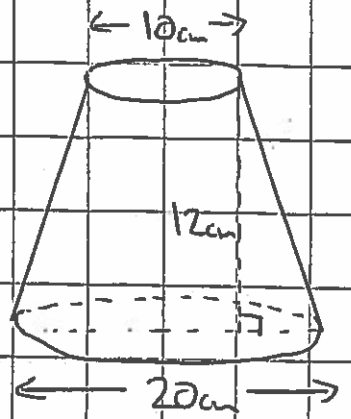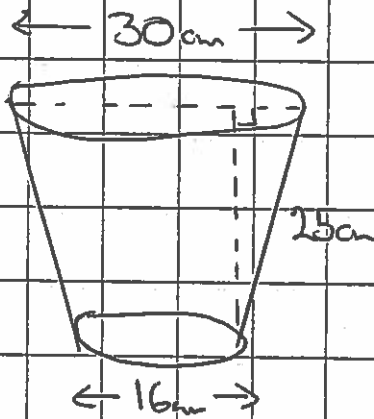
HOW TALL IS THE WASTE PAPER BIN?

(3)

B

C

6cm

F

E

15cm

A

13cm

D
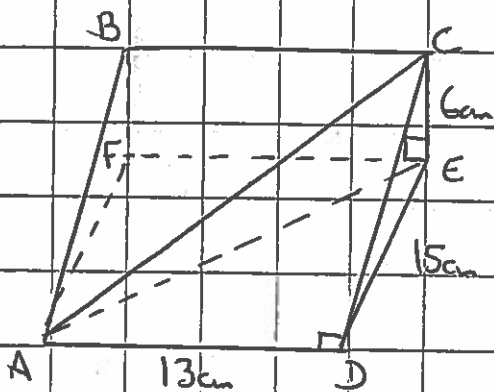
$AD = 13cm$ $DE = 15cm$ AND

$EC = 6cm$

CALCULATE THE LENGTH OF THE LINE AC

(4) ABCD EFGH IS A CUBOID.

$AE = 5cm$, $EH = 9cm$

AND $HG = 6cm$.

CALCULATE THE LENGTH OF THE LINE AG.

B

A

5cm

E

F

C

D

9cm

G

6cm

H
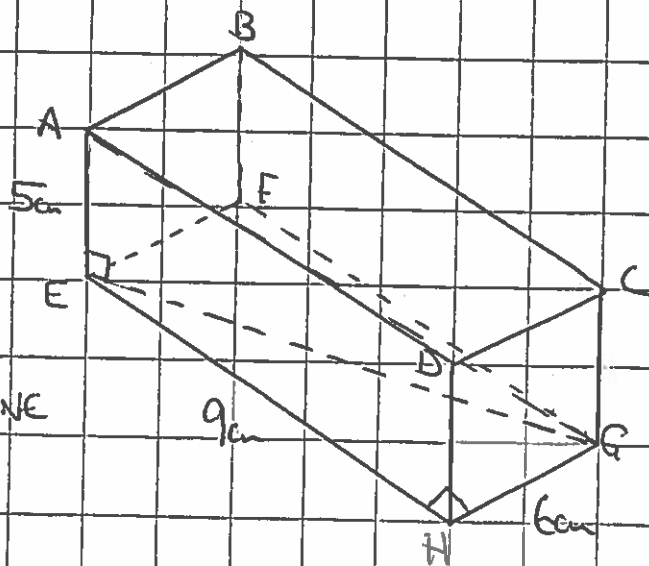
5

THE DIAGRAM SHOWS A PYRAMID PABCD

X IS AT THE CENTRE OF THE BASE

THE BASE IS A RECTANGLE WITH

$AB = CD = 12cm$

$BC = AD = 9cm$
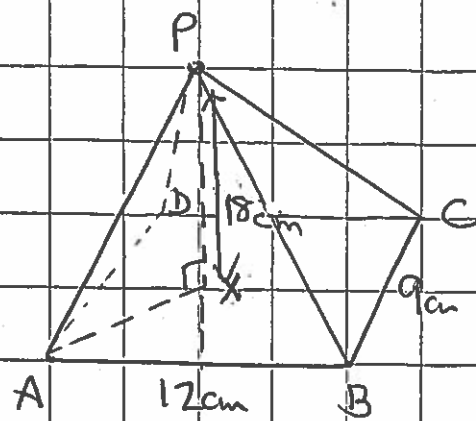
THE HEIGHT OF THE PYRAMID IS $PX = 18 cm$

CALCULATE THE LENGTH OF THE EDGE PA.

6

A TEPEE HAS HEIGHT 2m AND
THE CIRCUMFERENCE OF THE BASE
IS 7.54 m

CALCULATE THE LENGTH OF THE
SLOPING EDGE

7
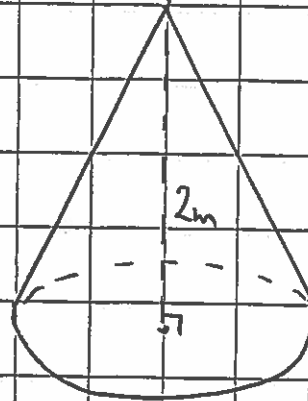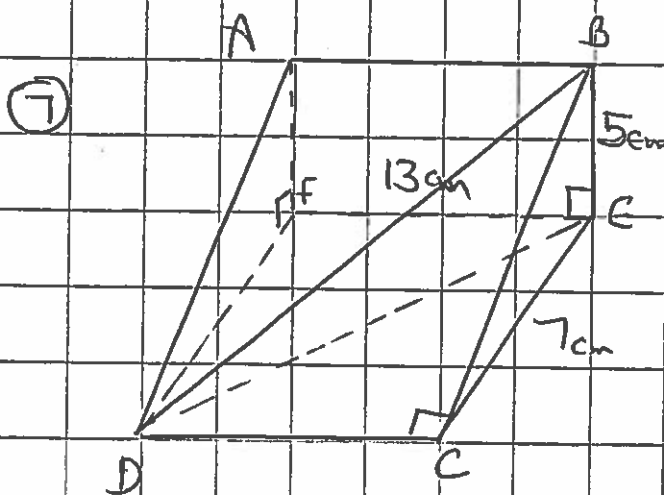
THE DIAGRAM SHOWS A
WEDGE ABCDEF.

$DB = 13cm$ $CE = 7cm$

AND $BE = 5cm$

CALCULATE THE LENGTH OF
THE LINE DC

EXERCISE 10.

USING THE CONVERSE OF PYTHAGORAS' THEOREM, DECIDE WHICH TRIANGLES ARE RIGHT-ANGLED :—

① Triangle ABC: AB = 12, BC = 5, AC = 10

② Triangle DEF: DE = 5, EF = 4, DF = 3

③ Triangle GHJ: GH = 6, HJ = 5, GJ = 4

④ Triangle KLM: KL = 8, KM = 17, LM = 15

⑤ Triangle PQR: PQ = 35, QR = 12, PR = 37

⑥ Triangle STV: ST = 13, TV = 4, SV = 9

⑦ Triangle WXY: WX = 24, WY = 26, XY = 10

⑧ Triangle NZJ: NZ = 14, ZJ = 11, NJ = 9

⑨ Triangle PUB: PU = 24, UB = 7, PB = 25

⑩ Triangle TCP: TC = 10, CP = 16, TP = 20

⑪ Triangle LEO: LE = 12, EO = 3.5, LO = 12.5

⑫ Triangle NED: NE = 9, ED = 18, ND = 16

18
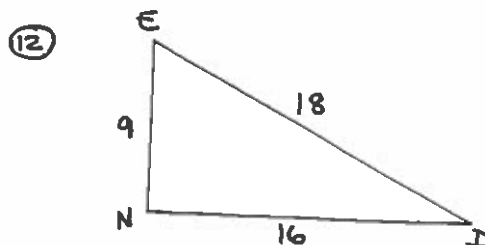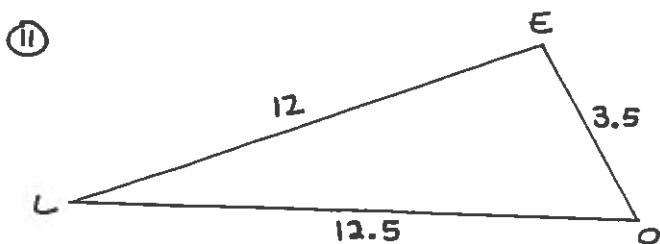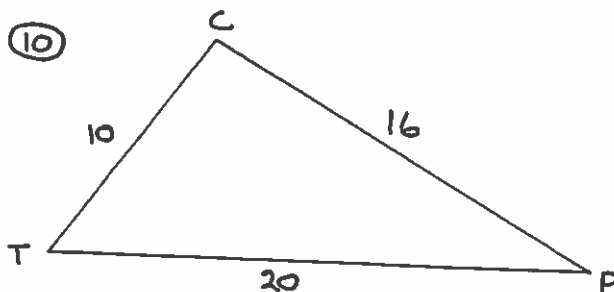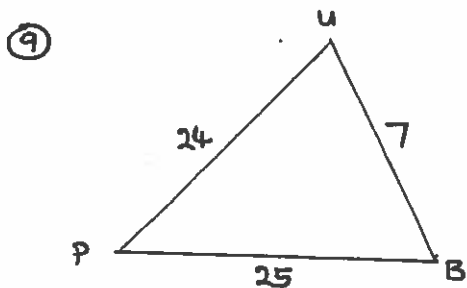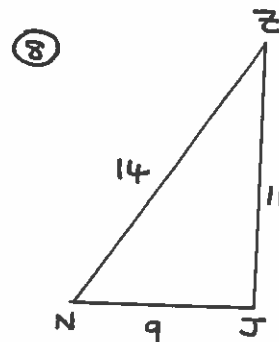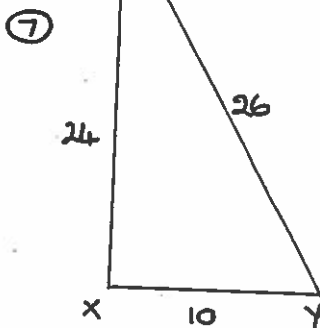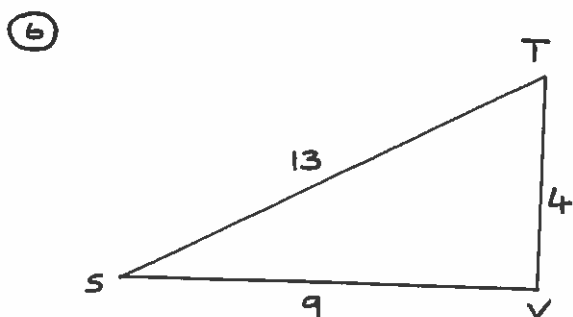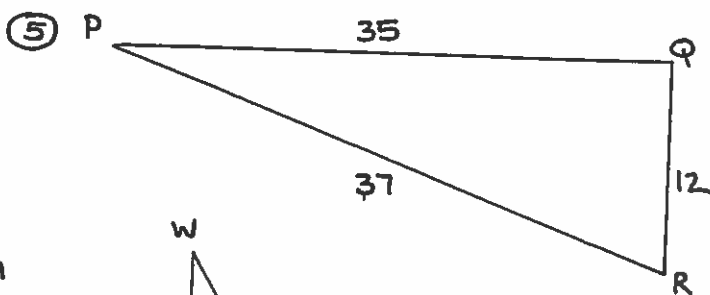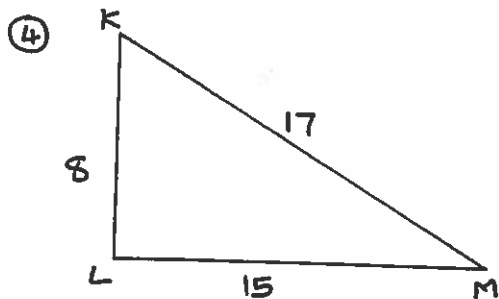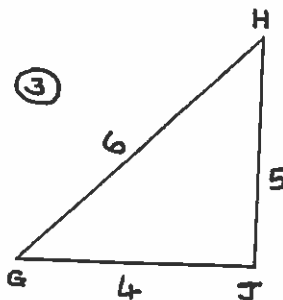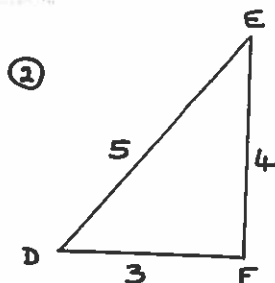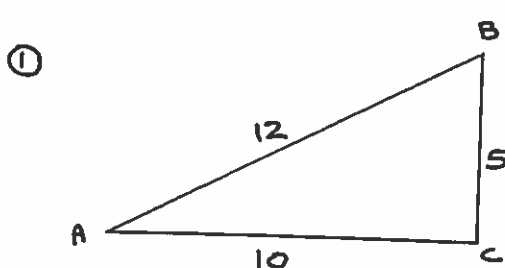
EXAMPLE PLOT THE POINTS $P(2,1)$, $Q(6,4)$ AND $R(9,0)$. PROVE THAT TRIANGLE PQR IS RIGHT-ANGLED.

IF TRIANGLE PQR IS RIGHT-ANGLED THEN :

$$PR^2 = PQ^2 + QR^2$$

$$PR^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$
$$PR^2 = (9-2)^2 + (0-1)^2$$
$$PR^2 = 7^2 + (-1)^2$$
$$PR^2 = 49 + 1$$
$$\underline{PR^2 = 50}$$
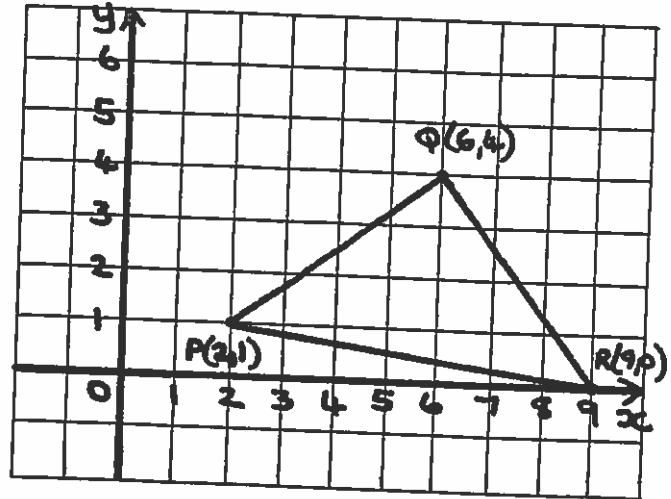
$$PQ^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$
$$PQ^2 = (6-2)^2 + (4-1)^2$$
$$PQ^2 = 4^2 + 3^2$$
$$PQ^2 = 16 + 9$$
$$\underline{PQ^2 = 25}$$

$$QR^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$
$$QR^2 = (9-6)^2 + (0-4)^2$$
$$QR^2 = 3^2 + (-4)^2$$
$$QR^2 = 9 + 16$$
$$\underline{QR^2 = 25}$$

SINCE $PR^2 = PQ^2 + QR^2$, THEN TRIANGLE PQR IS RIGHT-ANGLED AT Q.

⑬ PLOT THE POINTS $A(1,4)$, $B(4,6)$ AND $C(8,0)$. PROVE THAT TRIANGLE ABC IS RIGHT-ANGLED.

⑭ PLOT THE POINTS $K(-2,2)$, $L(3,4)$ AND $M(7,-6)$. PROVE THAT TRIANGLE KLM IS RIGHT-ANGLED.

⑮ PLOT THE POINTS $T(-5,1)$, $H(4,-1)$ AND $E(-4,-3)$. PROVE THAT TRIANGLE THE IS RIGHT-ANGLED.

⑯ PLOT THE POINTS $E(-7,0)$, $N(1,-2)$ AND $D(-2,3)$. PROVE THAT TRIANGLE END IS RIGHT-ANGLED.

⑲

PYTHAGORAS ANSWERS

Ex. 7.

① 17 cm   ② 17.66 m   ③ 18.79 km   ④ 83.52 m

⑤ 10.44 m   ⑥ 15 m   ⑦ 4.28 m   ⑧ 80 cm

3D

① 13 cm   ② 24 cm   ③ 20.7 cm   ④ 11.9 cm

⑤ 19.5 cm   ⑥ 23 m   ⑦ 9.75 cm

Ex 10   CONVERSE OF PYTHAGORAS

① No   ② YES   ③ NO   ④ YES   ⑤ YES   ⑥ NO

⑦ YES   ⑧ NO   ⑨ YES   ⑩ NO   ⑪ YES   ⑫ NO

⑬ $AB^2 = 13$ $BC^2 = 52$ $AC^2 = 65$, $13 + 52 = 65$ so $\triangle$ ABC is $\perp^d$

⑭ $KL^2 = 29$ $LM^2 = 116$ $KM^2 = 145$ $29 + 116 = 145$ so $\triangle$ KLM is $\perp^d$.

⑮ $TE^2 = 17$ $EN^2 = 68$ $TN^2 = 85$ $17 + 68 = 85$ so $\triangle$ TEN is $\perp^d$.

⑯ $DG^2 = 34$ $DN^2 = 34$ $EN^2 = 68$ $34 + 34 = 68$ so $\triangle$ DEN is $\perp^d$.

⑳

ANGLE PROPERTIES

Exercise 10

Find the sizes of the angles marked with letters:

(You may wish to sketch them first and fill in all the angles)

1.

65°

x

2.

75°

145°   y

3.

z

y

40°

125°

x

4.

43°

x

72°

5.

25°

z

6.

y

72°

28°

7.

52°

y

8.

z   46°

21

9.

10.

11.

12.

13.

14.
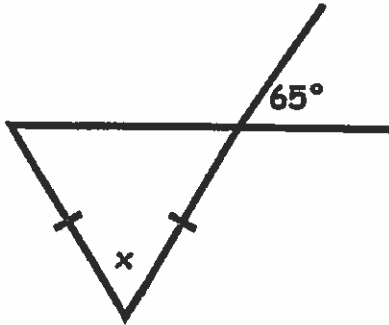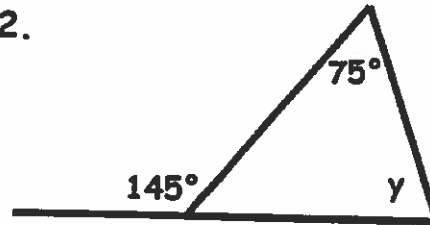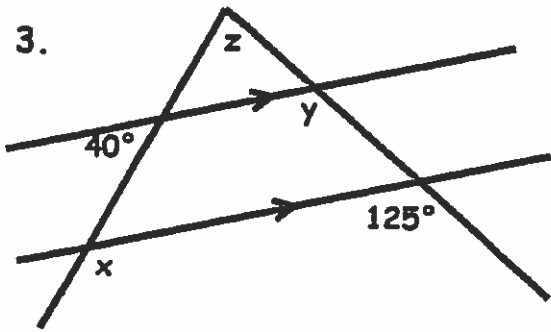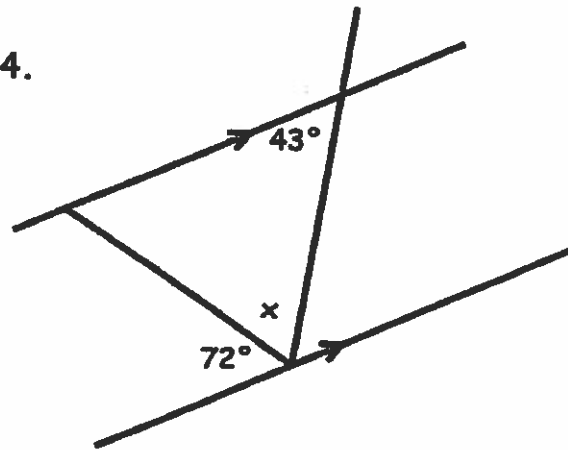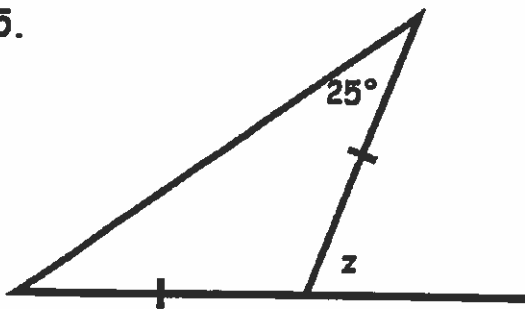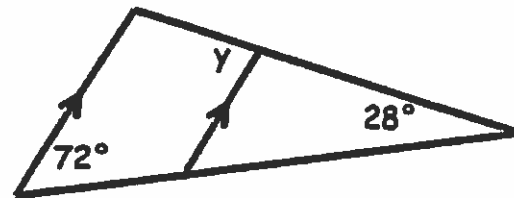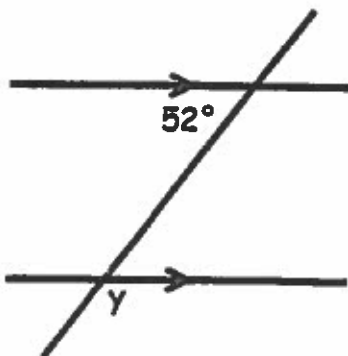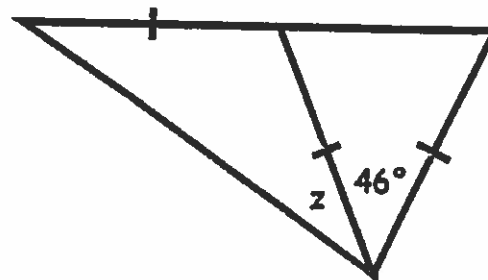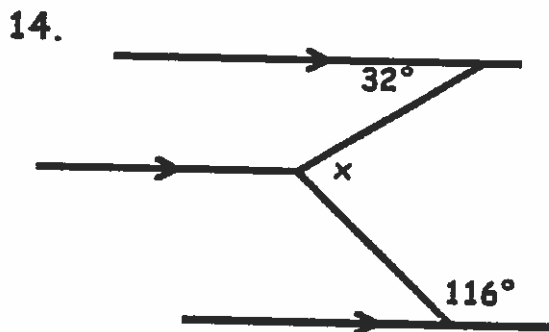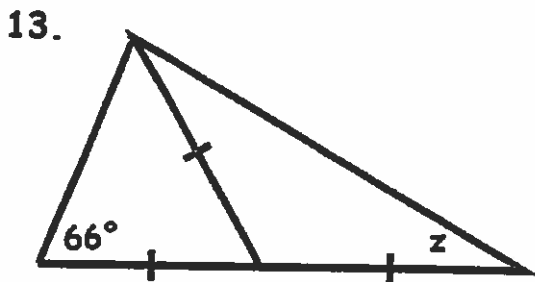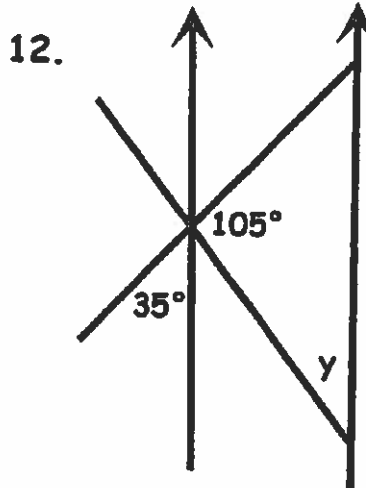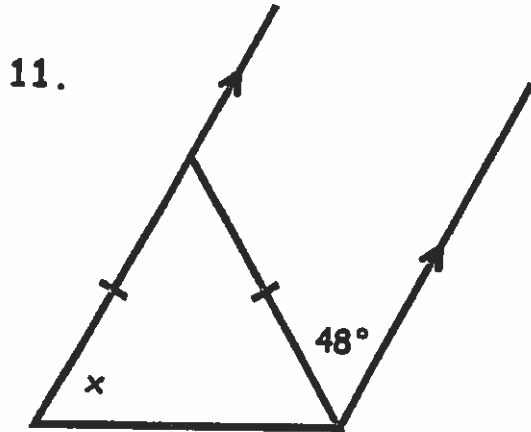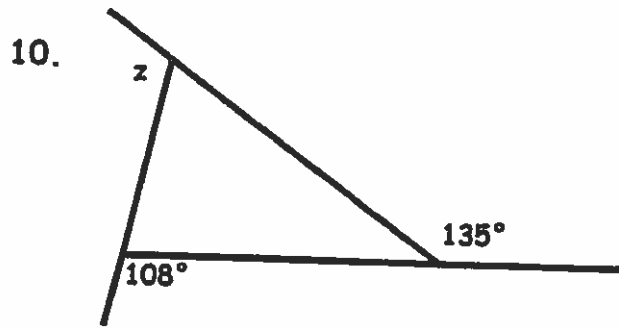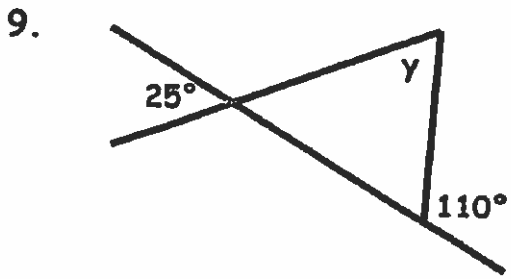
15. In the diagram opposite

- PQRS is a square
- PR is a diagonal of the square
- Triangle RST is equilateral

Calculate the size of x.
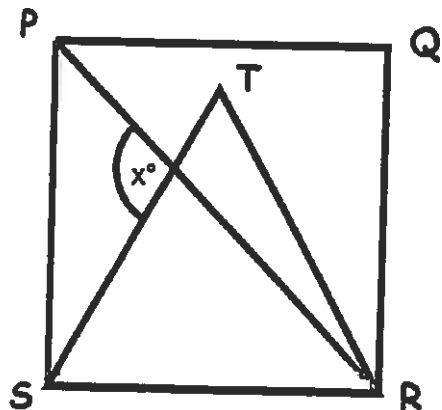
ANGLE PROPERTIES (ANSWERS)

Ex 10

① $x = 50°$ ② $y = 70°$ ③ $x = 140$ $y \pm 125°$, $z = 85°$

④ $x = 65°$ ⑤ $z = 50°$ ⑥ $y = 100°$ ⑦ $y = 128°$

⑧ $z = 56.5°$ ⑨ $y = 85°$ ⑩ $z = 117°$ ⑪ $x = 66°$

⑫ $y = 40°$ ⑬ $z = 24°$ ⑭ $x = 96°$ ⑮ $x = 105°$

6. A wedge of cheese is cut from a large circular block of radius 32 cm and height 12 cm. For the wedge, the angle at C, the centre, is 20°.

Cheese

Calculate:
(a) the area of the sector BCE.
(b) the volume of the wedge of cheese.

7. The area of this sector is 78·5 cm² and the radius of the circle from which it has been cut is 10 cm.

Calculate the size of angle POQ.
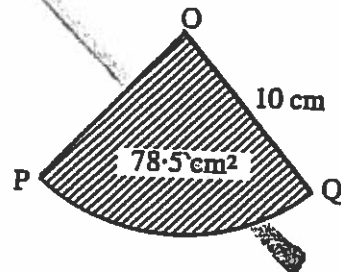
C. The relationship between tangent and radius

Exercise 3

1. Copy the diagrams below and fill in the sizes of the angles marked with a letter.
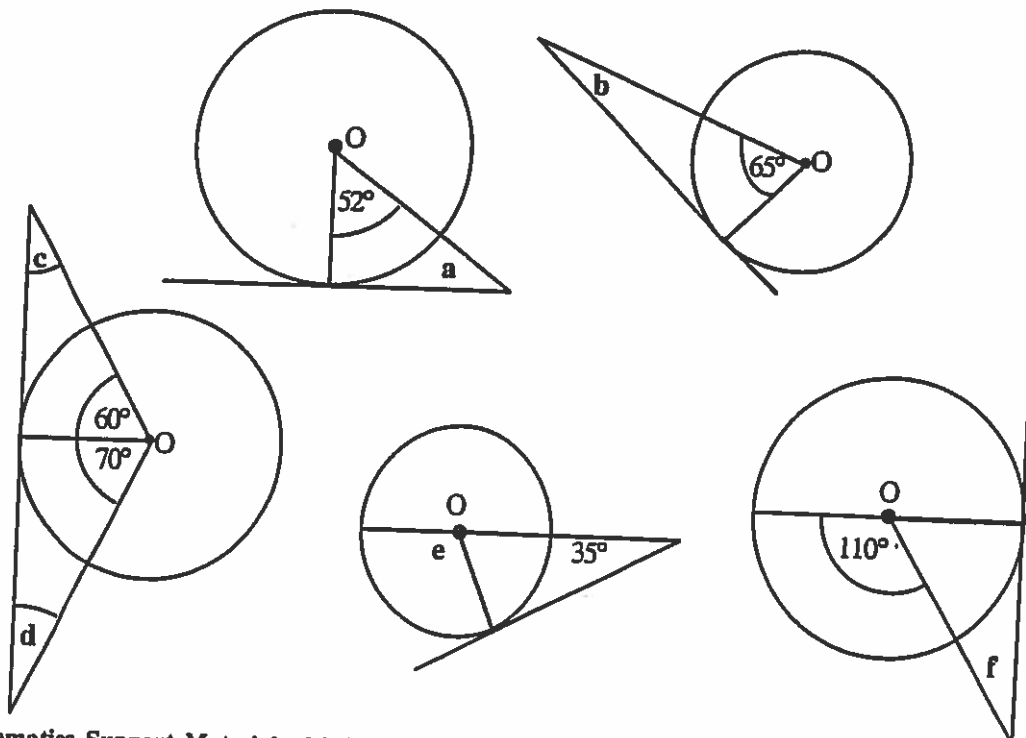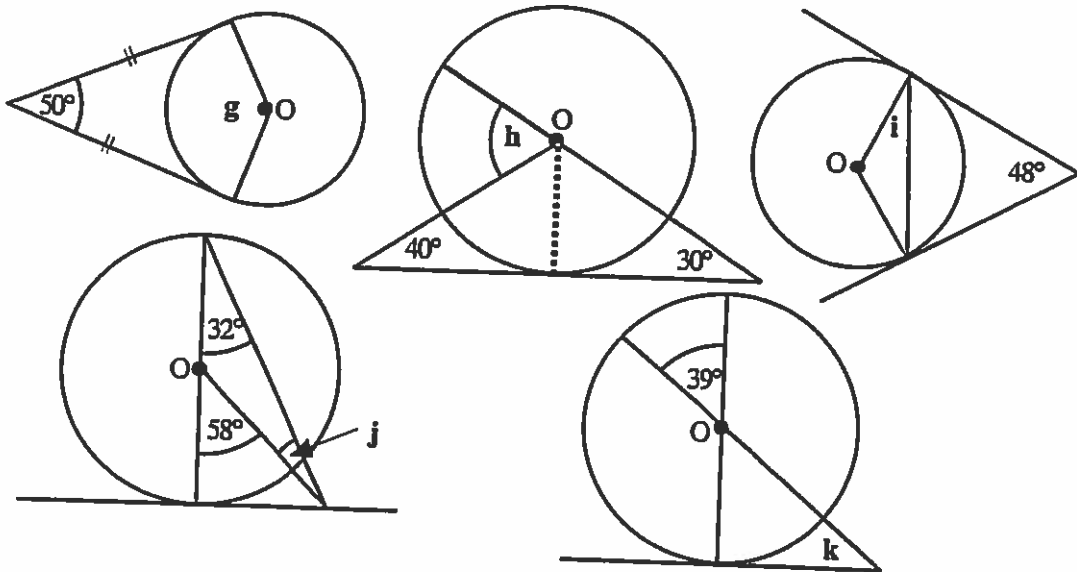
2. The largest possible circle, centre C, is drawn inside a square. The circle and square sit vertically, with one edge on the horizontal surface PQ.

    Triangle ABC is drawn with AB on the line PQ.

    Angle CAB = 27°

    Calculate the size of the angle marked $w°$.
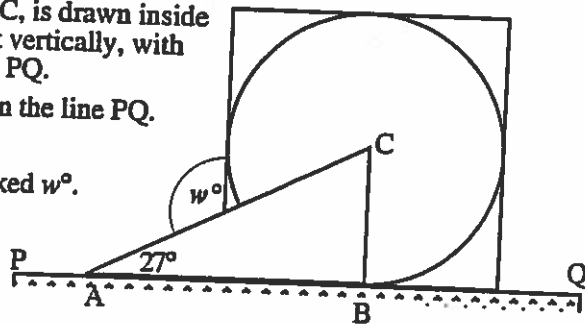
3. AB is a tangent to the circle with centre C.
    It meets the circle at the point P.
    Angle CBP = 36°.

    Calculate the value of the angle marked $x°$.
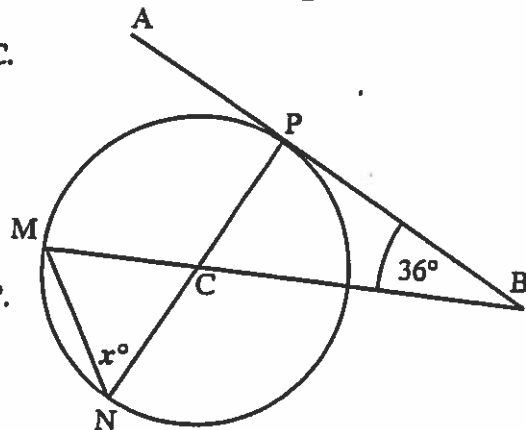
4. Calculate v and w. (The lengths are in centimetres.)
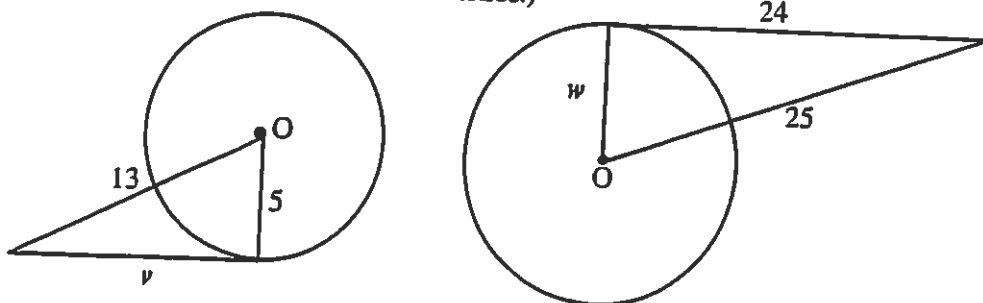
5. Calculate the sizes of the angles marked x, y and z correct to the nearest degree. (The lengths are in centimetres.)
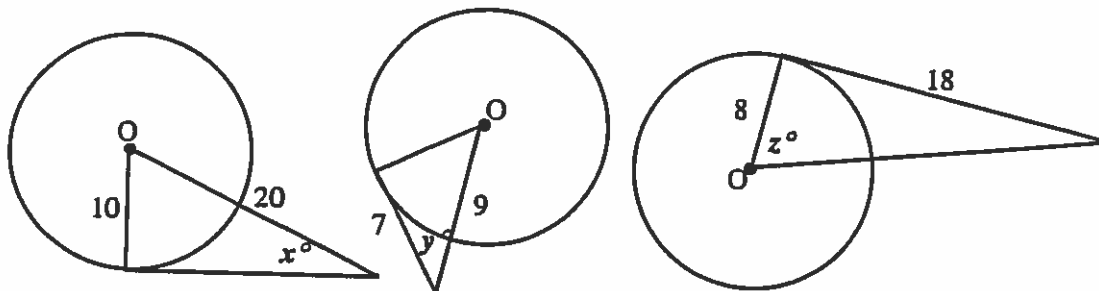
6.
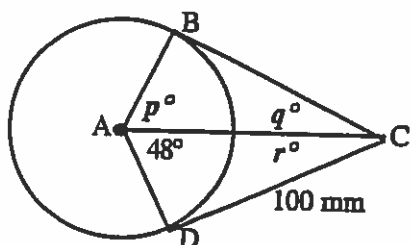
ABCD is a tangent kite.

Write down:
(a) the length of BC.
(b) the values of p, q and r.

D. Angle in a semi-circle

Exercise 4

1. Calculate the sizes of the angles marked a, b, c, d, e, f, and g.
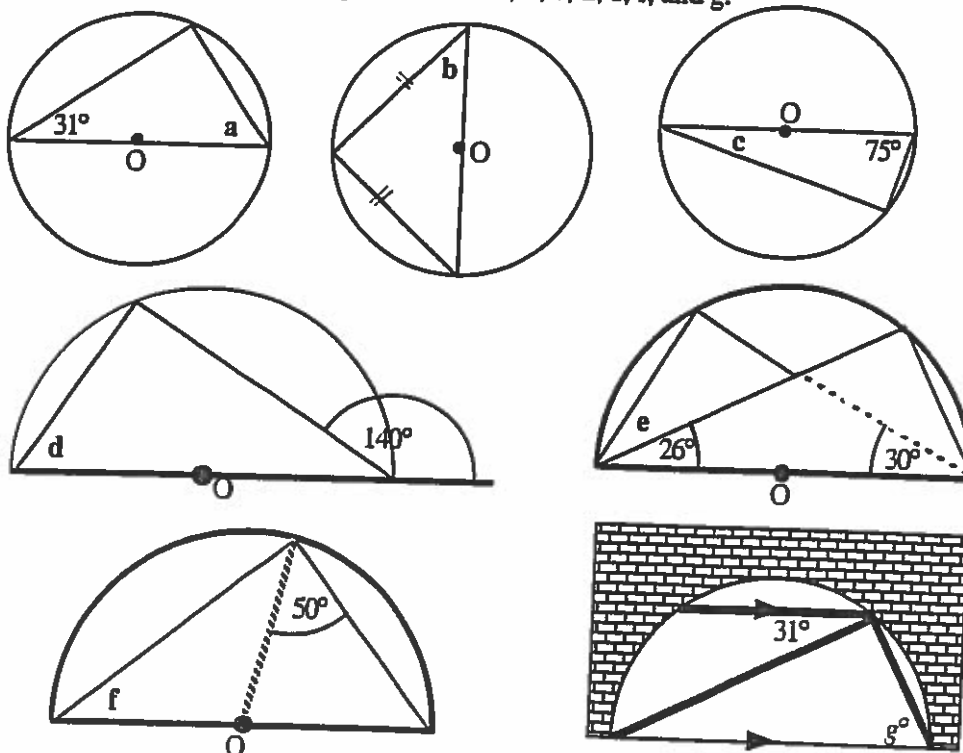
2. The semi-circular arch of a bridge is strengthened by a triangular metal structure as shown.

    (a) Calculate the size of ∠ABP.   140°
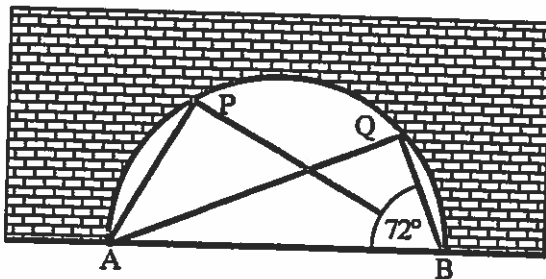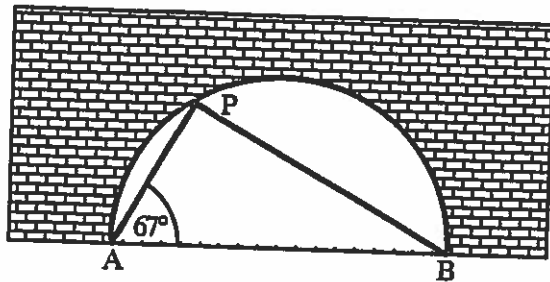
    (b) A second triangular structure is added. Calculate the size of ∠PAQ.

3. In the two diagrams below, calculate x, correct to 1 decimal place.

    (a)
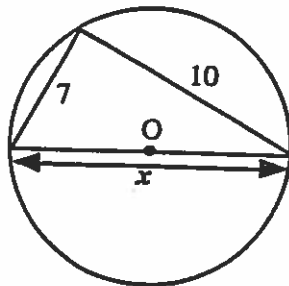
    (b)
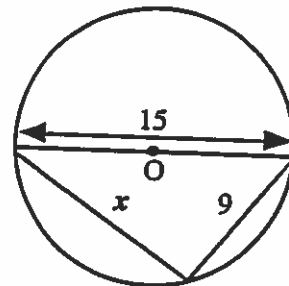
4. In these diagrams, calculate v and w correct to 1 decimal place.

    (a)

    (b)

    (c)

    (d)

28
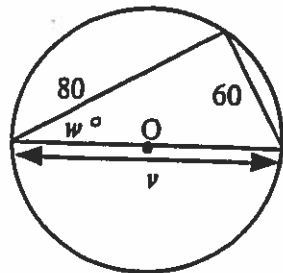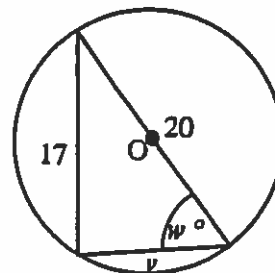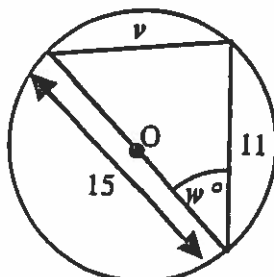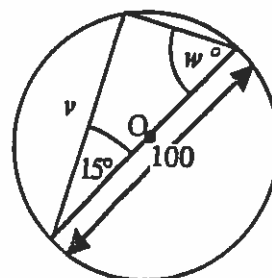
E. The interdependence of the centre, bisector of a chord and a perpendicular to a chord

Exercise 5

1. Copy the diagrams and fill in all the angles.

(a)

(b)

(c)

(d)
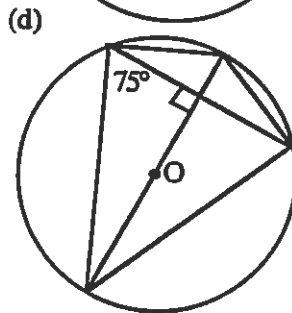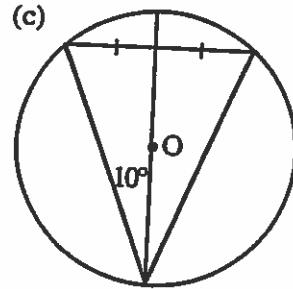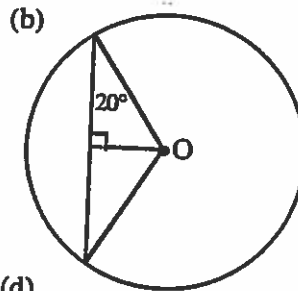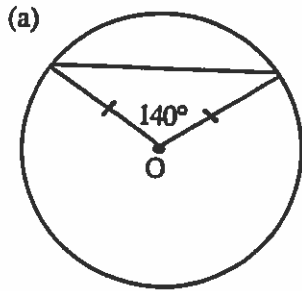
2. Calculate the distance from O to chord AB in each case. (All lengths are in centimetres.)

(a)

(b)

(c)
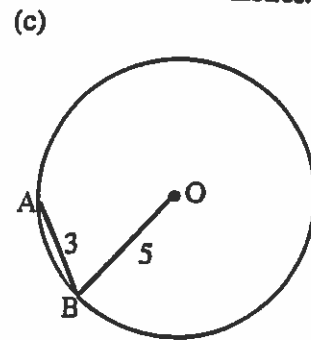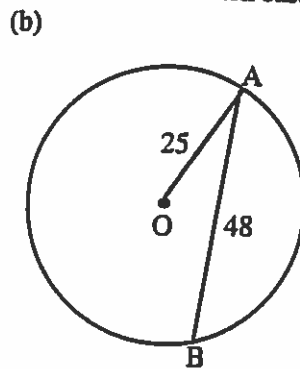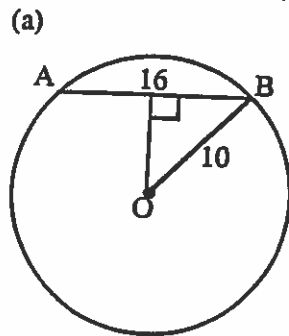
3. Calculate the length of the chord PQ in each case.

(a)

(b)
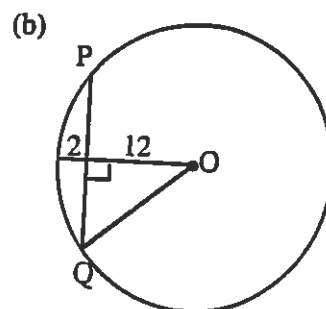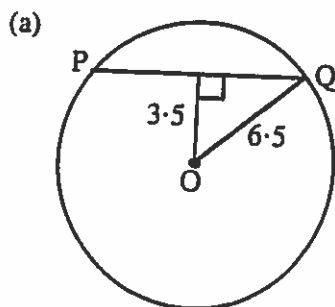
4. Calculate the value of the letters a, b, c, and d.

(i)

(ii)

(iii)
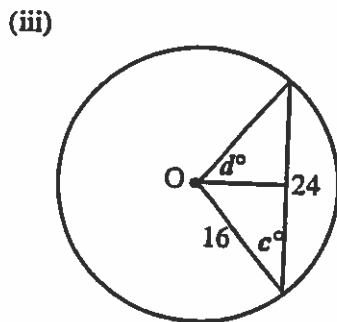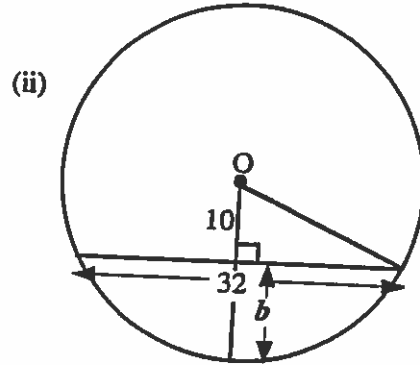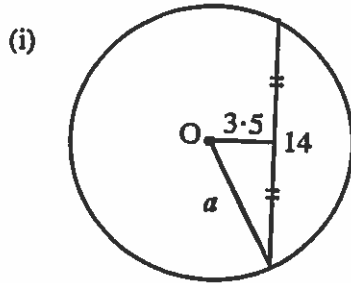
5. Given that the radius of the circle is 25 cm and AB = 48 cm, calculate the length of the line MT.

6.
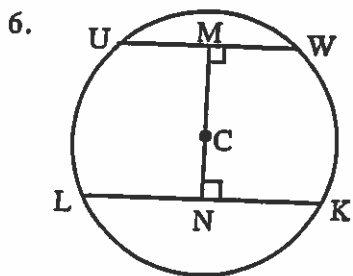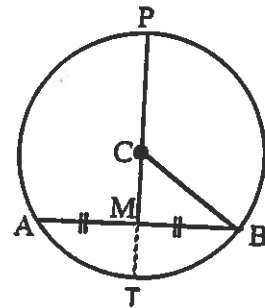
The diameter of the circle is 100 cm. UW = 62 cm and LK = 72 cm. and UW is parallel to LK.

Calculate the length of MN.

7. The diameter of a tank of waste product is 60 cm and the depth of the sludge is 25 cm.

Calculate the width AB of the surface of the waste sludge.
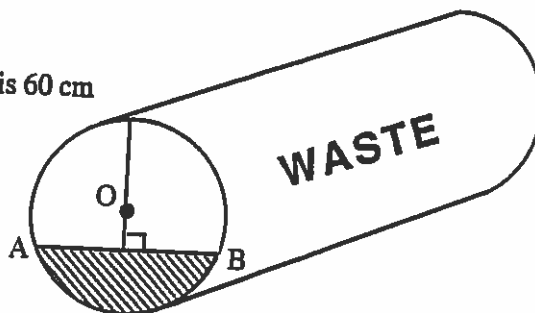
(m) $5w(2w - 3)(2w + 3)$ (n) $(y - 2)(y - 1)$ (o) $(a - 10)(a + 3)$

(p) $(y - 2)(y + 3)$ (q) $(12 - r)(2 + r)$ (r) $(x - 7)^2$

(s) $(2p - 3)(3p - 4)$ (t) $(2x + 1)^2$ (u) $2(q + 8)(q - 9)$

(v) $(2x - y)(x + 2y)$ (w) $2(a^2 + 1)(3a^2 - 2)$ (x) $(5y^2 + 3)(y^2 - 3)$

Properties of the Circle

Exercise 1

1. (i) 5·2 cm   (ii) 9·42 cm   (iii) 25·1 cm   (iv) 14·0 cm   (v) 47·1 cm   (vi) 44·7 cm
   (vii) 42·4 cm

2. 47·1 cm   3. 83·7 cm   4. 314 m   5. 45° 33·0 inches   6. 92°

Exercise 2

1. (a) 105 cm^2 (b) 177 cm^2 (c) 471 cm^2 (d) 236 cm^2
   (e) 367 cm^2 (f) 377 cm^2

2. 2152 cm^2 3. 1·64 m^2 4. 2261 cm^2

5. (a) 112 cm^2 (b) 134 cm^3 6. (a) 179 cm^2 (b) 2144 cm^3 7. 90°

Exercise 3

1. $a = 38$, $b = 25$, $c = 30$, $d = 20$, $e = 125$, $f = 20$, $g = 130$, $h = 70$, $i = 24$,
   $j = 26$, $k = 51$.

2. $w = 117$ 3. $x = 63$ 4. $v = 12$, $w = 7$

5. $x = 30$, $y = 38·9$, $z = 66$ 6. (a) 100mm (b) $p = 48$, $q = 42$, $r = 42$

Exercise 4

1. $a = 59$, $b = 45$, $c = 15$, $d = 50$, $e = 34$, $f = 40$, $g = 59$.

2. (a) 23°   (b) 49°   3. (a) 12·2   (b) 12·0

4. (a) $v = 100$, $w = 36·9$ (b) $v = 10·5$, $w = 58·2$ (c) $v = 10·2$, $w = 42·8$
   (d) $v = 96·6$, $w = 75$

Exercise 5

1. (a) 140° 20° 20°   (b) 20° 20° 70° 70°   (c) 10° 10° 80° 80° 90° 90° 90° 90°
   (d) 4 x 90°, 4 x 75°, 4 x 15°

2. (a) 6   (b) 7   (c) 4·77   3. (a) 11·0   (b) 14·4

4. $a = 7·8$, $b = 8·9$, $c = 41·4$, $d = 48·6$.

5. 18 cm   6. 73·9 cm   7. 59·2 cm

SIMILARITY

FOR EACH PAIR OF SIMILAR TRIANGLES DECIDE WHETHER THE
SECOND TRIANGLE IS AN ENLARGEMENT OR REDUCTION AND
FIND THE SIZES OF THE UNMARKED SIDES.

①

②

③

④

5

Q
9.1
5.5
P
12.0
R

E
D
7.2
F

6

V
4.0
W
5.8
6.8
U
L
8.4
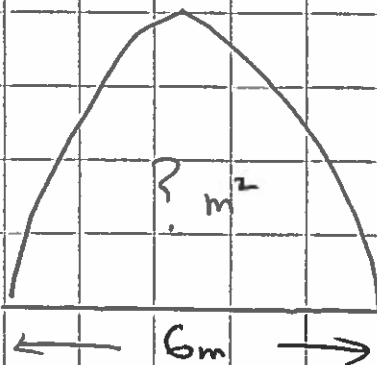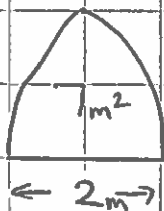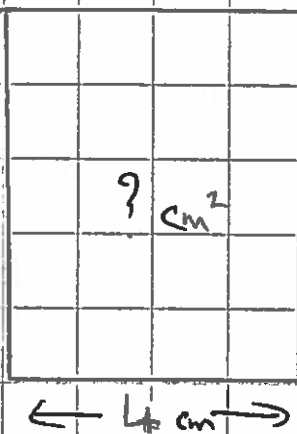N
M

7

E
16
10
D
7
F

Q
5
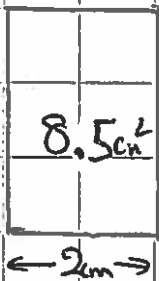P
R

8

5
4
6

Y
12
Z
X

32
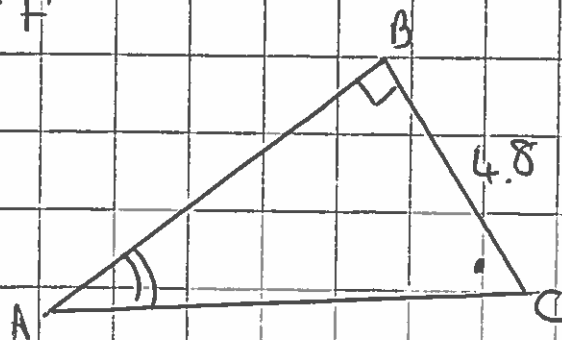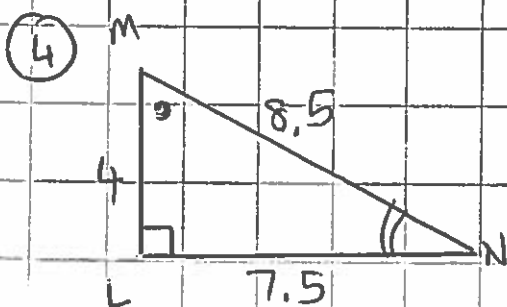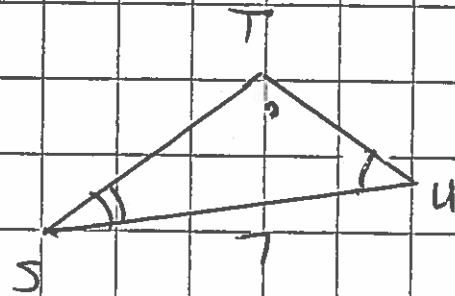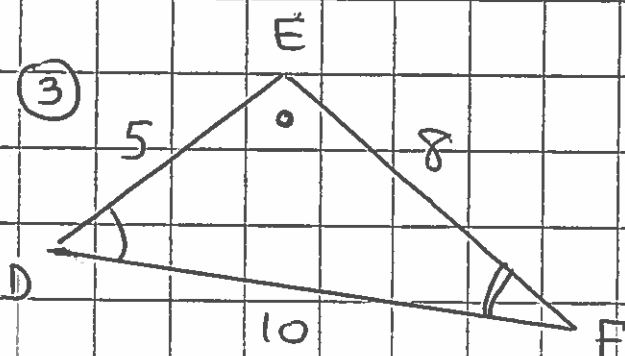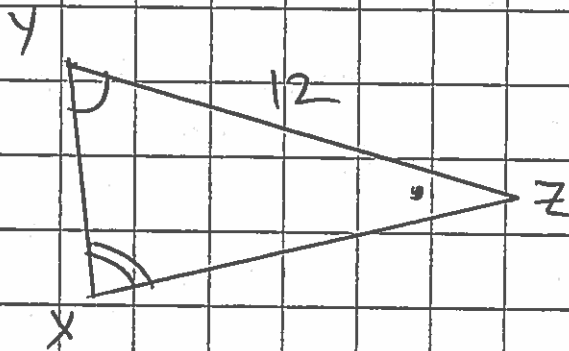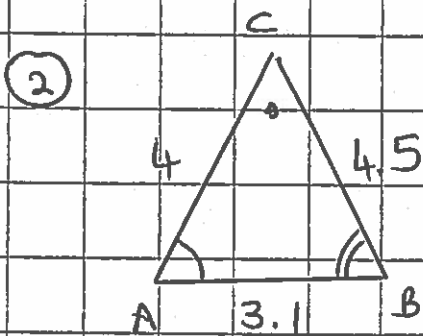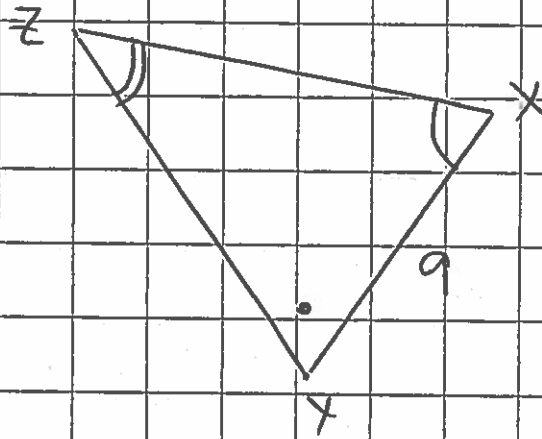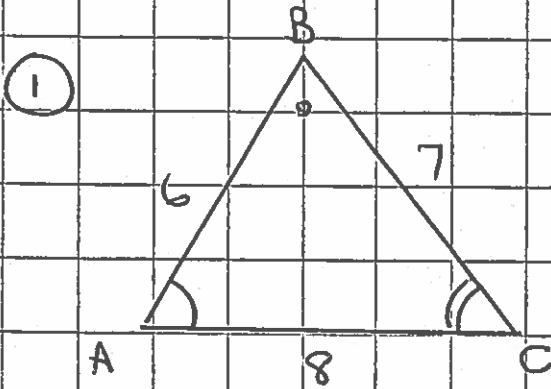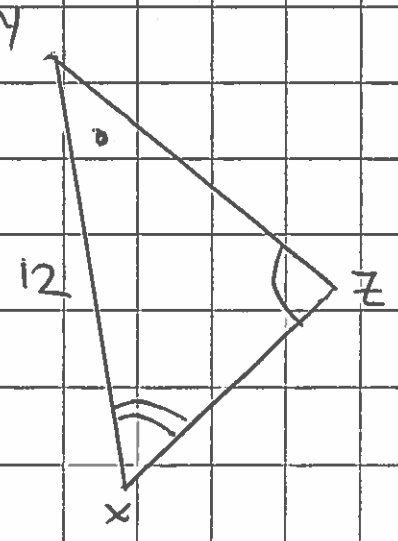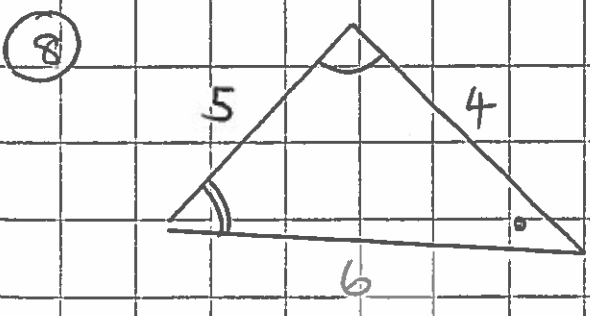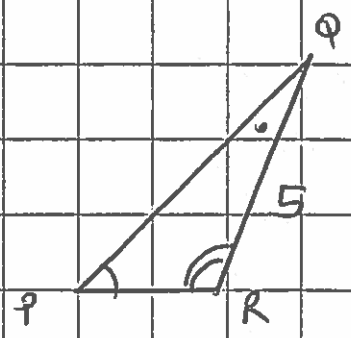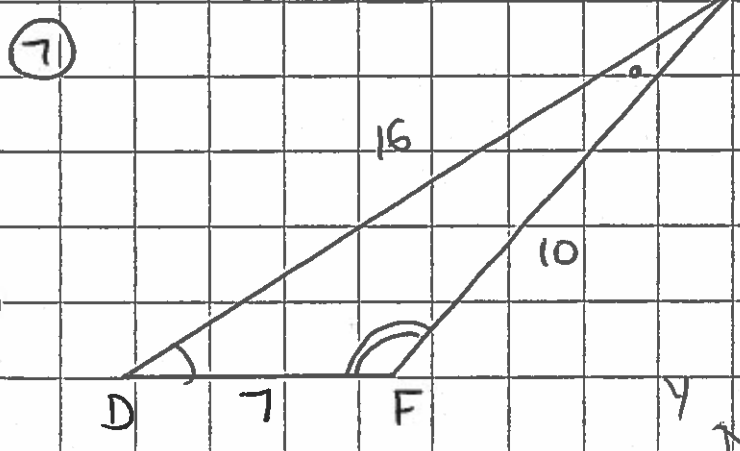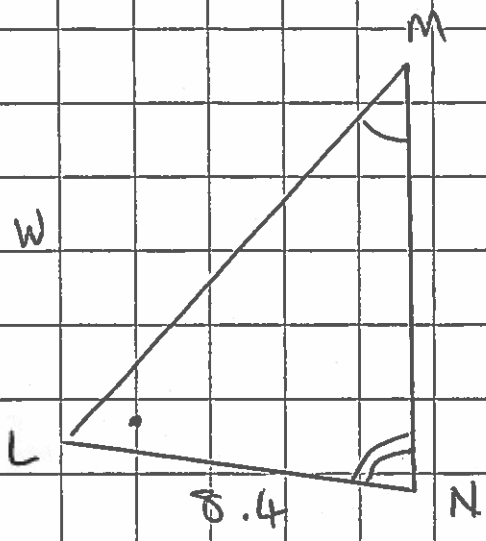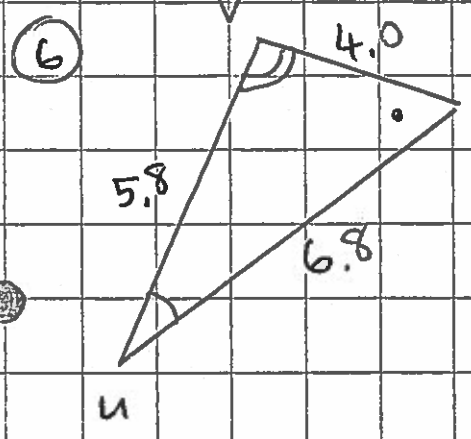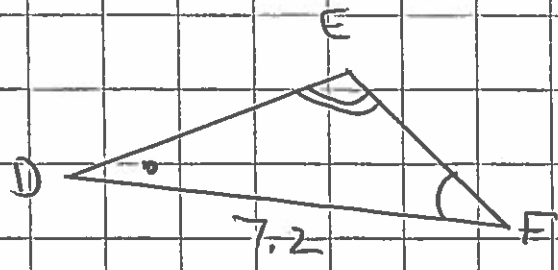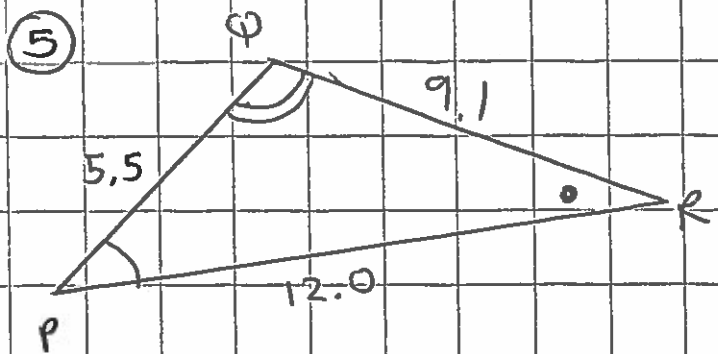
SIMILAR AREAS

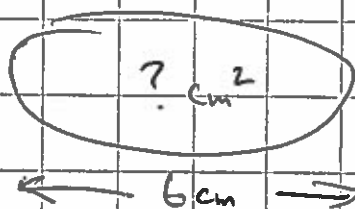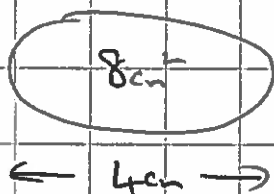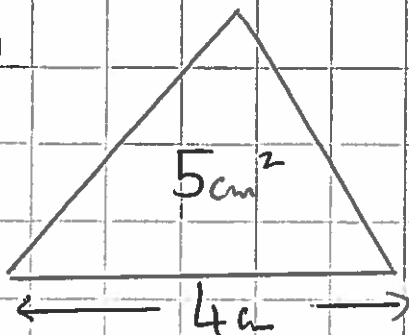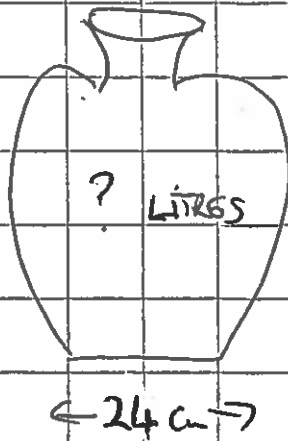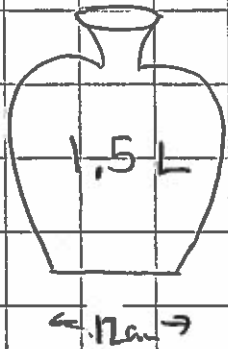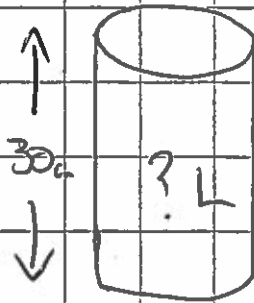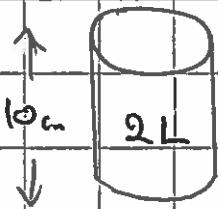CALCULATE THE AREA OF THE SECOND SHAPE IN EACH OF THE FOLLOWING:

①

$7 m^2$

← 2m →

$? m^2$

← 6m →

②

$8.5 cm^2$

← 2cm →

$? cm^2$

← 4 cm →

③

$8 cm^2$

← 4cm →

$? cm^2$

← 6cm →

④

$5 cm^2$

← 4cm →

$? cm^3$

← 2cm →

⑤

2cm

3cm

A = 3.14cm²

A = ? cm²

⑥

8cm²

2cm

? cm²

5cm

⑦

24cm²

8cm

96cm²

? cm

⑧

72cm²

12cm

8cm²

? cm

㉞
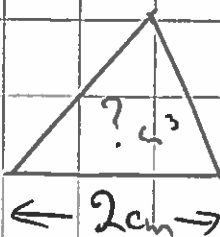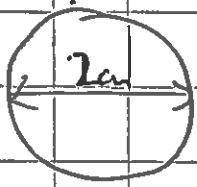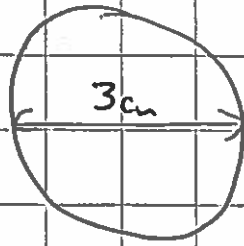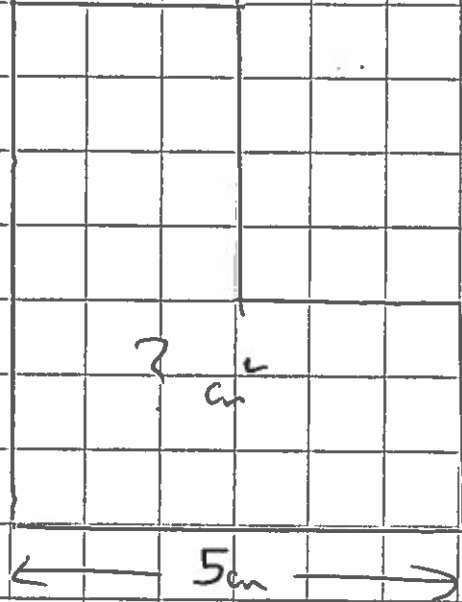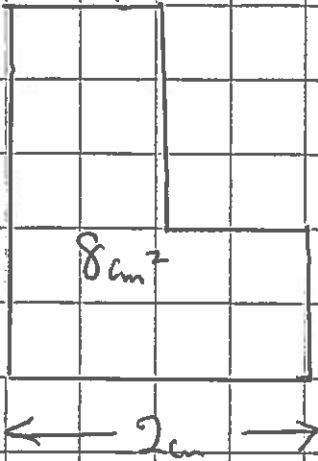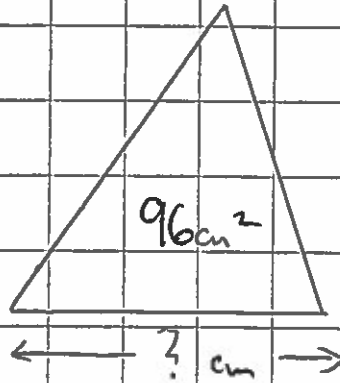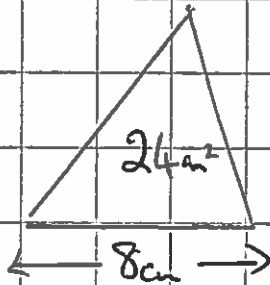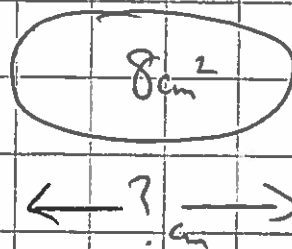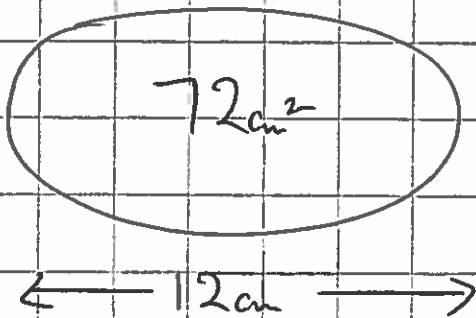
SIMILAR VOLUMES

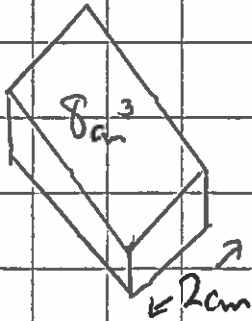CALCULATE THE VOLUME OF THE SECOND SHAPE IN EACH OF THE FOLLOWING?

①

1.5 L

← 12cm →

? LITRES

← 24 cm →

②

10cm

2 L
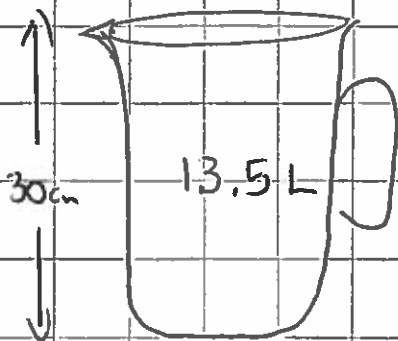
30cm

? L

③

8 cm³

← 2cm

? cm³

8 cm

④

30cm

13.5 L

10cm

? L

③⑤

⑤

72cm³

6cm

?

3cm

⑥

8000cm³
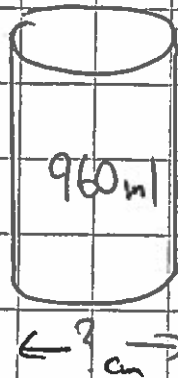
52 cm

?

3 cm

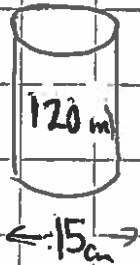⑦
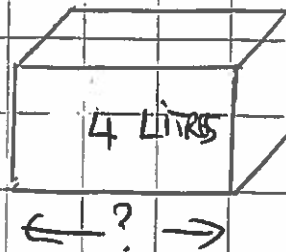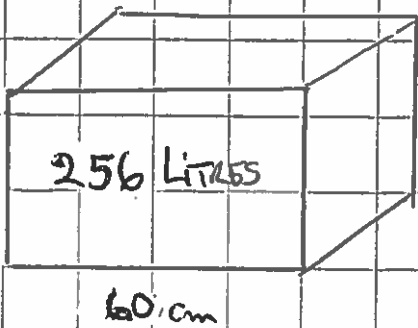
120 ml

15cm

960 ml

? cm

⑧

256 LITRES

60 cm

4 LITRES

?

㊱
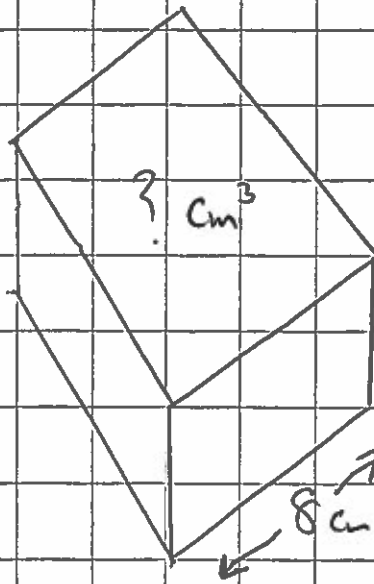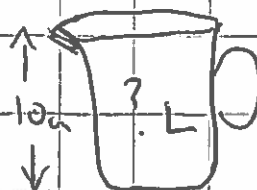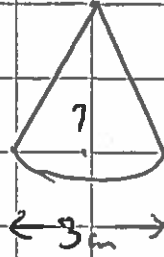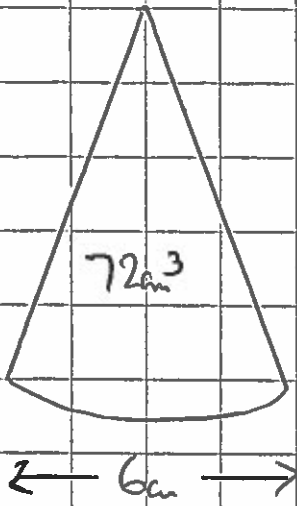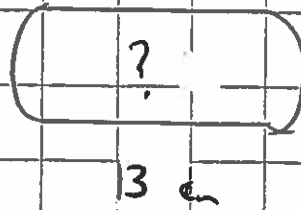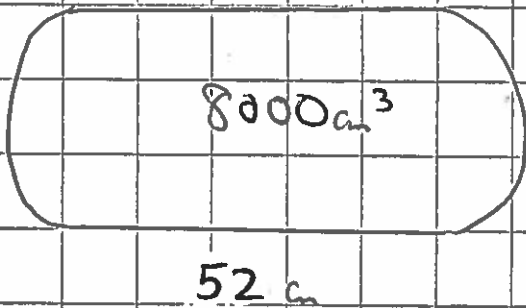
SIMILARITY (ANSWERS)

① XZ = 12
  YZ = 10.5

② XZ = 13.5
  YZ = 9.3

③ TU = 3.5
  TS = 5.6

④ AB = 9
  AC = 10.2

⑤ EF = 3.3
  DE = 5.46

⑥ LM = 14.28
  MN = 12.18

⑦ PR = 3.5
  PQ = 8

⑧ YZ = 8
  XZ = 10

SIMILAR AREA

① 63 m²

② 34 cm²

③ 18 cm²

④ 1.25 m²

⑤ 7.065 cm²

⑥ 50 cm²

⑦ 16 cm

⑧ 4 cm

SIMILAR VOLUMES

① 12 L

② 54 L

③ 512 cm³

④ 0.5 L
  (500 ml)

⑤ 9 cm³

⑥ 125 cm³

⑦ 30 ml

⑧ 15 cm

③⑦