N5 Maths - Relationships (Part 1)

In this booklet:

LINEAR RELATIONSHIPS

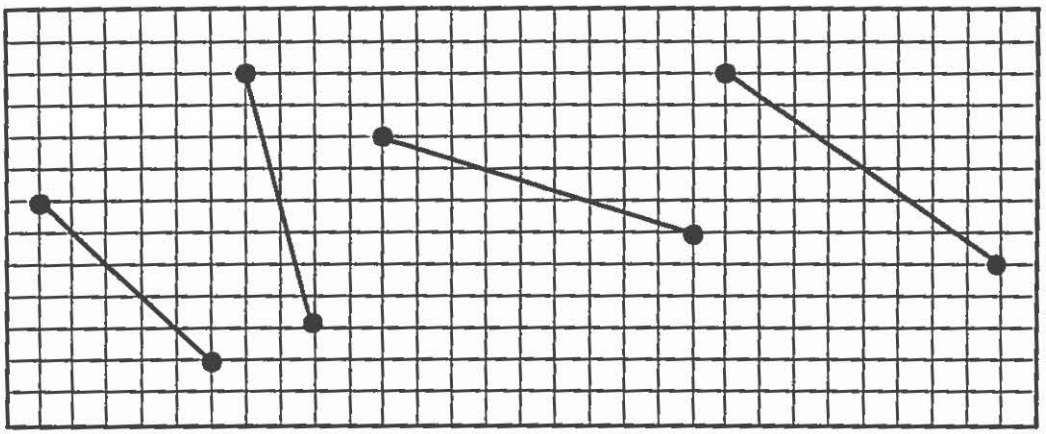
By the end of this set of exercises, you should be able to

(a) determine the gradient of a straight line

(b) sketch a straight line given its equation in the form $y = ax + b$

(c) determine the equation of a straight line in the form $y = ax + b$ from its graph

5. Calculate the gradient of each line.



6. Calculate the gradients of the lines joining the following points.
(Some are positive, some negative).

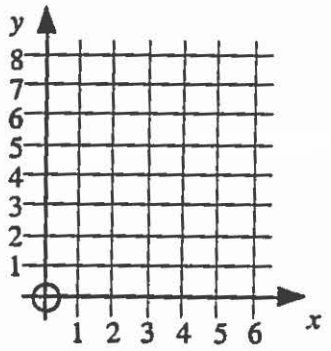
(a) A(1,6), B(6,1)	(b) D(0,7), E(2,3)	(c) G(–2,5), H(1,–4)
(d) J(–6,–3), K(3,0)	(e) M(–6,0), N(0,–4)	(f) P(1,–1), Q(3,1)
(g) S(–1,10), T(3,–2)	(h) V(–6,–10), W(2,–6)	(i) Y(–12, 5), Z(3,0)

7. (a) On a small coordinate diagram plot the two points A(1,3) and B(6,3).
(b) Find the gradient of the line joining A and B using your formula.
(c) Comment on the connection between the shape (slope) of the line drawn in part (a) and the corresponding value of its gradient as calculated in part (b).

B. Sketching Lines in the form $y = ax + b$

Exercise 2

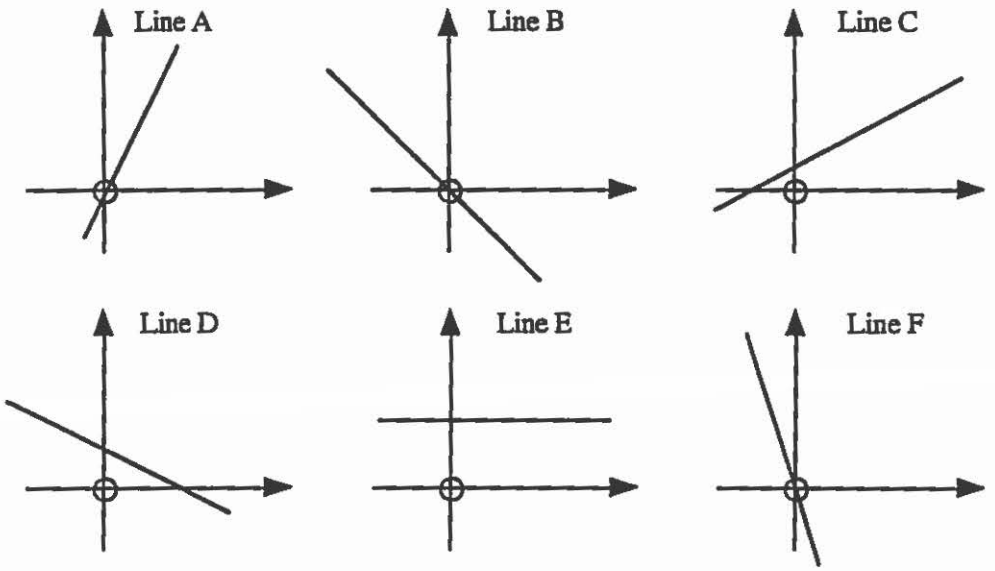
1. Drawing the line $y = 2x + 1$:
(a) Make a copy of this coordinate diagram.
(b) Where does the line $y = 2x + 1$ cut the y - axis? (plot this point).
(c) The gradient of the line is $\frac{2}{1}$. From your first plotted point, move 1 box right and 2 boxes up. Plot this 2nd point.
(d) Join your 2 points and extend the line.
(e) Label the line $y = 2x + 1$.



2. Draw the following lines, labelling each one carefully.

(a)	$y = 3x + 2$	(b)	$y = 4x - 3$
(c)	$y = x + 5$	(d)	$y = {}^1/_2x + 4$
(e)	$y = -2x + 1$	(f)	$y = -3x - 5$
(g)	$y = -x + 3$	(h)	$y = {}^3/_4x + 1$

3. Look at the 6 lines shown and the list of 6 gradients given below

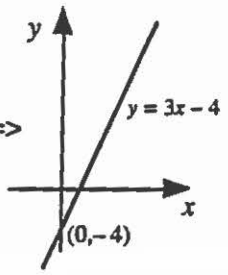


Gradients: $a_1 = {}^1/_2$, $a_2 = -3$, $a_3 = -{}^1/_2$, $a_4 = 0$, $a_5 = -1$, $a_6 = 2$

Match up the lines (A, B, C, D, E, F) with the gradients (a_1, a_2, a_3, a_4, a_5, a_6).

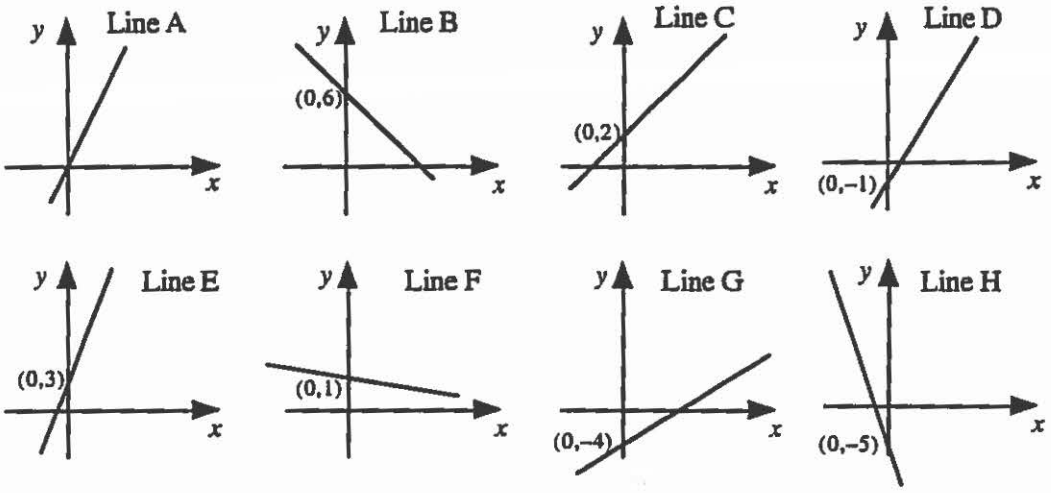
4. This time, simply make a neat <u>sketch</u> of the given line, indicating where it cuts the y - axis.

example: $y = 3x - 4$ =>



(a)	$y = x + 3$	(b)	$y = 2x - 3$
(c)	$y = {}^1/_2x + 6$	(d)	$y = -2x + 3$
(e)	$y = -x - 4$	(f)	$y = 6x - 6$
(g)	$y = {}^1/_5x + 2$	(h)	$y = -{}^1/_2x + 4$
(i)	$y = -4x - 3$	(j)	$y = {}^4/_3x - 1$

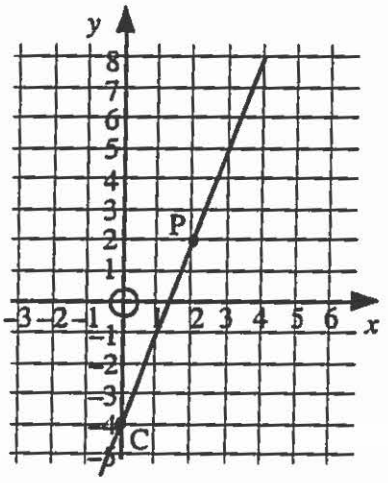
5. Look at the following sketches of 8 lines and the list of 8 equations.
Match each line to its corresponding equation.



Lines: $y = 2x - 1$, $y = 5x + 3$, $y = 3x$, $y = {}^1/_2x - 4$,
$y = -2x - 5$, $y = -x + 6$, $y = -{}^1/_4x + 1$, $y = x + 2$.

C. Determining the equation of a line in the form $y = ax + b$

Exercise 3



1. Determine the equation of the line shown opposite

Step 1 Start always with the general equation of any line:

=> $y = ax + b$

Step 2 Pick out the coordinates of where the line cuts the y - axis – (0,...)

Use this to begin to write down the line's equation:

=> $y = ax - ...$

Step 3 Find the gradient of the line by using any two points on the line e.g. C and P.

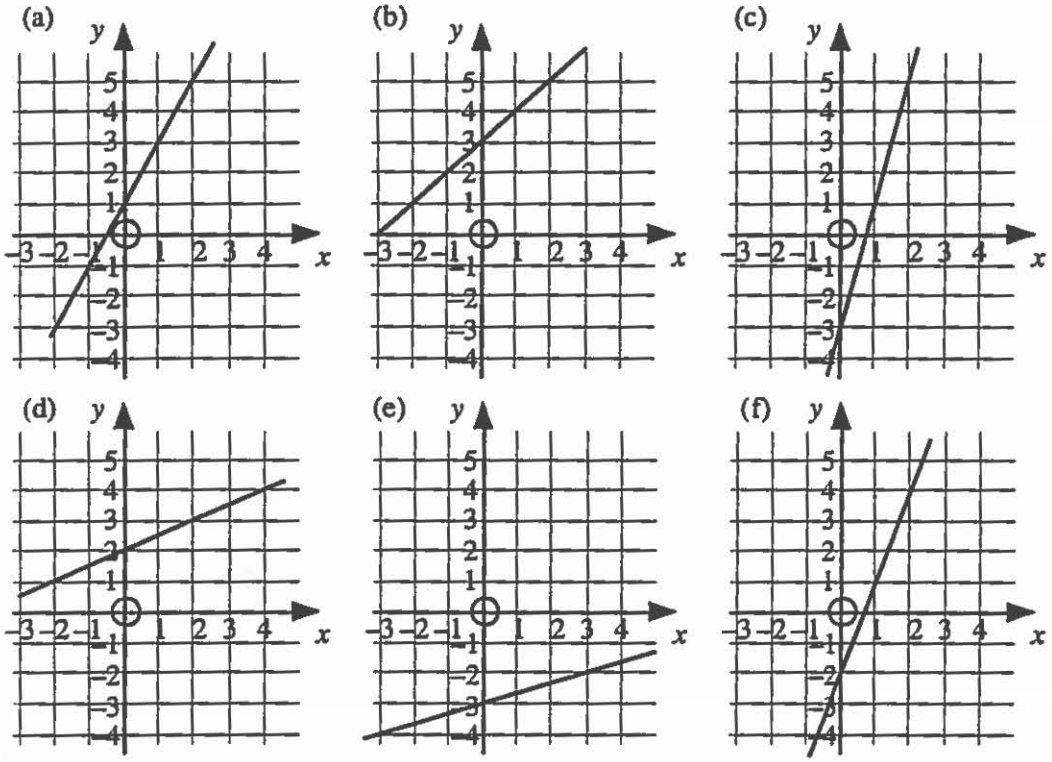
Use this to complete your equation:

=> $y = ..x - ...$

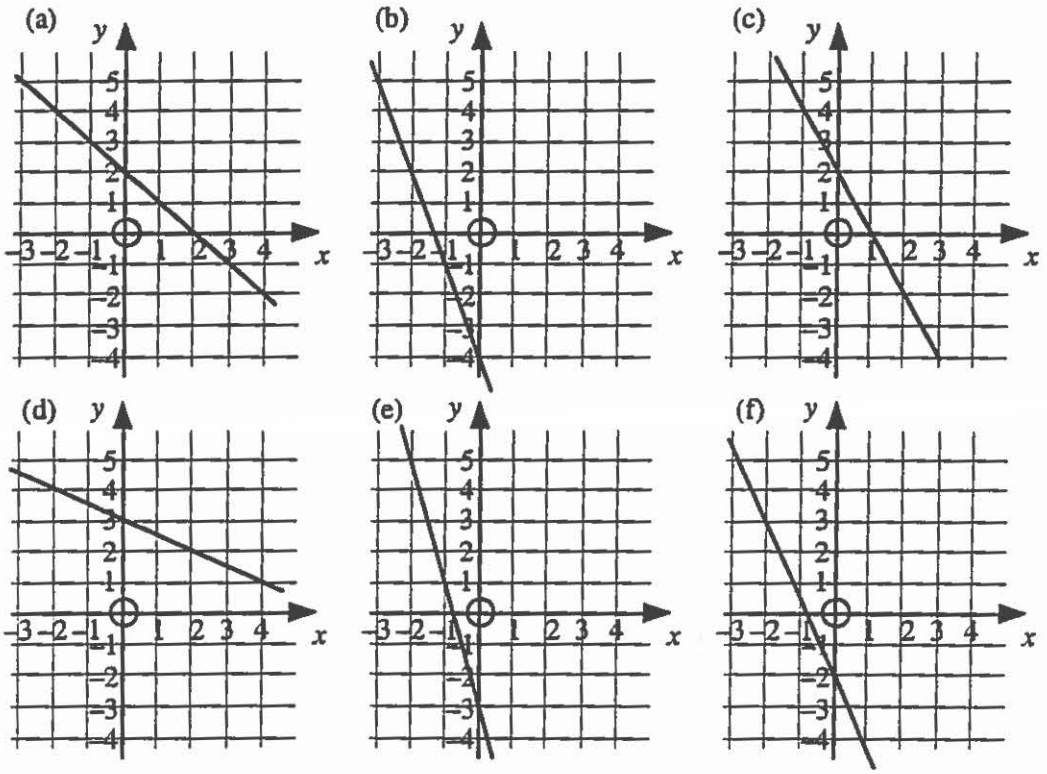
2. On the next page there are drawings of six lines.
Use the technique shown in question 1 to determine their nature.

cont'd ...

2. cont'd...



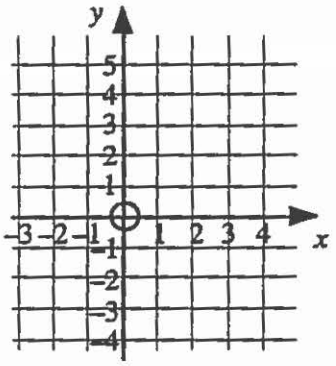
3. The following lines all have negative gradients. Use the same technique as shown in question 1 to determine their equations.



Checkup for Linear Relationships

1. (a) Given the two points A(2,–1) and B(4,7), calculate the gradient of the line AB.
   (b) Repeat for the line joining C(–7,2) and D(1,–4).

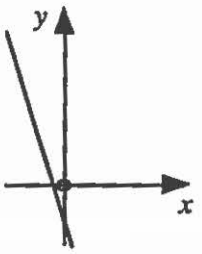
2. Make a copy of this coordinate diagram and draw the line $y = 2x - 2$.



3. Sketch the line $y = \frac{1}{2}x + 1$.

4. Sketch the line $y = -x - 1$.

5. Which of the following is most likely to be the equation of the line shown opposite?



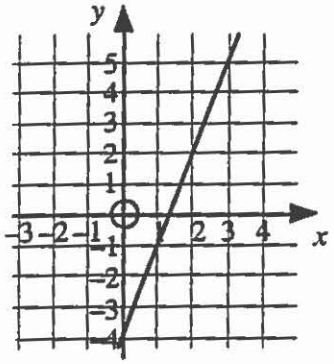
   (a) $y = 3x - 2$ (b) $y = -3x - 2$

   (c) $y = \frac{1}{3}x - 2$ (d) $y = \frac{1}{3}x + 2$

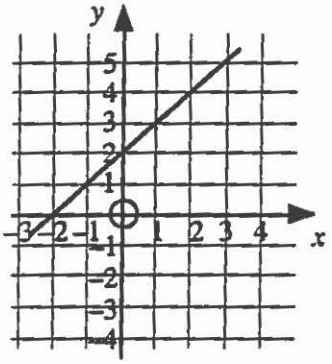
   (e) $y = -3x + 2$ (f) $y = -\frac{1}{3}x - 2$

6. Determine the equations of the following four lines in the form $y = ax + b$:

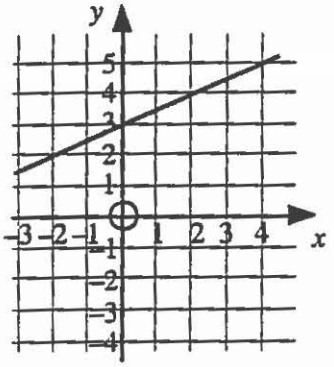
(a)



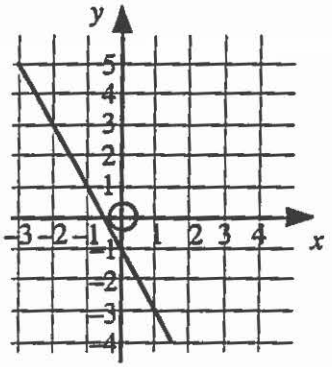
(b)



(c)



(d)



Checkup for Volumes of Solids

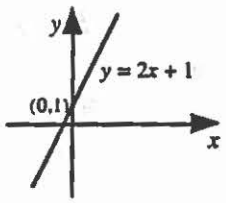
1. (a) 112·5 cm^3 (b) 168 cm^3 (c) 185 cm^3
2. (a) 459 cm^3 (b) 1001 cm^3 (c) 536·1 cm^3
3. (a) 2797·7 cm^3 (b) 769·3 cm^3 (c) 588·7 cm^3
4. 678·24 cm^3 + 3391·2 cm^3 + 452·16 cm^3 = 4521·6 cm^3

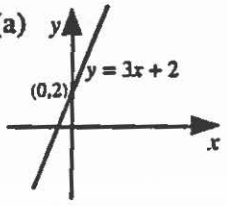
Linear Relationships

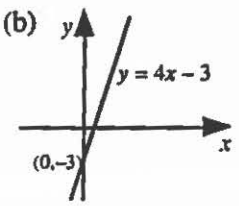
Exercise 1

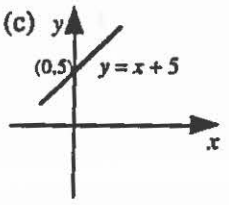
1. 2, 5, $^1/_2$, $^2/_3$
2. (a) 4 (b) 3 (c) $^1/_2$ (d) $^2/_3$
3. (a) $^1/_3$ (b) $^1/_3$ (c) 6 (d) 3
4. slopes downwards if gradient is negative as you move from left to right
5. –1, –4, $-^1/_3$, $-^3/_4$
6. (a) –1 (b) –2 (c) –3 (d) $^1/_3$ (e) $-^2/_3$
   (f) 1 (g) –3 (h) $^1/_2$ (i) $-^1/_3$
7. (a) sketch showing vertical line. (b) gradient doesn't exist (error)
   (c) gradient of a vertical line does not exist.

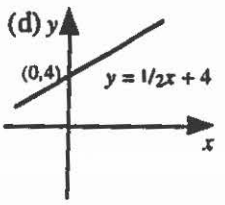
Exercise 2

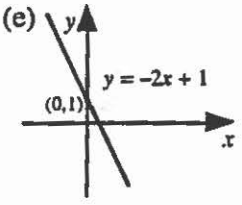
1. 

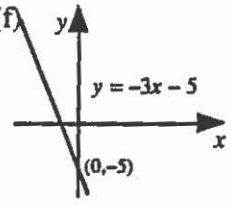
2. (a) 

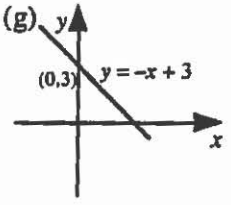
(b) 

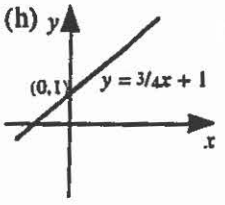
(c) 

(d) 

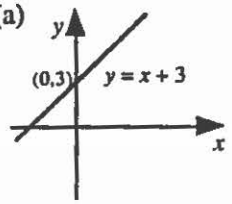
(e) 

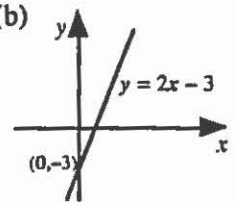
(f) 

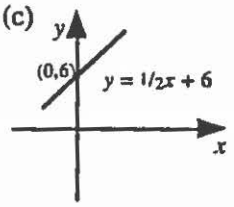
(g) 

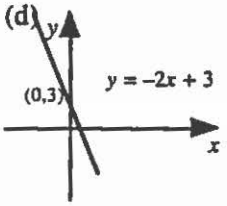
(h) 

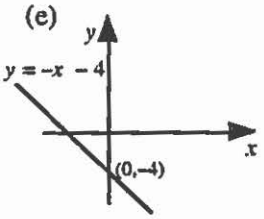
3. Line A – $a_6 = 2$ Line B – $a_5 = -1$ Line C – $a_1 = ^1/_2$
   Line D – $a_3 = -^1/_2$ Line E – $a_4 = 0$ Line F – $a_2 = -3$

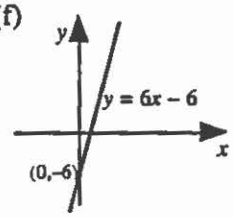
4. (a) 

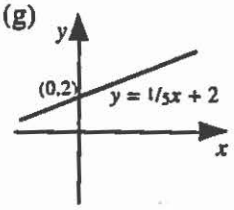
(b) 

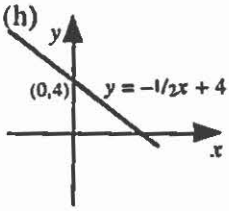
(c) 

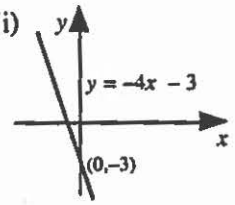
(d) 

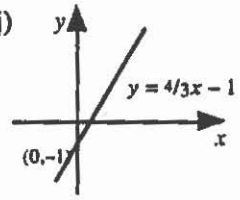
(e) 

(f) 

(g) 

(h) 

(i) 

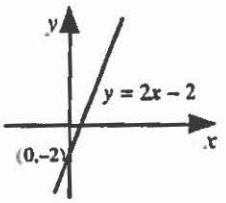
(j) 

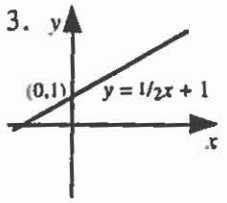
5. Line A – $y = 3x$ Line B – $y = -x + 6$ Line C – $y = x + 2$
   Line D – $y = 2x - 1$ Line E – $y = 5x + 3$ Line F – $y = -1/4x + 1$
   Line G – $y = 1/2x - 4$ Line H – $y = -2x - 5$

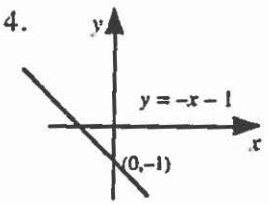
Exercise 3

1. Step 2: (0,–4); $y = ax - 4$ Step 3: gradient = 3; $y = 3x - 4$
2. (a) $y = 2x + 1$ (b) $y = x + 3$ (c) $y = 4x - 3$
   (d) $y = 1/2x + 2$ (e) $y = 1/3x - 3$ (f) $y = 3x - 2$
3. (a) $y = -x + 2$ (b) $y = -3x - 4$ (c) $y = -2x + 2$
   (d) $y = -1/2x + 3$ (e) $y = -4x - 3$ (f) $y = -5/2x - 2$

Check-up for Linear Relationships

1. (a) gradient = 4 (b) gradient = $-3/4$
2. 

3. 

4. 

5. $y = -3x - 2$
6. (a) $y = 3x - 4$ (b) $y = x + 2$ (c) $y = 1/2x + 3$ (d) $y = -2x - 1$

SIMULTANEOUS LINEAR EQUATIONS

By the end of this set of exercises, you should be able to

(a) Construct formulae to describe a linear relationship

(b) Understand the significance of the point of intersection of two graphs

(c) Solve simultaneous linear equations in two variables graphically

(d) Solve simultaneous linear equations in two variables algebraically

SIMULTANEOUS LINEAL EQUATIONS

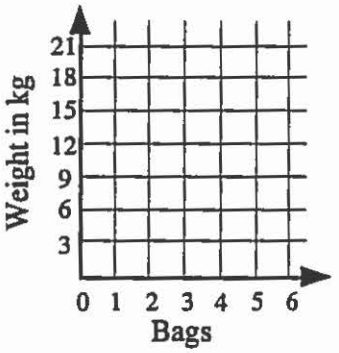
A. Construction of Formula

Exercise 1

1. A greengrocer sells Brussel Sprouts in 3 kilogram bags.
   The table compares the number of bags with the weight of sprouts sold.

Number of Bags (N)	1	2	3	4	5	6
Weight of sprouts (W)	3	6	9	12	15	18

(a) **Copy** and complete: Weight = ... x No. Bags
(b) Write a formula for the weight of sprouts.
(c) Use your formula to find the weight of sprouts in 10 bags.
(d) In your jotter, use your table to plot and join the points on a coordinate diagram like this :–
(e) Extend your graph to show a straight line which passes through the origin.



2. A confectioner sells jelly eels in packs of ten.
   (a) Copy and complete the table:

Number of packs (P)	1	2	3	4	5	6
Number of eels (E)	10					

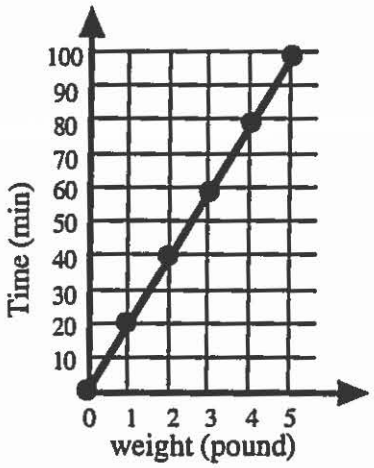
(b) **Copy** and complete :– Number of eels = ... x No. packs
(c) Write a formula for calculating the number of eels.
(d) Use your formula to find the number of eels in 9 packs.
(e) Use your table to plot and join the points on a coordinate diagram.
(f) Extend your graph to show a straight line which passes through the origin.

3. The graph shows cooking times for roast beef.

(a) **Copy** and complete the table:

Weight (W)	1	2	3	4	5	6
Time (T)	20					

(b) Write a formula for the time (T) taken to cook a roast if you know its weight (W).
(c) Use your formula to find the time taken to cook a 10 pound roast .



4. Mr. R. Highet called out Computer Fix to repair his computer. They have a 'call out' charge of £25 plus a charge of £8 per hour.

No. Hours (h)	1	2	3	4	5
Charge £ (C)	33	41	49		

(a) How much do Computer Fix charge for:
(i) 4 hours? (ii) 5 hours?

(b) Write a formula for the charge (C), given the number of hours worked (h).

5. To hire a cement mixer it costs a basic £8 plus £4 for each day you have the machine.

(a) Copy and complete the table:

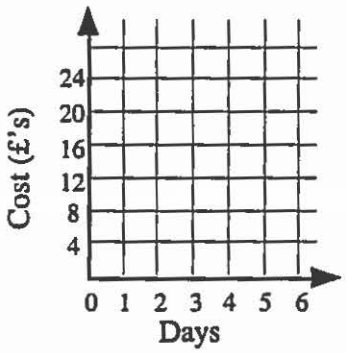
No. Days (D)	1	2	3	4	5
Charge £ (C)	12				

(b) Write a formula for the charge (C) given the number of days (D) for which you have the machine.

(c) In your jotter, use your table to plot and join the points on a coordinate diagram like this:

(d) Extend your graph to cut the vertical (C) axis and give the coordinates of the point where the line cuts that axis.

(e) Explain this point in relation to hiring a cement mixer.



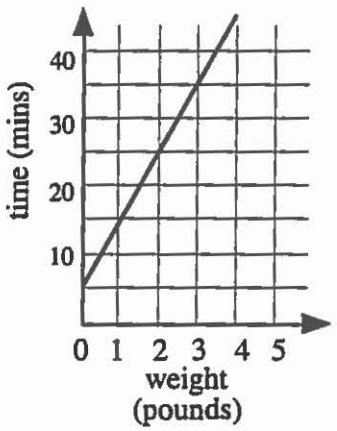
6. The graph shows defrosting times for a chicken.

(a) Using the graph, **copy** and complete the table.

Weight (W pounds)	1	2	3	4	5	6
Time (T min)	15					

(b) Write a formula for the time (T) taken to cook a chicken if you know its weight (W).

(c) Use your formula to find the time taken to cook a 10 pound chicken .

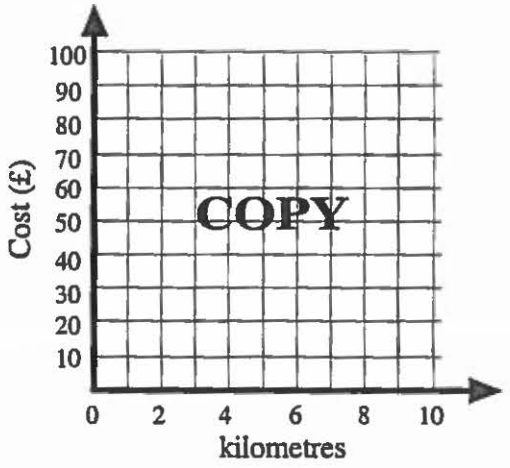


7. Fast Delivery charges £50, plus £5 per kilometre to deliver parcels.

   (a) Write down a formula for the charge £C for a delivery of k kilometres.

   (b) Calculate the charge for a 10 kilometre trip.

   (c) Draw a graph of charges up to 10km, using these scales.

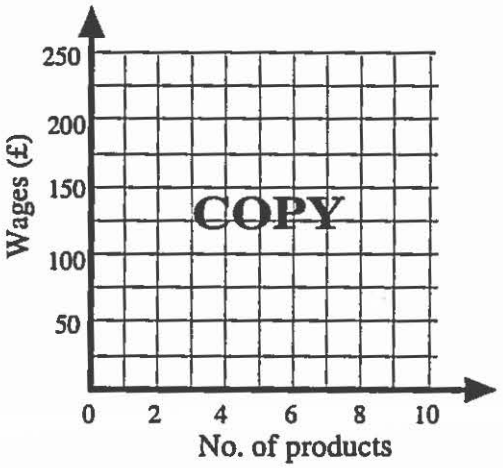


8. Mrs. Divers sells cosmetics.
   She gets paid a basic £80 per week plus £10 each time she sells a product from the new Opius Perfume range.

   (a) Write down a formula for her wage £W for a week in which she sells P products.

   (b) Work out her wage for a for a week in which she sells 20 products.

   (c) Draw a graph of her wages for up to 20 products, using these scales.



9. Mr. McGarrill, the school janitor, is ordering sweeping brushes at £10 each.
   If he pays quickly he finds that he can get a discount of £5 off his total bill.

   (a) **Copy** and complete the table:

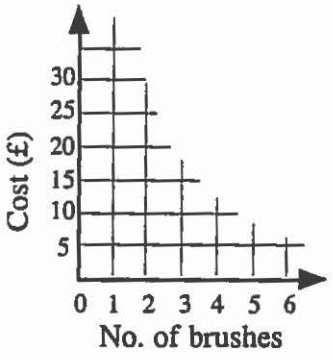
No. Brushes (B)	1	2	3	4	5
Cost £ (C)	5	15	25		

   (b) What is his bill for:

   (i) 4 brushes? (ii) 5 brushes?

   (c) Write a formula for the cost (C) for a number of brushes (B).

   (d) In your jotter, use your table to plot and join the points on a coordinate diagram like this:

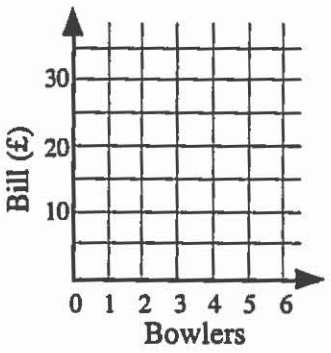


10. A group of adults are having a night out at a ten–pin bowling alley.
The cost is normally £6 each, but a midweek special is giving £4 off the total bill.

(a) Make up a table to show the total bill for 1, 2, 3, 4, 5, 6 bowlers.

(b) Write a formula for the total bill (£T) for a number of bowlers (B).

(c) In your jotter, use your table to plot and join the points on a coordinate diagram like this:



Revision:- Drawing Straight Lines

Exercise 2

For each of the following equations of a straight line:

- choose three points on the line
- plot the points on squared paper, each one on a separate diagram
- draw a straight line through them.

1. $y = x$	2. $y = 3x$	3. $y = x + 1$	4. $y = 2x + 3$
5. $y = 2x - 1$	6. $y = 2 - x$	7. $y = 5$	8. $x = 3$
9. $x + y = 6$	10. $x - y = -2$	11. $2x + y = 0$	12. $y = -x + 1$

B. Solving Simultaneous Linear Equations Graphically

Exercise 3

By drawing the graphs represented by the following equations on squared paper, solve each pair of simultaneous equations.

1. $x + y = 6$ $y = x$	2. $x + y = 4$ $x + 2y = 6$	3. $x - y = 4$ $x - 2y = 6$
4. $x + y = 8$ $x - y = 2$	5. $x + 2y = 5$ $x - y = -1$	6. $y = x + 2$ $y = -x - 4$
7. $x + 3y = 7$ $x - 3y = 1$	8. $y = 2x + 2$ $y = -x - 4$	9. $2x - y = 3$ $y = 5$
10. $2x + y = 4$ $3x + 2y = 9$	11. $3x - 3y = -6$ $3x - 2y = 0$	12. $x + 3y = 8$ $2x - y = -5$

Exercise 4A

1.

Goudie's Car Hire
£40 Deposit + £10 a day

Henry's Rent a Car
£20 per day

(a) Copy and complete the tables showing the charges for the two car hire companies.

Goudie's

Number of days	0	1	2	3	4	5	6	7
Cost (£)	40	50	60					

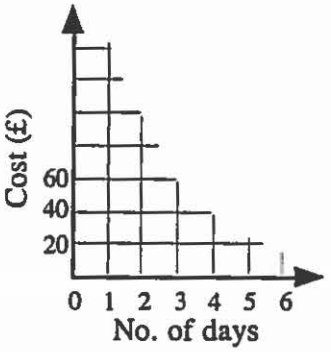
Henry's

Number of days	0	1	2	3	4	5	6	7
Cost (£)	0	20	40					

(b) Draw the straight line graph for both car hire companies on the same coordinate diagram.

(c) The two companies charge the same amount only once. For how many days is this?

(d) Up to how many days is Henry's cheaper?



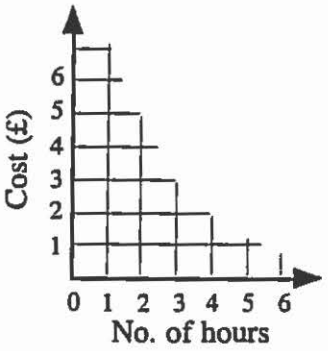
2. 'Hire a bike in Millport.'
Mr. Dawes charges **£1 deposit plus 50p per hour.**
Mr. Beckham charges **No deposit, £1 per hour.**

(a) Make two tables to show the prices for up to 6 hours hire at Dawes' and Beckham's.

(b) Draw the straight line graph for both bicycle hire companies on the same coordinate diagram.

(c) For what number of hours hire is the cost the same at both shops?

(d) If you wanted to hire a bike for 4 hours, which shop would you go to in order to save money?



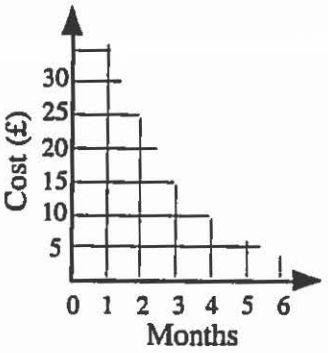
3. RENT A COMPUTER are offering computers for
£20 deposit, plus £5 per month.
COMPU HIRE are offering similar computers for
£10 per month, with no deposit.

(a) Make two tables to show the prices for up to 5 months at each place.

(b) Draw the straight line graph for both computer rental companies on the same coordinate diagram.

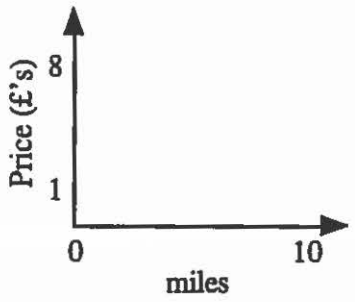
(c) (i) For what number of months is the cost the same at both shops?

(ii) What price is this?



4. BLACK CAB TAXI COMPANY charge **50p** per mile.
RED TAXIS charge **£2** for any journey up to **4 miles**, then **£1** per mile for each additional mile.

(a) Make two tables to show the prices for up to a 10 mile journey at both firms.
(b) Draw the straight line graph for both taxi companies on the same coordinate diagram.
(c) For how many miles is the cost the same at both firms?
(d) You are travelling only 2 or 3 miles – which taxi company would you phone to save money?



Exercise 4B

1. Third Lanark v Leith Athletic

Adult Charge £x
Child Charge £y

One adult and one child paid £8 to attend this football match.
$x + y = 8$
Two adults and one child paid £13.
$2x + y = 13$

(a) Draw the lines $x + y = 8$ and $2x + y = 13$ on the same coordinate diagram using suitable points on each line.
(b) Write down the coordinates of the point of intersection.
(c) What is significant about this point in terms of prices to get into the match?
(d) What was the charge for 10 adults and 10 children at this match?

2. The professional at Worthwent Golf Club prices her goods as follows:

Golf Balls £x Golf Gloves £y

Arnold bought 2 golf balls and 1 golf glove for £8. $2x + y = 8$
Tiger bought 4 golf balls and 1 golf glove for £12. $4x + y = 12$

(a) Draw the lines $2x + y = 8$ and $4x + y = 12$ on the same coordinate diagram using suitable points on each line.
(b) Write down the coordinates of the point of intersection.
(c) What was the cost of a golf ball?
(d) What was the cost of a golf glove?
(e) What does the professional charge for 3 golf balls and 3 golf gloves?

3. 2 jotters and 2 pencils cost 80p. 1 jotter and 3 pencils cost 60p.
Let the cost of a jotter be x pence and the cost of a pencil be y pence.
One equation from the data given is $2x + 2y = 80$.
(a) Write down the other equation in terms of x and y.
(b) Draw the two straight lines which the equations represent on the same coordinate diagram using suitable points on each line.
(c) Use your graph to find the cost of a jotter and the cost of a pencil.

4. 1 packet of Weedo and 1 packet of slug pellets costs £5.
   1 packet of Weedo and 3 packets of slug pellets costs £9.
   Let the cost of a packet of Weedo be £x and the cost of a packet of slug pellets be £y.
   (a) Write down two equations in terms of x and y.
   (b) Draw the two straight lines which the equations represent on the same coordinate diagram using suitable points on each line.
   (c) Use your graph to find the cost of a packet of Weedo and the cost of a bottle of slug pellets.

5. Mary bought 3 T–shirts and 2 bottles of colour dye for £12.
   Sally bought 2 of the T–shirts and 5 bottles of colour dye for £30.
   Let the cost of a T–shirt be £x and the cost of a bottle of colour dye be £y.
   (a) Write down two equations in terms of x and y.
   (b) Draw the two straight lines which the equations represent on the same coordinate diagram using suitable points on each line.
   (c) Use your graph to find the cost of a T–shirt and the cost of a bottle of colour dye.

6. The total cost of two books is £10 and the difference in their cost is £2.
   Let the cost of a one book be £x and the cost of the other book be £y .
   (a) Write down two equations in terms of x and y.
   (b) Draw the two straight lines which the equations represent on the same coordinate diagram using suitable points on each line.
   (c) Use your graph to find the cost of each book.

C. Solving Simultaneous Linear Equations Algebraically

Exercise 5A

Solve these simultaneous equations by eliminating x or y, etc.

1. $x + y = 12$
   $x - y = 8$

2 $x + y = 6$
   $x - y = 4$

3. $x + y = 10$
   $x - y = 8$

4. $x + 2y = 6$
   $x - 2y = 2$

5. $a + 4d = 9$
   $a - 4d = 1$

6. $3r + t = 10$
   $3r - t = 2$

7. $5p + q = 4$
   $2p + q = 1$

8. $6u + 6w = 6$
   $4u + 6w = 6$

9. $7x - 3y = 1$
   $4x - 3y = -2$

10. $4g - 5h = 13$
    $3g - 5h = 11$

11. $5e - 2f = 8$
    $-e + 2f = 0$

12. $-3x - 4y = 3$
    $3x + y = 6$

Exercise 5B

Solve these simultaneous equations by first multiplying both sides of the equations by suitable numbers.

1. $x + 2y = 4$
   $2x - y = 3$

2. $3a + d = 9$
   $a - 2d = 3$

3. $4e - f = 11$
   $e + 2f = 5$

4. $g + 2h = 7$
   $2g - h = 9$

5. $m + 3n = 2$
   $2m - n = 4$

6. $5p + q = 3$
   $p - 2q = 5$

7. $3r + 2s = 1$
   $r + s = 0$

8. $4t + 2u = 4$
   $t + u = 0$

9. $3v - 4w = 13$
   $v + w = 2$

10. $x - y = 4$
    $3x - 2y = 8$

11. $5x - 2y = -1$
    $x - 3y = 5$

12. $x - 3y = 1$
    $2x - y = 7$

Exercise 5C

Solve these simultaneous equations by first multiplying both sides of the equations by suitable numbers.

1. $2p - 3q = 1$
   $3p + 2q = 8$

2. $2x + 4y = 14$
   $7x + 3y = 27$

3. $2v + 3w = 0$
   $v - w = 5$

4. $7a + 4d = 1$
   $5a + 2d = -1$

5. $2r - 3s = 12$
   $3r - 2s = 13$

6. $5x - 8y = 0$
   $4x - 3y = -17$

7. $3g + 2h - 6 = 0$
   $g - h - 1 = 1$

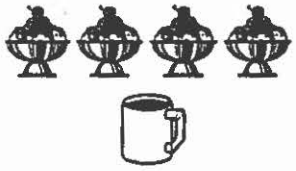
8. $3m + 5n - 23 = 0$
   $5m + 2n - 13 = 0$

9. $3f - 5g - 11 = 2$
   $2f + 4g - 9 = 7$

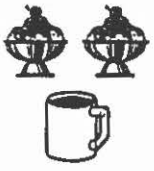
Exercise 5D

Write down a pair of simultaneous equations for each picture, then solve them to answer the question. (Use £x and £y to represent the cost of one of each item each time).

1.



Total cost £9



Total cost £5

Find the cost of: (a) one ice cream sundae. (b) one mug of cocoa.

2.

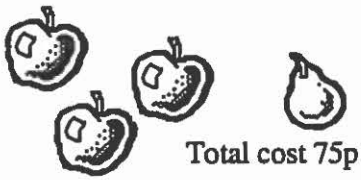
Total cost £24

Total cost £21

Find the cost of: (a) one hammer. (b) one spanner.

3. 

Total cost 55p



Total cost 75p

Find the cost of: (a) one apple. (b) one pear.

4. 

Total cost £3·50

Total cost £2·50

Find the cost of: (a) one frothy drink. (b) one slice of cake.

5. 

Total cost £90

Total cost £110

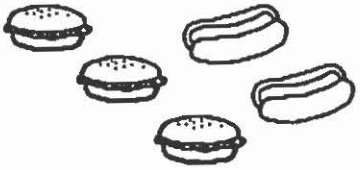
Find the cost of: (a) one football. (b) one rugby ball.

6. 

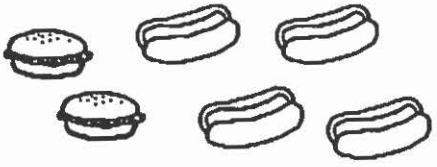
Total cost £7

Total cost £6·50

Find the cost of: (a) one disk. (b) one calculator.

7. 

Total cost £6·50



Total cost £7

Find the cost of: (a) one hot dog. (b) one hamburger.

8. At a supermarket, a lady paid £2·70 for 6 red peppers and 5 corn on the cobs.
At the same supermarket, a man paid £1·20 for 3 red peppers and 2 corn on the cobs.

Find the cost of: (a) one pepper. (b) one corn stick.

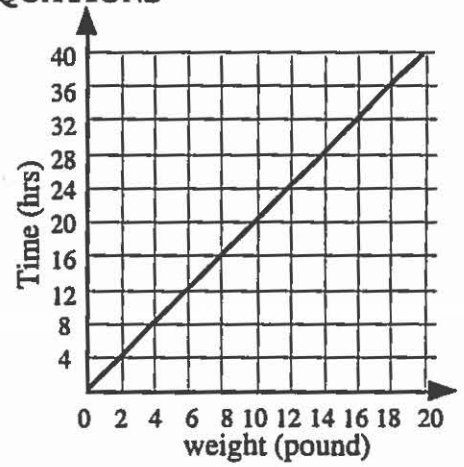
9. At a newsagent, a boy paid £1·10 for 2 memo pads and 7 pencils.
At the same shop, a girl paid £1·60 for 7 memo pads and 2 pencils.

Find the cost of: (a) one memo pad. (b) one pencil.

10. An adult's ticket for the cinema is £3 more than a child's.
The adult's ticket is also twice that of the child's.
Let the price of an adult's ticket be £x and the price of a child's ticket be £y.
Form a pair of simultaneous equations and solve them to find the price of each ticket.

CHECKUP FOR SIMULTANEOUS LINEAR EQUATIONS

1. The graph shows defrosting times at room temperature for Christmas turkey.



(a) Copy and complete the table:

Weight (W)	0	2	4	6	8	10	12	14	16	18	20
Time (T)	0	4									

(b) Write a formula for the time (T) taken to defrost a turkey if you know its weight (W).

(c) Use your formula to find the time taken to defrost a 15 pound turkey.

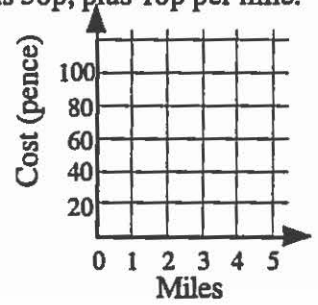
2. Pizza Point will deliver pizzas to your door. The charge is 50p, plus 10p per mile.

(a) Write down a formula for the charge C pence for a delivery of M miles.

(b) Work out the charge for a 5 mile delivery.

(c) Draw a graph of charges up to 5 miles, using the scales shown.

(d) What would be the charge for a 10 mile delivery ?



3. By drawing graphs of these equations on squared paper, solve each pair of simultaneous equations.

(a) $x + y = 8$
$y = x$

(b) $x + 2y = 7$
$4x - y = 10$

(c) $x + 3y = 0$
$x - 2y = 5$

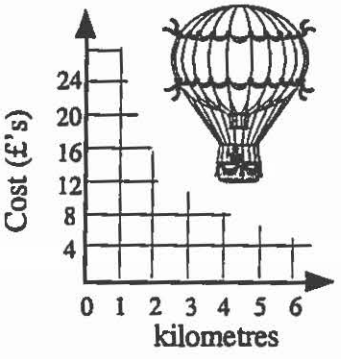
4. HIGH FLY offer balloon trips at £10 basic, plus £2 per kilometre travelled.
FLIGHT BALLOONS offer the same trips at £4 per kilometre, with no other charges.

(a) Make two tables to show the prices for up to a trip of 6 km with both companies.

(b) Draw the straight line graph for both companies on the same coordinate diagram.

(c) (i) How many kilometres can you travel for the same price at both businesses?

(ii) What price is this?



5. Terry bought a bottle of shampoo and a bottle of conditioner for £6.
Lesley bought 4 bottles of shampoo and a bottle of conditioner for £12.
Let the cost of a bottle of shampoo be £x and the cost of a bottle of conditioner be £y.

(a) Write down two equations in terms of x and y.

(b) Draw the two straight lines which the equations represent on the same coordinate diagram using suitable points on each line.

(c) Use your graph to find the cost of a bottle of shampoo and the cost of a bottle of conditioner.

6. Solve these simultaneous equations algebraically:

(a) $x + y = 20$
$x - y = 4$

(b) $x - 3y = -1$
$x + 3y = 11$

(c) $2x + y = 10$
$-2x + y = -10$

(d) $v + 3w = 7$
$2v - w = 0$

(e) $2p + 3q = 19$
$4p - 7q = -27$

(f) $2x - 3y = 1$
$3x + 2y = -5$

(g) $5s + 3t = 19$
$7s - 2t = 8$

(h) $4x - 3y - 1 = 4$
$3x + 4y - 10 = 0$

7. Write down a pair of simultaneous equations for each picture, then solve them to answer the question. (Use £x and £y to represent the cost of one of each item).

(a)

Total cost £36

Total cost £28

Find the cost of: (i) one spider. (ii) one turtle.

(b) 5 pairs of compasses and 2 pairs of scissors together cost £2·30.
3 pairs of compasses along with 3 pairs of scissors cost £2·10.

Find the cost of: (i) one pair of compasses. (ii) one pair of scissors .

8. The sum of two whole numbers is 112, and their difference is 36.
Form a pair of simultaneous equations and solve them to find the two numbers.

Exercise 4B

1. 87·3°
2. (a) 40·8° (b) 83·9° (c) 47° (d) 61·7° (e) 54·0° (f) 26·7°
3. 100·3°
4. (a) 109·5° (b) 111·8° (c) 113·3°
5. $x = 40{\cdot}1°$, $y = 57{\cdot}4°$
6. (a) x = 56·9° (b) Area = 214·8 cm^2

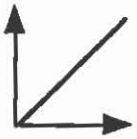
Checkup for Trigonometry

1. (a) –0·342 (b) –0·839 (c) –0·087
2. 91.6 cm^2
3. 50·7 cm^2
4. (a) 11·4 cm (b) 11·1 cm (c) 12·5 cm (d) 9·3 cm (e) 8·9 cm
   (f) 22·2 cm (g) 80·4° (h) 72·3° (i) 131·6°
5. 118·7°
6. 94·5 km
7. 199 m
8. 117·9 cm^2

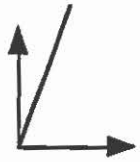
Simultaneous Linear Equations

Exercise 1

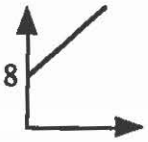
1. (a) 3 (b) W = 3N (c) 30kg (d)(e)



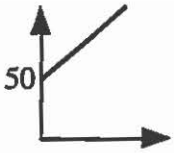
2. (a) 1/10 2/20 3/30 4/40 5/50 6/60 in table (b) 10 (c) E = 10P (d) 90
   (e) (f)



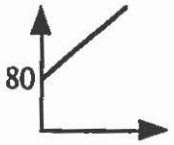
3. (a) 1/20 2/40 3/60 4/80 5/100 6/120 in table (b) T = 20W (c) 200 mins
4. (a) £57 £65 (b) C = 8h + 25
5. (a) 1/12 2/16 3/20 4/24 5/28 in table (b) C = 4D + 8 (c)
   (d) (0,8) (e) Costs £8 before even paying for any days !!



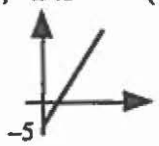
6. (a) 1/15 2/25 3/35 4/45 5/55 6/65 in table (b) T = 10W + 5
   (c) 105 mins
7. (a) C = 5k + 50 (b) £100 (c)



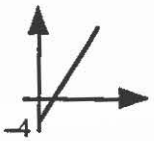
8. (a) W = 10P + 80 (b) £280 (c)



9. (a) 1/5 2/15 3/25 4/35 5/45 in table
   (b) £35; £45 (c) C = 10B – 5
   (d)



10. (a) 1/2 2/8 3/14 4/20 5/26 6/32 in table (b) T = 6B – 4 (c)



Exercise 2

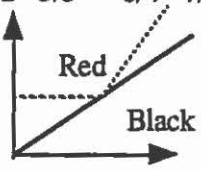
1. Graph of a straight line through (0,0), (1,1) (2,2) etc.
2. Graph of a straight line through (0,0), (1,3) (2,6) etc.
3. Graph of a straight line through (0,1), (1,2) (2,3) etc.
4. Graph of a straight line through (0,3), (1,5) (2,7) etc.
5. Graph of a straight line through (0,–1), (1,1) (2,3) etc.
6. Graph of a straight line through (0,2), (1,1) (2,0) etc.
7. Graph of a straight line through (0,5), (1,5) (2,5) etc.
8. Graph of a straight line through (3,0), (3,1) (3,2) etc.
9. Graph of a straight line through (0,6), (1,5) (2,4) etc.
10. Graph of a straight line through (0,2), (1,3) (2,4) etc.
11. Graph of a straight line through (0,0), (1,–2) (2,–4) etc.
12. Graph of a straight line through (0,1), (1,0) (2,–1) etc.

Exercise 3

1. (3,3) 2. (2,2) 3. (2,–2) 4. (5,3) 5. (1,2) 6. (–3,–1) 7. (4,1)
8. (–2,–2) 9. (4,5) 10. (–1,6) 11. (4,6) 12. (–1,3)

Exercise 4A

1. (a) Goudies 0/40 1/50 2/60 3/70 4/80 5/90 6/100 7/110
   Henry's 0/0 1/20 2/40 3/60 4/80 5/100 6/120 7/140
   (b) Straight lines crossing at (4,80)
   (c) 4 days (d) 3 days
2. (a) Dawes 0/1 1/1·50 2/2 3/2·50 4/3 5/3·50 6/4
   Beckams 0/0 1/1 2/2 3/3 4/4 5/5 6/6
   (b) Straight lines crossing at (2,2) (c) 2 hours (d) Dawes
3. (a) Rent a Computer 0/20 1/25 2/30 3/35 4/40 5/45
   Compu Hire 0/0 1/10 2/20 3/30 4/40 5/50
   (b) Straight lines crossing at (4,40) (c) 4 £40
4. (a) Black 0/0 1/0·5 2/1 3/1·5 4/2 5/2·5 6/3 7/3·5 8/4 9/4·5 10/5
   Red 0/2 1/2 2/2 3/2 4/2 5/3 6/4 7/5 8/6 9/7 10/8
   (b) Lines crossing at (4,2)
   (c) 4 miles
   (d) Black Cab



Exercise 4B

1. (a)(b) Straight lines crossing at (5,3) (c) £5 adult £3 child (d) £80
2. (a)(b) Straight lines crossing at (2,4) (c) £2 (d) £4 (e) £18
3. (a) $x + 3y = 60$ (b) Straight lines crossing at (30,10) (c) jotter 30p pencil 10p
4. (a) $x + y = 5$ $x + 3y = 9$ (b) Straight lines crossing at (3,2) (c) Weedo £3 slug £2
5. (a) $3x + 2y = 12$ $2x + 5y = 30$ (b) Straight lines crossing at (0,6) (c) shirt free dye £6
6. (a) $x + y = 10$ $x - y = 2$ (b) Straight lines crossing at (6,4) (c) £6 and £4

Exercise 5A

1. (10,2)	2. (5,1)	3. (9,1)	4. (4,1)	5. (5,1)
6. (2,4)	7. (1,–1)	8. (0,1)	9. (1,2)	10. (2,–1)
11. (2,1)	12. (3,–3)			

Exercise 5B

1. (2,1)	2. (3,0)	3. (3,1)	4. (5,1)	5. (2,0)
6. (1,–2)	7. (1,–1)	8. (2,–2)	9. (3,–1)	10. (0,–4)
11. (–1,–2)	12. (4,1)			

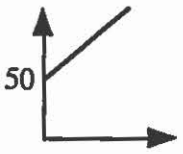
Exercise 5C

1. (2,1)	2. (3,2)	3. (3,–2)	4. (–1,2)	5. (3,–2)
6. (–8,–5)	7. (2,0)	8. (1,4)	9. (6,1)	

Exercise 5D

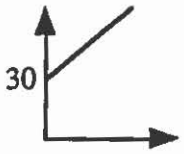
1.	$4x + y = 9$	$2x + y = 5$	ice cream £2	cocoa £1
2.	$3x + 2y = 24$	$2x + 3y = 21$	hammer £6	spanner £3
3.	$2x + y = 55$	$3x + y = 75$	apple 20p	pear 15p
4.	$2x + y = 3{\cdot}50$	$x + 2y = 2{\cdot}50$	drink £1·50	cake 50p
5.	$5x + 2y = 90$	$5x + 3y = 110$	football £10	rugby ball £20
6.	$2x + 3y = 7$	$5x + 2y = 6{\cdot}50$	disk 50p	calculator £2
7.	$3x + 2y = 6{\cdot}50$	$2x + 4y = 7$	hot dog £1	hamburger £1·50
8.	$6x + 5y = 2{\cdot}70$	$3x + 2y = 1{\cdot}20$	pepper 20p	corn 30p
9.	$2x + 7y = 1{\cdot}10$	$7x + 2y = 1{\cdot}60$	pad 20p	pencil 10p
10.	$x - y = 3$	$x = 2y$ or equivalent	adult £6	child £3

Checkup for Simultaneous Linear Equations

1. (a) 0/0 2/4 4/8 6/12 8/16 10/20 12/24 14/28 16/32 18/36 20/40 in table
   (b) T = 2W (c) 30 hours
2. (a) C = 10M + 50 (b) 100p (c) 

(d) 150p
3. (a) (4,4) (b) (3,2) (c) (3,–1)
4. (a) High Fly 0/10 1/12 2/14 3/16 4/18 5/20 6/22 in table
   Flight Balloons 0/0 1/4 2/8 3/12 4/16 5/20 6/24 in table
   (b) Straight lines crossing at (5,20) (c) 5km £20
5. (a) $x + y = 6$ $4x + y = 12$ (b) Straight lines crossing at (2,4) (c) Sham £2 Cond £4
6. (a) (12,8) (b) (5,2) (c) (5,0) (d) (1,2) (e) (2,5)
   (f) (–1,–1) (g) (2,3) (h) (2,1)
7. (a) $3x + y = 36$ $2x + y = 28$ spider £8 turtle £12
   (b) $5x + 2y = 2{\cdot}30$ $3x + 3y = 2{\cdot}10$ compasses 30p scissors 40p
8. 74 & 38

Specimen Assessment Questions

1. 25·4 cm^2
2. (a) 16·1 cm (b) 26·2° (c) 7·1 cm
3. 124 m
4. 58·0°
5. (a) £50 £70 £90 £110 £130 (b) C = 20h + 30 (c) 

6. (a) (6,4) (b) (20,30)
7. (a) $x + y = 4$ $2x + 4y = 10$ (b) Straight lines crossing at (3,1) (c) Adult £3 Child £1
8. (a) (1,–1) (b) (5,2) (c) (4,2)
9. (a) $3x + y = 2{\cdot}60$ $x + 2y = 2{\cdot}20$ coke 60p chips 80p

4. By finding a common denominator with letters, work out these additions/subtractions:

(a) $\frac{2}{x}+\frac{3}{y}$ (b) $\frac{5}{a}-\frac{2}{b}$ (c) $\frac{4}{c}+\frac{1}{d}$ (d) $\frac{1}{p}-\frac{2}{q}$

(e) $\frac{2}{v}+\frac{2}{w}$ (f) $\frac{1}{g}-\frac{1}{h}$ (g) $\frac{7}{k}+\frac{1}{n}$ (h) $\frac{1}{x}-\frac{8}{y}$

5. Add or subtract these fractions:

(a) $\frac{x+1}{3}+\frac{x+1}{2}$ (b) $\frac{x+2}{4}+\frac{x-1}{5}$ (c) $\frac{x+3}{2}+\frac{x+1}{4}$ (d) $\frac{2x-3}{5}+\frac{x+1}{3}$

(e) $\frac{x+1}{2}-\frac{x+1}{3}$ (f) $\frac{x+2}{2}-\frac{x+1}{5}$ (g) $\frac{2x+1}{2}-\frac{x+1}{4}$ (h) $\frac{x+1}{2}-\frac{x-1}{5}$

C. Changing the subject of a formula

Exercise 3A

This exercise has a mixed selection of formulae.
Change the subject of each formula to the letter shown in the brackets.
ALL WORKING and ALL STEPS SHOULD BE SHOWN.

1. $x+2=c$ (x)
2. $x-4=c$ (x)
3. $x+p=q$ (x)
4. $x-p=q$ (x)
5. $x/2=a$ (x)
6. $x/7=a$ (x)
7. $x/y=a$ (x)
8. $x/p=m$ (x)
9. $x/r=s$ (x)
10. $4x=20$ (x)
11. $4x=a$ (x)
12. $gx=h$ (x)
13. $nx=t$ (x)
14. $2x+1=5$ (x)
15. $2x+1=b$ (x)
16. $2x+c=b$ (x)
17. $ax+c=b$ (x)
18. $px+q=r$ (x)
19. $vx-w=y$ (x)
20. $D=S \times T$ (S)
21. $C=\pi d$ (d)
22. $x^2=16$ (x)
23. $x^2=y$ (x)
24. $A=\pi r^2$ (r)
25. $T=D/S$ (S)
26. $A=y^2$ (y)
27. $P=3\pi r^2$ (π)
28. $P=5\pi r^2$ (r)
29. $h-p=q$ (h)
30. $h-p=q$ (p)
31. $2h-5p=q$ (h)
32. $2h-5p=q$ (p)
33. $b-c=ax$ (x)

Exercise 3B

1. Change the subject of each formula to h.

(a) $g=hf$ (b) $e=g+h$ (c) $k=h/f$ (d) $e=g-h$

2. Change the subject of each formula to r.

(a) $Q=r^2$ (b) $N=\pi r^2$ (c) $M=2\pi r^2$ (d) $P=\pi r^2 w$

3. Change the subject of each formula to m.

(a) $A = klm$ (b) $B = Km$ (c) $C = \pi mr^2$ (d) $D = \frac{1}{3pm}$

4. Change the subject to x.

(a) $p = q + x$ (b) $r = s - x$ (c) $r = s - 5x$ (d) $r = 7x - 3$

(e) $m = 2(x + 1)$ (f) $m = {}^1/_2(x - 5)$ (g) $n = {}^1/_2(x + 2)$ (h) $p = {}^1/_2(x + q)$

5. Change the subject of the formula to the letter in brackets.

(a) $P/Q = R$ (P) (b) $t = {}^1/_s$ (s) (c) $M = P/Q^2$ (Q) (d) $v = \sqrt{\frac{w}{z}}$ (w)

(e) $d = \frac{e}{5f}$ (f) (f) $\frac{K}{mn} = T$ (n) (g) $R = \frac{7}{9s^2}$ (s) (h) $a^2 + b^2 = c^2$ (a)

6. Harder examples. Change the subject of the formula to the letter in brackets.

(a) $A + d = V/T$ (T) (b) $px + qx = r$ (x) (c) $ax = bx + c$ (x)

(d) $m = \frac{r - s}{s}$ (s) (e) $x = \frac{v - w}{v + w}$ (w) (f) $p = 2\sqrt{r} - 1$ (r)

D. Simplifying surds

Exercise 4

1. Express each of the following in its simplest form:

(a) $\sqrt{8}$ (b) $\sqrt{12}$ (c) $\sqrt{27}$ (d) $\sqrt{20}$ (e) $\sqrt{50}$ (f) $\sqrt{28}$

(g) $\sqrt{18}$ (h) $\sqrt{24}$ (i) $\sqrt{200}$ (j) $\sqrt{75}$ (k) $\sqrt{45}$ (l) $\sqrt{72}$

(m) $\sqrt{300}$ (n) $\sqrt{147}$ (o) $\sqrt{54}$ (p) $7\sqrt{8}$ (q) $5\sqrt{32}$ (r) $6\sqrt{40}$

2. Add or subtract the following:

(a) $3\sqrt{2} + 5\sqrt{2}$ (b) $6\sqrt{5} - 5\sqrt{5}$ (c) $8\sqrt{10} + 5\sqrt{10}$ (d) $9\sqrt{20} - 9\sqrt{20}$

(e) $\sqrt{6} - 3\sqrt{6}$ (f) $\sqrt{3} + \sqrt{3} - 3\sqrt{3}$ (g) $5\sqrt{7} - 8\sqrt{7} + 3\sqrt{7}$ (h) $10\sqrt{2} + 10\sqrt{3}$

3. Simplify:

(a) $\sqrt{8} - \sqrt{2}$ (b) $\sqrt{18} - \sqrt{2}$ (c) $\sqrt{125} + 5\sqrt{5}$ (d) $\sqrt{48} + \sqrt{12}$

(e) $\sqrt{45} + \sqrt{20}$ (f) $\sqrt{63} - \sqrt{28}$ (g) $\sqrt{50} + \sqrt{18}$ (h) $\sqrt{72} - \sqrt{32}$

Exercise 5

1. Simplify:

(a) $\sqrt{3} \times \sqrt{3}$ (b) $\sqrt{5} \times \sqrt{5}$ (c) $\sqrt{6} \times \sqrt{6}$ (d) $\sqrt{1} \times \sqrt{1}$

(e) $\sqrt{x} \times \sqrt{x}$ (f) $\sqrt{3} \times \sqrt{2}$ (g) $\sqrt{4} \times \sqrt{5}$ (h) $\sqrt{16} \times \sqrt{a}$

(i) $\sqrt{2} \times \sqrt{c}$ (j) $\sqrt{x} \times \sqrt{y}$ (k) $\sqrt{2} \times \sqrt{8}$ (l) $\sqrt{2} \times \sqrt{32}$

(m) $\sqrt{6} \times \sqrt{3}$ (n) $\sqrt{20} \times \sqrt{10}$ (o) $3\sqrt{2} \times \sqrt{2}$ (p) $3\sqrt{2} \times 2\sqrt{3}$

contd.

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SOLVING INEQUATIONS INVOLVING BRACKETS

TO SOLVE (IN)EQUATIONS INVOLVING BRACKETS WE MUST:-

1. MULTIPLY OUT ANY BRACKETS FIRST OF ALL.
2. TAKE ANY LETTERS TO THE LEFT-HAND SIDE OF THE (IN)EQUALITY SIGN.
3. TAKE ANY NUMBERS TO THE RIGHT-HAND SIDE OF THE (IN)EQUALITY SIGN.

EXAMPLES

(i) SOLVE $2(x-5) < 4$

EXPAND THE BRACKET

$\Rightarrow \quad 2x - 10 < 4$

TAKE THE -10 ACROSS TO THE RIGHT HAND SIDE AND CHANGE IT TO $+10$.

$\Rightarrow \quad 2x < 4 + 10$

$\Rightarrow \quad 2x < 14$

$2x$ MEANS 2 TIMES x SO WE TAKE THE 2 ACROSS THE INEQUALITY SIGN AND DIVIDE BY IT.

$\Rightarrow \quad x < \frac{14}{2}$

$\Rightarrow \quad \underline{\underline{x < 7}}$

(ii) SOLVE $3(y+2) - 5y > 12$

EXPAND THE BRACKET

$\Rightarrow \quad 3y + 6 - 5y > 12$

TAKE THE $+6$ ACROSS TO THE RIGHT HAND SIDE AND CHANGE IT TO -6.

$\Rightarrow \quad 3y - 5y > 12 - 6$

$\Rightarrow \quad -2y > 6$

$-2y$ MEANS -2 TIMES y SO WE TAKE THE -2 ACROSS THE INEQUALITY SIGN AND DIVIDE BY IT. REMEMBER TO CHANGE THE INEQUALITY SIGN.

$\Rightarrow \quad y < \frac{6}{-2}$

$\Rightarrow \quad \underline{\underline{y < -3}}$

(iii) SOLVE $-2(3x+5) \leq 3(4x+2)$

EXPAND THE BRACKETS

$\Rightarrow -6x - 10 \leq 12x + 6$

WE TAKE THE -10 OVER TO THE RIGHT-HAND SIDE AND CHANGE IT TO $+10$. THE $12x$ MOVES TO THE LEFT-HAND SIDE AND BECOMES $-12x$.

$\Rightarrow -6x - 12x \leq 6 + 10$

$\Rightarrow -18x \leq 16$

$-18x$ MEANS -18 TIMES x SO WE TAKE THE -18 ACROSS THE INEQUALITY SIGN AND DIVIDE BY IT. REMEMBER TO CHANGE THE INEQUALITY SIGN ROUND.

$\Rightarrow x \geqslant \frac{16}{-18}$

$\Rightarrow x \geqslant -\frac{8}{9}$

CANCEL DOWN

EXERCISE 15.

SOLVE THE FOLLOWING EQUATIONS OR INEQUATIONS :—

① $4(x-2) < 12$.

② $3(x+2) = -9$.

③ $2(2x+1) \geqslant 10$.

④ $7(1-2y) = -21$.

⑤ $4(3-x) \leqslant 8$.

⑥ $5(1-3t) < -25$.

⑦ $4(k-1) + k > 16$.

⑧ $3(2p+3) - 4p = 13$.

⑨ $4(3m-2) - 5m > 20$.

⑩ $2(5-3z) + 2z < -10$.

⑪ $8x - 2(3x-8) \leqslant 24$.

⑫ $5y - (3-4y) \geqslant 15$

⑬ $2(t+4) < 4t+7$

⑭ $y+7 \geqslant 5(y+3)$

⑮ $2(4x-3) = 3(5x-9)$

⑯ $5(2y-4) - (y-2) \leqslant 0$

⑰ $2(5-3x) + 11 > 3(2x-5)$

⑱ $4(8-z) = 9z - (z-4)$

8. (IN)EQUATIONS INVOLVING FRACTIONS.

BEFORE WE SOLVE EQUATIONS AND INEQUATIONS INVOLVING FRACTIONS WE MUST GO BACK OVER A TOPIC THAT WAS COVERED AT THE BEGINNING OF THE COURSE :

THE LOWEST COMMON MULTIPLE (L.C.M.) MEANS THE SMALLEST NUMBER THAT THE NUMBERS YOU ARE TALKING ABOUT WILL DIVIDE INTO EXACTLY.

EXAMPLES

(i) THE L.C.M. OF 2 AND 3 is 6.

(ii) THE L.C.M. OF 5 AND 8 is 40.

(iii) THE L.C.M. OF 4 AND 8 is 8.

(iv) THE L.C.M. OF 2, 3 AND 5 is 30.

EXERCISE 16.

WRITE DOWN THE L.C.M. OF THE FOLLOWING SETS OF NUMBERS:—

① 3 AND 4. ② 4 AND 5. ③ 3 AND 5.

④ 4 AND 7. ⑤ 2 AND 5. ⑥ 6 AND 4.

⑦ 2 AND 4. ⑧ 3 AND 6. ⑨ 2, 4 AND 5.

⑩ 2, 4 AND 6. ⑪ 3, 4 AND 6. ⑫ 4, 6 AND 8.

SOLVING (IN)EQUATIONS INVOLVING FRACTIONS

TO SOLVE (IN)EQUATIONS INVOLVING FRACTIONS WE MUST:-

1. GET RID OF ANY FRACTIONS BY MULTIPLYING THROUGHOUT BY THE L.C.M. OF THE DENOMINATORS.
2. THEN, MULTIPLY OUT ANY BRACKETS, IF THERE ARE ANY.
3. TAKE ANY LETTERS TO THE LEFT-HAND SIDE OF THE INEQUALITY SIGN AND ANY NUMBERS TO THE RIGHT-HAND SIDE.

EXAMPLE

SOLVE $\frac{2x}{3} + 5 = 7.$

$\Rightarrow \cancel{3}^{1} \times \frac{2x}{\cancel{3}_{1}} + 3 \times 5 = 3 \times 7.$ ← THERE IS ONLY 1 NUMBER IN THE DENOMINATOR WHICH IS 3. MULTIPLY EVERYTHING BY 3 AND CANCEL DOWN WHERE NECESSARY.

$\Rightarrow 2x + 15 = 21$ ← TAKE THE +15 ACROSS THE EQUALS SIGN AND CHANGE IT TO −15.

$\Rightarrow 2x = 21 - 15$

$\Rightarrow 2x = 6$ ← 2x MEANS 2 TIMES x SO WE TAKE THE 2 ACROSS AND DIVIDE BY IT.

$\Rightarrow x = \frac{6}{2}$

$\Rightarrow \underline{\underline{x = 3}}$

EXERCISE 17.

SOLVE THE FOLLOWING EQUATIONS OR INEQUATIONS:-

① $\frac{5x}{4} + 2 < 7.$ ② $\frac{2y}{3} - 1 \geq 3$ ③ $\frac{3z}{2} + 1 = \frac{11}{2}$

④ $3 - \frac{x}{5} \leq 2$ ⑤ $\frac{4p}{7} - 1 < -5$ ⑥ $12 - \frac{3t}{4} > 3$

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EXAMPLE

SOLVE $\frac{5}{4} - \frac{2x}{3} < 2$

$\Rightarrow {}^{3}12 \times \frac{5}{\cancel{4}_{1}} - {}^{4}\cancel{12} \times \frac{2x}{\cancel{3}_{1}} < 12 \times 2$

THE L.C.M OF 4 AND 3 IS **12**. SO WE MULTIPLY **EVERYTHING** BY 12 AND CANCEL DOWN WHERE NECESSARY.

$\Rightarrow 3 \times 5 - 4 \times 2x < 12 \times 2$

$\Rightarrow (15) - 8x < 24$

TAKE THE +15 ACROSS THE INEQUALITY SIGN AND CHANGE IT TO −15.

$\Rightarrow -8x < 24 - 15$

$\Rightarrow -8x < 9$

−8x MEANS −8 **TIMES** x SO WE TAKE THE −8 ACROSS AND **DIVIDE** BY IT. REMEMBER TO **CHANGE** THE INEQUALITY SIGN.

$\Rightarrow x > \frac{9}{8}$

$\Rightarrow \underline{\underline{x > -\frac{9}{8}}}$

SINCE 8 DOES NOT DIVIDE INTO 9 EXACTLY, LEAVE AS AN IMPROPER FRACTION.

SOLVE THE FOLLOWING EQUATIONS OR INEQUATIONS:—

(7) $\frac{x}{2} + \frac{3}{5} < 1$

(8) $\frac{3x}{4} + \frac{1}{2} = 5$

(9) $\frac{2t}{3} - \frac{5}{4} > -2$

(10) $\frac{4y}{5} - \frac{1}{2} = \frac{2}{3}$

(11) $\frac{3}{2} - \frac{2p}{7} \leq -2$

(12) $\frac{3}{2} - \frac{z}{4} > \frac{1}{3}$

(13) $\frac{y}{2} + \frac{3y}{5} < 4$

(14) $\frac{2x}{3} - \frac{x}{2} = -\frac{4}{5}$

(15) $\frac{2x}{5} - \frac{6x}{7} \geq \frac{1}{2}$

(16) $\frac{5y}{4} - \frac{2y}{6} < \frac{5}{12}$

(17) $\frac{3t}{4} \leq 1 - \frac{5t}{6}$

(18) $\frac{2}{3} \leq \frac{x}{2} - \frac{3x}{5}$

(19) $\frac{7p}{4} > 5 - \frac{3p}{2}$

(20) $\frac{2k}{5} < \frac{k}{4} - \frac{3}{10}$

EXAMPLE

SOLVE $\frac{2(5-3x)}{2} - \frac{7x}{6} < -5\frac{1}{2}$ ← NOTICE: SOMETHING CANCELS STRAIGHTAWAY.

$\Rightarrow \frac{(5-3x)}{2} - \frac{7x}{6} < -\frac{11}{2}$

THE L.C.M OF 6 AND 2 IS 6. SO WE MULTIPLY **EVERYTHING** BY 6 AND CANCEL DOWN WHERE NECESSARY.

$\Rightarrow \frac{3}{6} \times \frac{(5-3x)}{2_1} - \frac{1}{6} \times \frac{7x}{6_1} < -\frac{11}{2_1} \times 6^3$

$\Rightarrow 3(5-3x) - 7x < -11 \times 3$

$\Rightarrow 15 - 9x - 7x < -33$

TAKE THE +15 ACROSS THE INEQUALITY SIGN AND CHANGE IT TO −15.

$\Rightarrow -9x - 7x < -33 - 15$

$\Rightarrow -16x < -48$

−16x MEANS −16 TIMES x SO WE TAKE THE −16 ACROSS AND **DIVIDE** BY IT. REMEMBER TO **CHANGE** THE INEQUALITY SIGN.

$\Rightarrow x > \frac{-48}{-16}$

$\Rightarrow \underline{\underline{x > 3}}$

SOLVE THE FOLLOWING EQUATIONS AND INEQUATIONS :-

(21) $\frac{3x}{2} - \frac{(x-4)}{3} > 3$

(22) $\frac{2(5-x)}{5} + \frac{4x}{3} = 3$

(23) $3x - \frac{(x+5)}{2} < 4\frac{1}{3}$

(24) $\frac{3(y-2)}{4} + \frac{5y}{2} \geq \frac{5}{6}$

(25) $\frac{2p}{3} \leq \frac{(3p-1)}{2}$

(26) $\frac{(y-2)}{4} - \frac{2}{3} = \frac{(y-4)}{6}$

(27) $x - \frac{(x-2)}{7} \geq \frac{x}{2} + 1$

(28) $\frac{5t}{4} - \frac{3(4-3t)}{2} < 0$

(29) $\frac{2(4z-1)}{3} - \frac{5(1-2z)}{4} \leq -1\frac{1}{4}$

(30) $\frac{4w}{7} + \frac{(5-3w)}{2} > \frac{4(2-w)}{7}$

PG 22. EX12.

① $x<4$ 

② $y \geq 6$ 

③ $t \leq 2$ 

④ $z > 2$ 

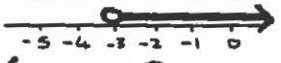
⑤ $p > 6$ 

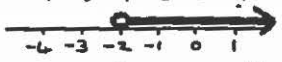
⑥ $w < 0$ 

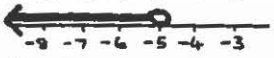
⑦ $b \geq 3$ 

⑧ $q < 7$ 

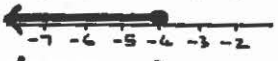
⑨ $p \leq -6$ 

⑩ $w > -3$ 

⑪ $m > -2$ 

⑫ $c < -5$ 

⑬ $k > 4$ 

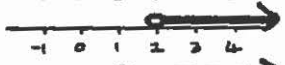
⑭ $y \leq -4$ 

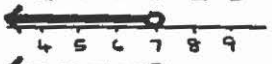
⑮ $g \geq 1$ 

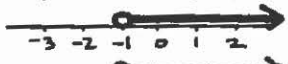
⑯ $f \leq 0$ 

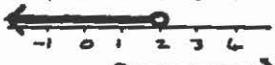
⑰ $y < 4$ 

⑱ $y < 1$ 

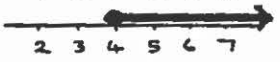
⑲ $z > 2$ 

⑳ $y < 7$ 

㉑ $x > -1$ 

㉒ $x < 2$ 

㉓ $y \geq -6$ 

㉔ $y \geq 4$ 

PG 24. EX13.

① $x<2$ ② $x>5$ ③ $y=-6$ ④ $w \leq 3$ ⑤ $t<9$ ⑥ $z \geq -4$ ⑦ $p=-3$ ⑧ $b=-2$
⑨ $k=1$ ⑩ $y<0$ ⑪ $s<\frac{5}{3}$ ⑫ $m>-2$ ⑬ $y=-6$ ⑭ $z=-5$ ⑮ $x=-6$ ⑯ $t>1$
⑰ $k \leq -2$ ⑱ $p>3$ ⑲ $w>-3$ ⑳ $x \geq -2$ ㉑ $y=-2$ ㉒ $p=0$ ㉓ $t \leq -1$ ㉔ $m=-3$
㉕ $k \geq -2$ ㉖ $y \geq 5$ ㉗ $x>13$ ㉘ $w<-1$ ㉙ $z>\frac{1}{2}$ ㉚ $y \leq \frac{1}{2}$

PG 25. EX14.

① $3x+6$ ② $5x-5$ ③ $4t+20$ ④ $7y-21$ ⑤ $10x-6$ ⑥ $6y-12$ ⑦ $15k-5$ ⑧ $12b+24$
⑨ $-2x-8$ ⑩ $-3y-3$ ⑪ $-2b+6$ ⑫ $-t-2$ ⑬ $-8y+4$ ⑭ $-10+5t$ ⑮ $-3z+1$ ⑯ $-49+14p$

PG 27. EX15.

① $x<5$ ② $x=-5$ ③ $x \geq 2$ ④ $y=2$ ⑤ $x \geq 1$ ⑥ $t>2$ ⑦ $k>4$ ⑧ $p=2$
⑨ $m>4$ ⑩ $z>5$ ⑪ $x \leq 4$ ⑫ $y \geq 2$ ⑬ $t>\frac{1}{2}$ ⑭ $y \leq -2$ ⑮ $x=3$ ⑯ $y \leq 2$
⑰ $x<3$ ⑱ $z=\frac{7}{3}$

PG 28. EX16.

① 12 ② 20 ③ 15 ④ 28 ⑤ 10 ⑥ 12 ⑦ 4 ⑧ 6
⑨ 20 ⑩ 12 ⑪ 12 ⑫ 24

PG 29. EX17.

① $x<4$ ② $y\geq 6$ ③ $z=3$ ④ $x\geq 5$ ⑤ $p<-7$ ⑥ $t<12$ ⑦ $x<\frac{4}{5}$ ⑧ $x=6$
⑨ $t>-\frac{9}{8}$ ⑩ $y=\frac{35}{24}$ ⑪ $p\geq\frac{49}{4}$ ⑫ $z<\frac{14}{3}$ ⑬ $y<\frac{40}{11}$ ⑭ $x=\frac{-24}{5}$ ⑮ $x=\frac{35}{32}$ ⑯ $y<\frac{5}{11}$
⑰ $t\geq\frac{12}{19}$ ⑱ $x\leq\frac{-3}{5}$ ⑲ $p>\frac{20}{13}$ ⑳ $k<-2$ ㉑ $x>\frac{10}{7}$ ㉒ $x=\frac{15}{4}$ ㉓ $x<\frac{41}{15}$ ㉔ $y\geq\frac{28}{39}$
㉕ $p\geq\frac{3}{5}$ ㉖ $y=6$ ㉗ $x\geq\frac{-2}{5}$ ㉘ $t<\frac{24}{23}$ ㉙ $z\leq\frac{4}{31}$ ㉚ $w<\frac{19}{5}$.

PG 33. EX18.

① $y+2$ ② $t-3$ ③ $5k$ ④ $b-7$ ⑤ $x+10$ ⑥ $3w$ ⑦ $p-1$ ⑧ $6z$
⑨ $4h+7$ ⑩ $3x-4$ ⑪ $15+k$ ⑫ $130-x$ ⑬ $p+4$ ⑭ $6k$ ⑮ $z-5$ ⑯ $t+9$
⑰ $x-7$ ⑱ $16-y$

PG 34. EX19.

① $p+6=14$ ② $w+3=7$ ③ $y-7<14$ ④ $12-x=7$
⑤ $t-9\geq 6$ ⑥ $4z=24$ ⑦ $2g=14$ ⑧ $xy<18$
⑨ $3x-4>25$ ⑩ $p+q=5$ ⑪ $17-5y\leq 21$ ⑫ $3b+6=48$
⑬ $3k\geq 15$ ⑭ $5y-4=16$ ⑮ $q=p+2$ ⑯ $10-2y>0$
⑰ $10<w<15$ ⑱ $m\geq 72$ ⑲ $p\leq 58$ ⑳ $F\geq 150$

PG 37. EX 20.

① 5 ② 3 ③ 2 ④ LESS THAN 5 ⑤ >10 ⑥ 6 < 30 ⑦ 6 ⑧ 5 < 10
⑨ 19 < 1 ⑩ > 2 ⑪ 7 ⑫ 0, 1, 2 ⑬ $0<L<8$ ⑭ $0<B<6$ and $7<L<13$ ⑮ 24, 26, 28 ⑯ 17, 18, 19
⑰ YES, -1, 2, 3 ⑱ 4, 12, 14 ⑲ P = 10, R = 27 ⑳ 14, 17 < 34

PG 40. REVISION EXERCISE

① (a) $2<5$ (b) $6>1$ (c) $p>3$ (d) $a\leq 0$ (e) $y<-2$ (f) $z\geq 6$

② (a) 2 is LESS THAN 4. (b) 3 is GREATER THAN −1. (c) Y is LESS THAN 10.
(d) t is GREATER THAN 0. (e) M is LESS THAN OR EQUAL TO 5. (f) p is GREATER THAN OR EQUAL TO 6.

③ (a) 9 LIES TO THE RIGHT OF 3, so $9>3$. (b) −5 LIES TO THE LEFT OF −2, so $-5<-2$.
(c) 7 LIES TO THE RIGHT OF 1, so $7>1$. (d) −3 LIES TO THE RIGHT OF −8, so $-3>-8$.
(e) −6 LIES TO THE LEFT OF −4, so $-6<-4$. (f) 4 LIES TO THE RIGHT OF −8, so $4>-8$.
(g) x LIES TO THE LEFT OF −5, so $x<-5$. (h) t LIES TO THE RIGHT OF 4, so $t>4$.

QUADRATIC FUNCTIONS

By the end of this unit, you should be able to:

(a) recognise quadratics of the form $y = kx^2$ and
$y = (x + a)^2 + b$; $a, b \in Z$, from their graphs

(b) identify the nature and coordinates of the turning points and the equation of the axis of symmetry of a quadratic of the form
$$y = k(x + a)^2 + b; \quad a, b \in Z, \ k = \pm 1$$

(c) know the meaning of 'root of an quadratic equation' and solve a quadratic equation graphically

(d) solve quadratic equations by factorisation

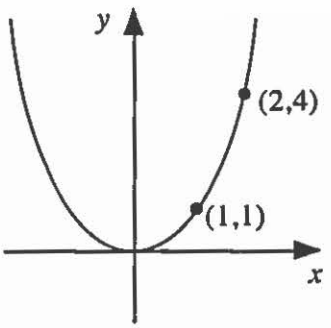
(e) solve quadratic equations by using the quadratic formula

A. Recognising quadratics from their graphs

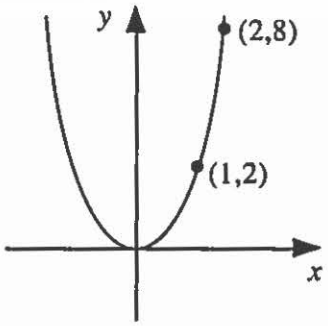
Exercise 1

1. Each of the following graphs represents a simple parabola of the form $y = kx^2$.
   Find k each time and hence write down the equation representing each parabola.

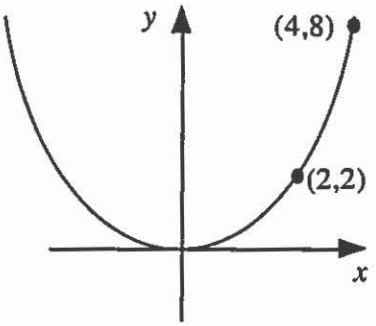
(a)



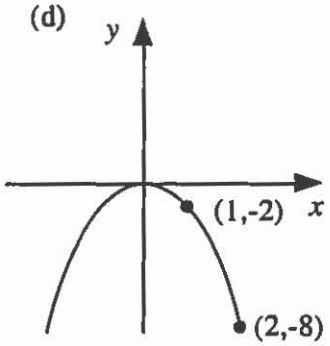
(b)



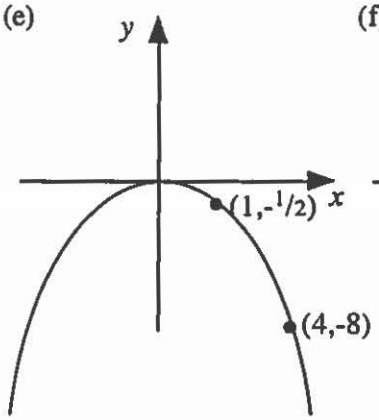
(c)



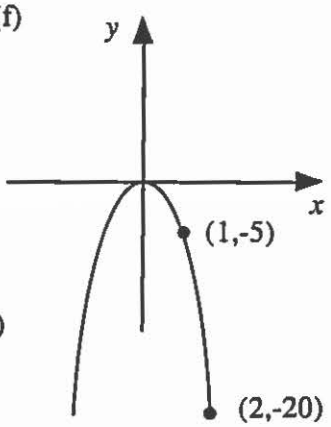
(d)



(e)

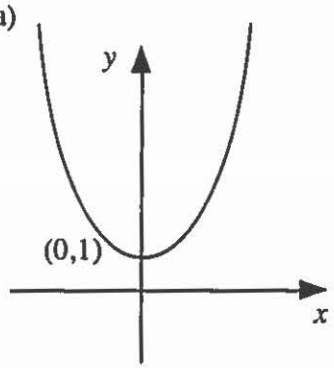


(f)

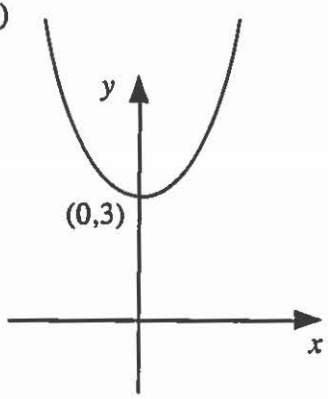


2. Each of the following parabolas can be represented by the equation
   $y = x^2 + b$ or $y = -x^2 + b$, where b is an integer. Write down their equations.

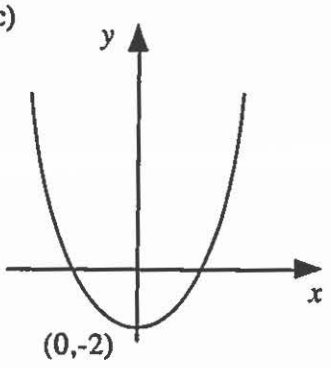
(a)



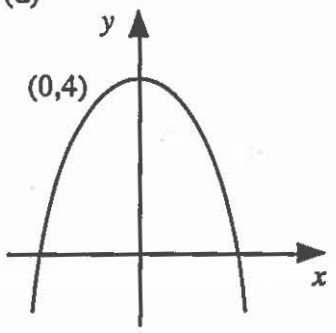
(b)



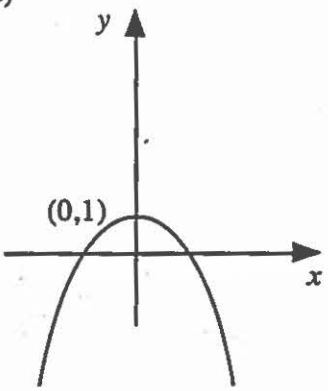
(c)



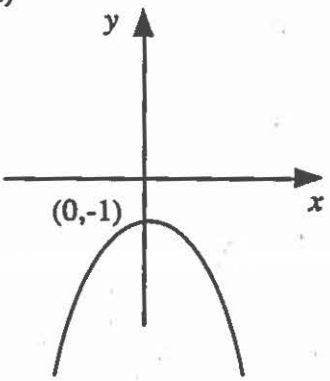
(d)



(e)

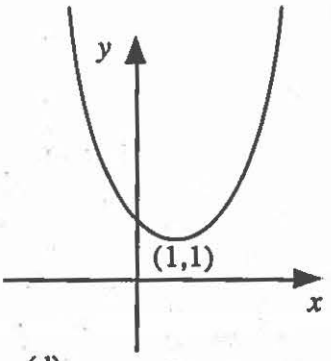


(f)

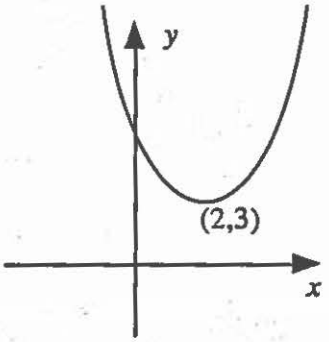


3. Each of the following parabolas can be represented by an equation of the form $y = (x + a)^2 + b$, (where a and b are integers). Write down the equation of each one.

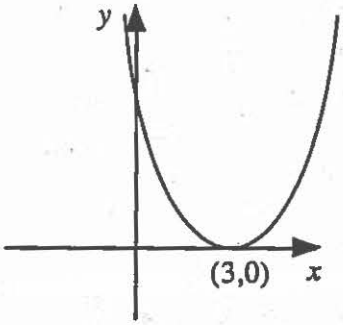
(a)



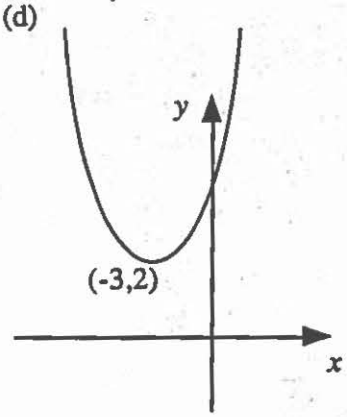
(b)



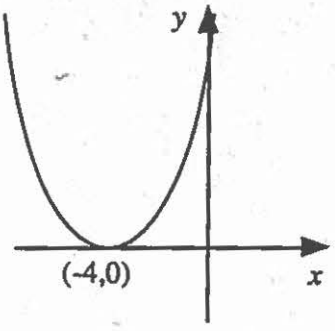
(c)



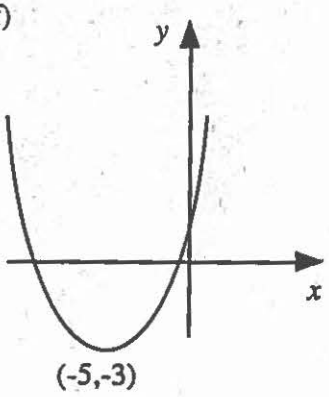
(d)



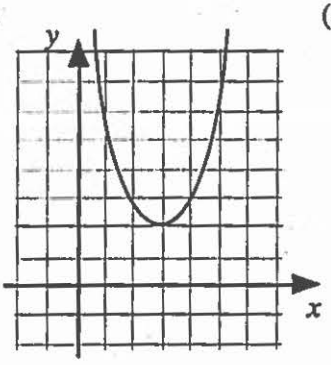
(e)



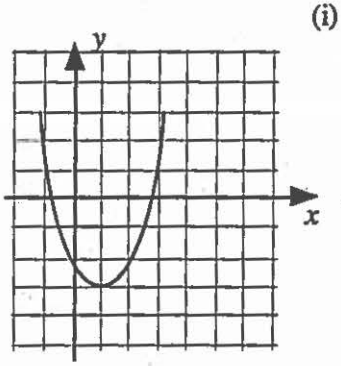
(f)



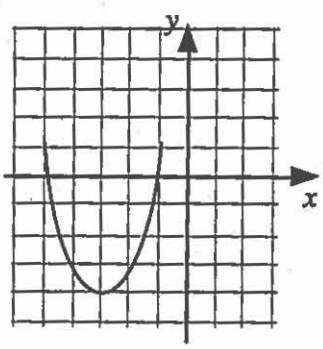
(g)



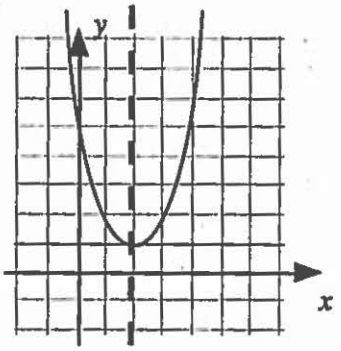
(h)



(i)



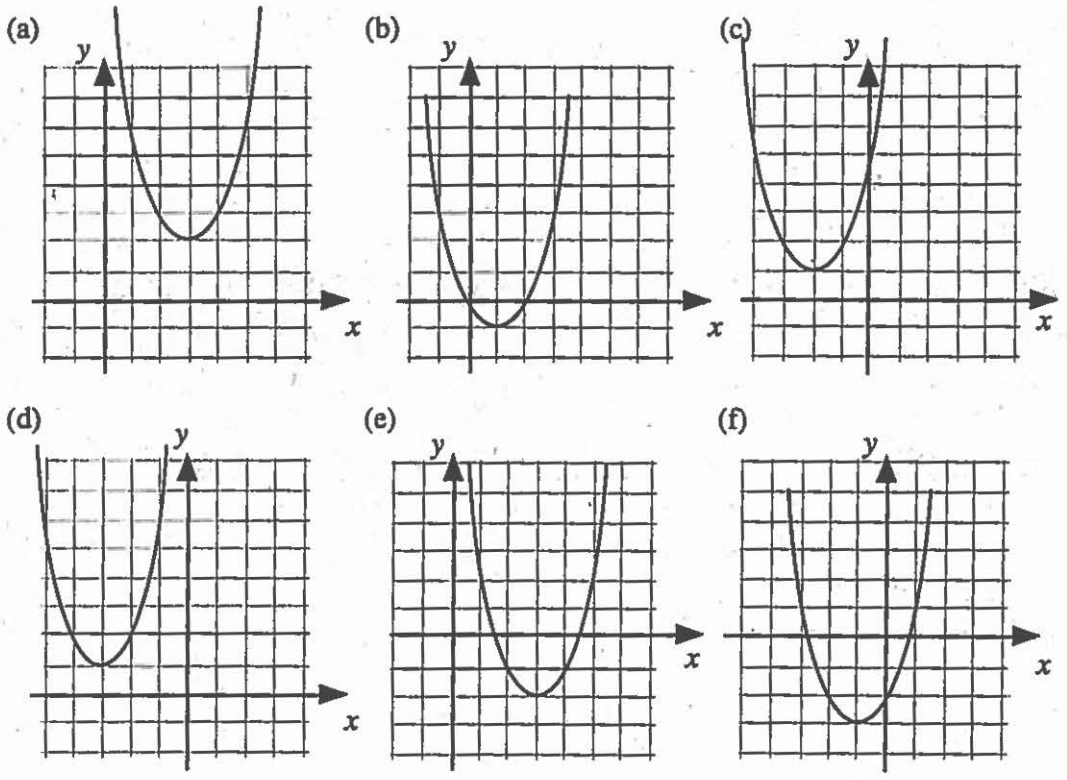
B. The nature and coordinates of turning points



Exercise 2

1. (a) Write down the coordinates of the minimum turning point of this parabola.
   (b) Write down the equation of the axis of symmetry. (the dotted line).
   (c) The equation of the parabola is of the form
   $$y = (x + a)^2 + b.$$
   Find the values of a and b and hence write down the equation of the parabola.

2. For each of the following parabolas:
   (i) write down the coordinates of its minimum turning point.
   (ii) write down the equation of its axis of symmetry.
   (iii) write down the equation of the function represented by the parabola.



3. Without making a sketch, write down the coordinates of the minimum turning points and the equation of the axes of symmetry of these parabolas.

(a) $y = (x - 4)^2 + 1$	(b) $y = (x - 2)^2 + 7$	(c) $y = (x - 8)^2 + 3$
(d) $y = (x + 1)^2 + 2$	(e) $y = (x - 1)^2 - 3$	(f) $y = (x + 3)^2 - 7$
(g) $y = (x - 5)^2$	(h) $y = (x + 2)^2$	(i) $y = x^2 + 3$

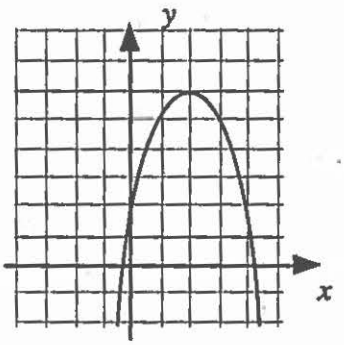
4. This parabola is of the form

$$y = -(x + a)^2 + b$$

(a) Write down the coordinates of the maximum turning point.

(b) Write down the equation of the axes of symmetry.

(c) Find the values of a and b and hence write down the equation of the function represented by the parabola.



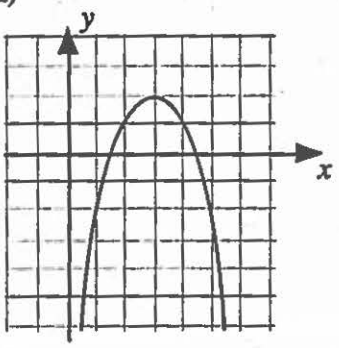
5. For each of the following parabolas:

(i) write down the coordinates of its maximum turning point.

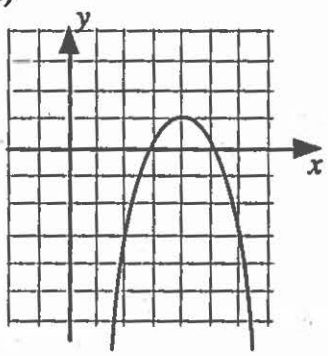
(ii) write down the equation of its axis of symmetry.

(iii) write down the equation of the function represented by the parabola.

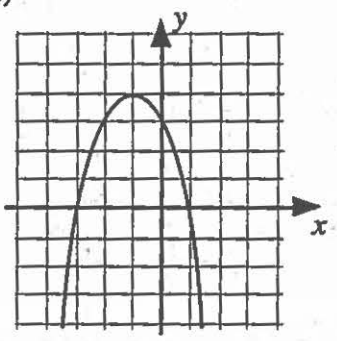
(a)



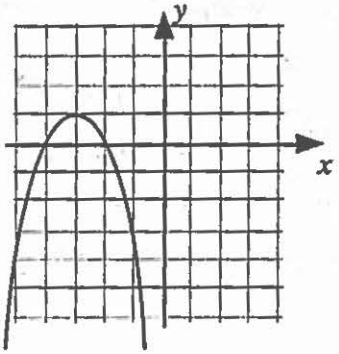
(b)



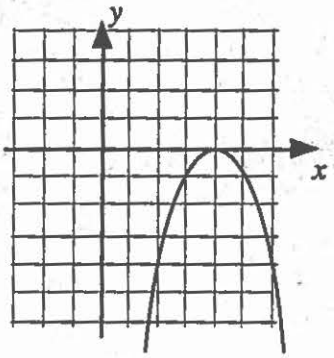
(c)



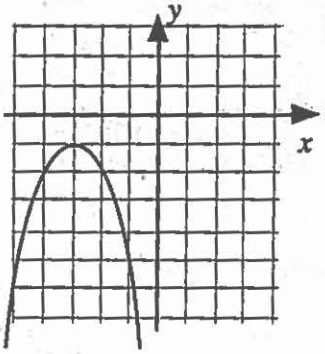
(d)



(e)



(f)



6. Without a sketch, write down the coordinates of the maximum turning points and the equation of the axes of symmetry of these parabolas.

(a) $y = -(x - 2)^2 + 6$ (b) $y = -(x - 5)^2 + 1$ (c) $y = -(x - 6)^2 - 2$

(d) $y = -(x + 1)^2 + 7$ (e) $y = -(x + 4)^2 - 5$ (f) $y = -(x + 3)^2$

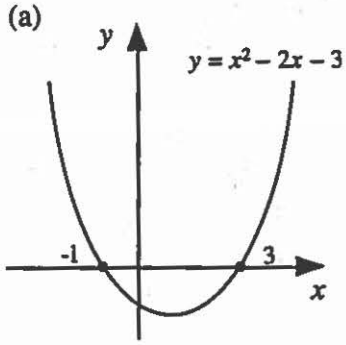
(g) $y = 7 - (x - 1)^2$ (h) $y = 1 - (x - 8)^2$ (i) $y = -2 - (x + 5)^2$

C. Solve quadratic equations graphically

Exercise 3

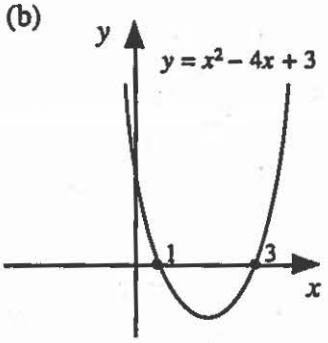
1. In each of the following, the graph has been sketched for you.
   Solve the quadratic equation associated with it.

(a)



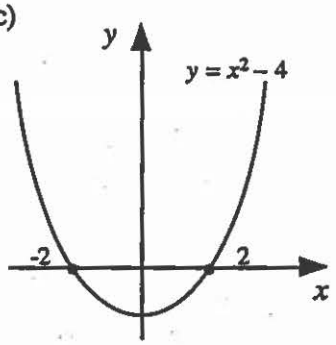
Solve $x^2 - 2x - 3 = 0$

(b)



Solve $x^2 - 4x + 3 = 0$

(c)

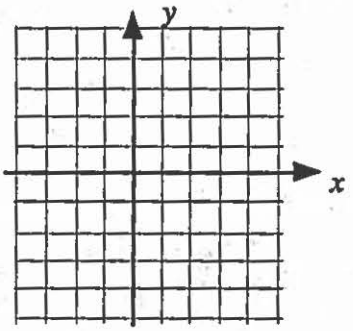


Solve $x^2 - 4 = 0$

2. For the quadratic function $y = x^2 - x - 2$

   (a) Copy and complete this table.

x	–2	–1	0	1	2	3
$y = x^2 - x - 2$	4	…	–2	…	…	…



   (b) Draw a set of axes, plot the 6 points above and draw a smooth (parabolic) curve through them.

   (c) Use your graph to solve the quadratic equation,
   $x^2 - x - 2 = 0$ [i.e. find the 2 roots]

3. Solve each of the following quadratic equations by:
   (i) completing the table
   (ii) plotting the points and drawing the smooth parabola
   (iii) reading off the roots from the graph.

(a) $x^2 - 4x = 0$

x	–1	0	1	2	3	4	5
$y = x^2 - 4x$	5	…	–3	…	…	…	…

(b) $x^2 + x - 2 = 0$

x	–3	–2	–1	0	1	2
$y = x^2 + x - 2$	4	…	–2	…	…	…

(c) $x^2 - 6x + 8 = 0$

x	0	1	2	3	4	5
$y = x^2 - 6x + 8$	8	…	…	…	…	…

(d) $6 - x - x^2 = 0$

x	–4	–3	–2	–1	0	1	2	3
$y = 6 - x - x^2$	–6	…	4	…	…	…	…	…

(e) $x^2 - 4 = 0$

x	–3	–2	–1	0	1	2	3
$y = x^2 - 4$	5	…	…	–4	…	…	…

(f) $5 + 4x - x^2 = 0$

x	–2	–1	0	1	2	3	4	5	6
$y = 5 + 4x - x^2$	…	…	…	…	…	…	…	…	…

D. Solve quadratic equations by factorising

Exercise 4

In this exercise you are going to solve quadratic equations by factorisation.

1. By considering a common factor, factorise and solve the following:

(a) $x^2 - 4x = 0$ (b) $x^2 - 10x = 0$ (c) $8x - x^2 = 0$

(d) $x^2 + 6x = 0$ (e) $x^2 + x = 0$ (f) $x^2 - x = 0$

(g) $2x^2 - 6x = 0$ (h) $5x^2 + 15x = 0$ (i) $12x^2 - 18x = 0$

2. By considering the difference of 2 squares, factorise and solve the following quadratic equations:

(a) $x^2 - 4 = 0$ (b) $x^2 - 9 = 0$ (c) $x^2 - 25 = 0$

(d) $16 - x^2 = 0$ (e) $x^2 - 100 = 0$ (f) $x^2 = 49$

(g) $x^2 = 81$ (h) $x^2 - 9 = 0$ (i) $25x^2 - 16 = 0$

3. Factorise the following trinomials and solve the quadratic equations:

(a) $x^2 + 3x + 2 = 0$ (b) $x^2 - 5x + 6 = 0$ (c) $x^2 + 6x + 5 = 0$

(d) $x^2 - 9x + 20 = 0$ (e) $x^2 + 7x + 10 = 0$ (f) $x^2 - 6x + 9 = 0$

(g) $x^2 - 7x + 12 = 0$ (h) $x^2 - 8x + 7 = 0$ (i) $x^2 - 13x + 42 = 0$

(j) $x^2 + 3x - 10 = 0$ (k) $x^2 - 3x - 4 = 0$ (l) $x^2 + 2x - 8 = 0$

(m) $x^2 - x - 20 = 0$ (n) $x^2 + x - 12 = 0$ (o) $x^2 + 2x - 35 = 0$

(p) $x^2 + 4x - 12 = 0$ (q) $x^2 + 3x - 18 = 0$ (r) $x^2 + 21x + 20 = 0$

(s) $x^2 - 9x + 8 = 0$ (t) $x^2 - 10x - 24 = 0$ (u) $x^2 + 5x - 24 = 0$

(v) $x^2 - 2x - 24 = 0$ (w) $x^2 - 23x - 24 = 0$ (x) $x^2 - 15x + 54 = 0$

4. The following are harder and take a little longer to do.
Solve:

(a) $2x^2 + 7x + 3 = 0$ (b) $2x^2 + 5x + 3 = 0$ (c) $3x^2 + 7x + 2 = 0$

(d) $2x^2 - 9x + 9 = 0$ (e) $3x^2 + 11x + 6 = 0$ (f) $5x^2 + 11x + 2 = 0$

(g) $3x^2 - 2x - 8 = 0$ (h) $3x^2 - 5x - 2 = 0$ (i) $3x^2 + 2x - 1 = 0$

(j) $2x^2 - 7x - 4 = 0$ (k) $5x^2 + 13x - 6 = 0$ (l) $2x^2 + 9x + 10 = 0$

5. Rearrange the following into quadratic equations of the form $ax^2 + bx + c = 0$ and solve them:

(a) $x(x + 2) = 3$ (b) $x(x - 1) = 20$ (c) $x(x - 3) = 10$

(d) $x(x - 5) = 6$ (e) $x(x + 3) = 70$ (f) $x(x + 1) = 56$

(g) $(x + 1)(x + 2) = 12$ (h) $(x - 1)(x + 2) = 28$ (i) $(x + 3)(x - 1) = 5$

(j) $(x - 1)(x + 1) = 8$ (k) $(x - 2)(x + 2) = 21$ (l) $(x - 2)(x - 3) = 2$

(m) $2x^2 + 3x - 1 = x^2 - x - 4$ (n) $3x^2 + 5x - 8 = 2x^2 + 2x + 2$

(o) $2x(x + 2) = 16$ (p) $x(2x - 3) = 20$ (q) $x(3x - 1) = 10$

(r) $x(5x + 2) = 7$ (s) $(2x - 1)(x + 3) = 30$ (t) $(x + 1)(3x - 2) = 12$

E. Solve quadratic equations using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise 5

1. Solve the following quadratic equations using the formula, correct to 2 decimal places:

(a) $x^2 + 4x + 1 = 0$ (b) $x^2 + 3x + 1 = 0$ (c) $x^2 + 6x + 2 = 0$

(d) $x^2 + 10x + 7 = 0$ (e) $x^2 - 4x + 2 = 0$ (f) $x^2 - 5x + 5 = 0$

(g) $2x^2 + 7x + 2 = 0$ (h) $3x^2 + 10x + 5 = 0$ (i) $4x^2 - 7x + 2 = 0$

2. Solve the following quadratic equations using the formula, correct to 3 decimal places:

(a) $x^2 + 2x - 2 = 0$ (b) $x^2 + x - 5 = 0$ (c) $x^2 + 3x - 6 = 0$

(d) $x^2 - 2x - 5 = 0$ (e) $x^2 - 3x - 1 = 0$ (f) $x^2 + 8x - 2 = 0$

(g) $2x^2 + 3x - 4 = 0$ (h) $3x^2 - 2x - 2 = 0$ (i) $8x^2 - 3x - 1 = 0$

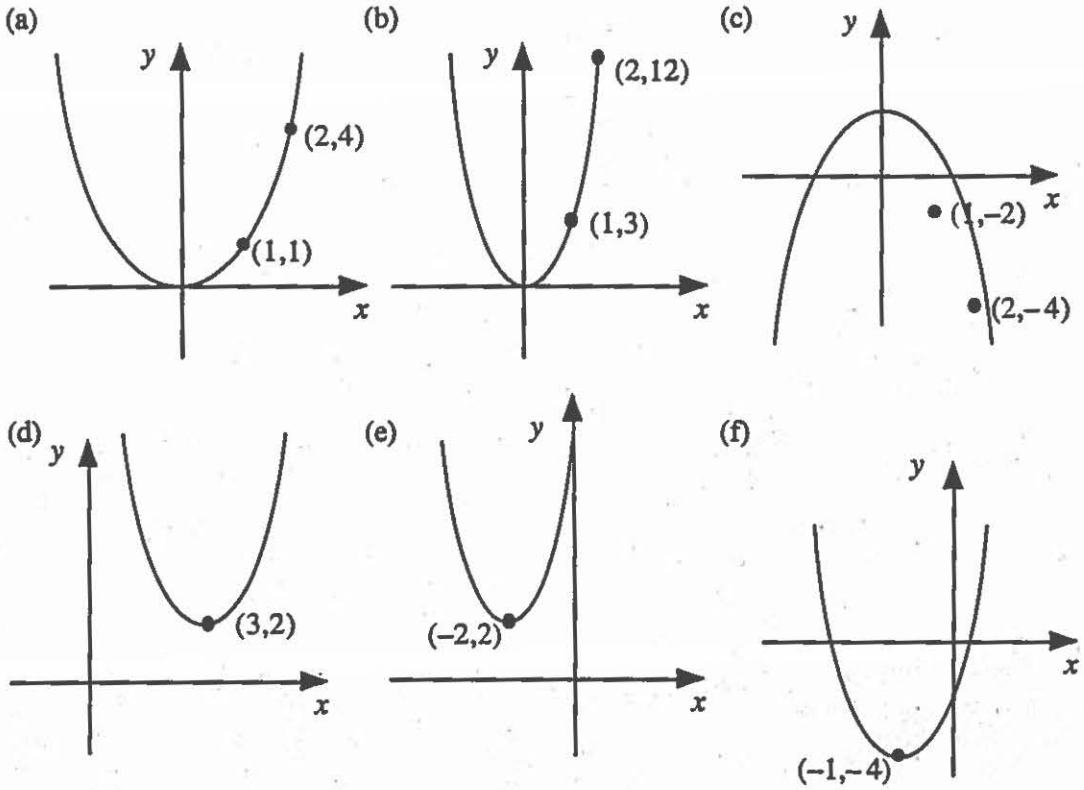
3. Rearrange the following and solve each equation giving the roots correct to 3 significant figures (3 figures of accuracy):

(a) $x(x + 5) = 2$ (b) $x(x - 3) = 1$ (c) $2x(x + 3) = 5$

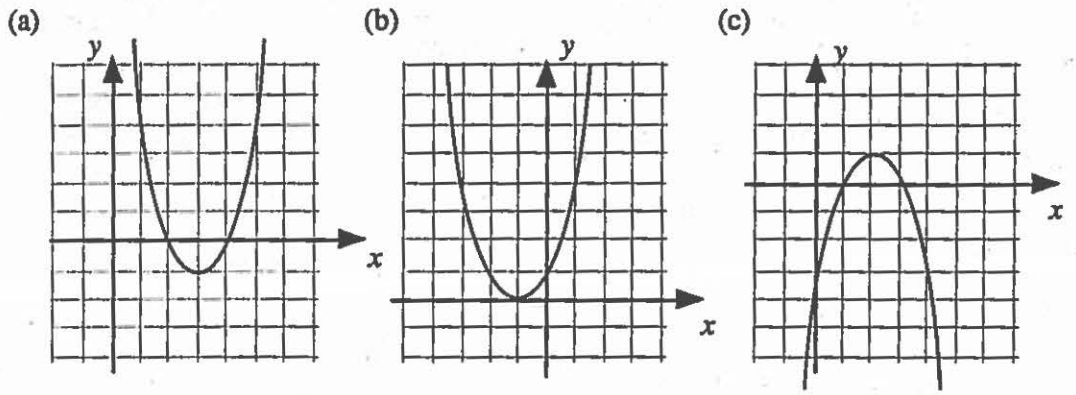
(d) $3x(x - 1) = 10$ (e) $(x + 3)(x + 4) = 7$ (f) $(x - 1)^2 = 5$

Checkup for quadratic equations

1. Write down the equations of the quadratics corresponding to these parabolas:
   (each one is of the form $y = kx^2$ or $y = (x + a)^2 + b$)



2. (i) Write down the coordinates of the turning point for each of the following, stating whether it is a maximum or a minimum.
   (ii) Write down the equation of the axes of symmetry for each one.



3. Write down the coordinates of the turning point of each of the following.
State whether each is a maximum or a minimum.
Give also the equation of the axes of symmetry.

(a) $y = (x - 21)^2 + 5$ (b) $y = 3(x + 2)^2 - 1$ (c) $y = -2(x - 3)^2 + 2$

4. Solve each of the following quadratic equations by
   (i) completing the table.
   (ii) plotting the points and drawing the smooth parabola.
   (iii) reading off the roots from the graph.

(a) $x^2 - 2x - 8 = 0$

x	–3	–2	–1	0	1	2	3	4	5
$y = x^2 - 2x - 8$	7	…	…	–8	…	…	…	…	…

(b) $x^2 + 2x - 3 = 0$

x	–4	–3	–2	–1	0	1	2
$y = x^2 + 2x - 3$	…	…	–3	…	…	…	…

5. Solve the following quadratic equations by factorising:

(a) $x^2 - 7x = 0$ (b) $x^2 - 9 = 0$ (c) $x^2 + 8x + 12 = 0$

(d) $6x^2 + 9x = 0$ (e) $25 - x^2 = 0$ (f) $x^2 - x - 30 = 0$

(g) $4x^2 - 9 = 0$ (h) $x^2 - 7x + 10 = 0$ (i) $2x^2 + 7x - 15 = 0$

(j) $x(x + 5) = 14$ (k) $(x - 1)(x + 2) = 18$ (l) $(x - 2)^2 = 16$

6. Solve the following quadratic equations using the formula.
(Give the answers correct to 2 decimal places):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(a) $x^2 + 6x + 3 = 0$ (b) $x^2 - 4x + 1 = 0$ (c) $3x^2 + 10x + 6 = 0$

(d) $x^2 + 5x - 2 = 0$ (e) $x^2 - 6x - 4 = 0$ (f) $2x^2 - x - 5 = 0$

Checkup for algebraic operations

1. (a) $1/2$ (b) $3a/b$ (c) $1/(x-3)$ (d) $v-w$ (e) $\frac{x+y}{2x+3y}$ (f) $1/(a+8)$ (g) $(v+1)/(v-2)$
2. (a) $b^2/2$ (b) a (c) $3z$ (d) a (e) $3c/8$ (f) $(15u-8v)/20$ (g) $(3m+k)/km$ (h) $(3x+1)/10$
3. (a) $x=1$ (b) $x=w+a$ (c) $x=(p-m)/a$ (d) $x=pz/w$ (e) $x=\sqrt{(N/2\pi)}$ (f) $x=(T-3)/4$ (g) $x=2w-y$ (h) $x=\sqrt{(5/8M)}$
4. (a) $4\sqrt{2}$ (b) $10\sqrt{10}$ (c) $6\sqrt{5}$ (d) $\sqrt{5}$ (e) $2\sqrt{2}$ (f) $11\sqrt{2}$ (g) 3/a (h) $3\sqrt{2}$
5. (a) 4 (b) −10 (c) 8
6. (a) $\sqrt{5}/5$ (b) $4\sqrt{2}$ (c) $3\sqrt{5}$ (d) $\sqrt{3}/3$
7. (a) 5^{14} (b) x^2 (c) $6m^4$ (d) a^6 (e) $16a^6b^2$ (f) a^4 (g) 1 (h) x^5-x^4
8. (a) $1/5^2$ (b) a/b^3 (c) y^6 (d) $(x^2)/4$
9. (a) $\sqrt{b}$ (b) $1/\sqrt{c^3}$
10. (a) $x^{4/3}$ (b) $a^{-3/2}$
11. (a) 216 (b) $1/4$ (c) x^4 (d) $1/y^7$ (e) $1/a-1/a^4$ (f) $s/2$ (g) x

Quadratic functions

Exercise 1

1. (a) $y=x^2$ (b) $y=2x^2$ (c) $y=1/2x^2$
   (d) $y=-2x^2$ (e) $y=-1/2x^2$ (f) $y=-5x^2$
2. (a) $y=x^2+1$ (b) $y=x^2+3$ (c) $y=x^2-2$
   (d) $y=-x^2+4$ (e) $y=-x^2+1$ (f) $y=-x^2-1$
3. (a) $y=(x-1)^2+1$ (b) $y=(x-2)^2+3$ (c) $y=(x-3)^2$
   (d) $y=(x+3)^2+2$ (e) $y=(x+4)^2$ (f) $y=(x+5)^2-3$
   (g) $y=(x-3)^2+2$ (h) $y=(x-1)^2-4$ (i) $y=(x+3)^2-4$

Exercise 2

1. (a) (2,1) (b) $x=2$ (c) $y=(x-2)^2+1$
2. (a) (i) (3,2) (ii) $x=3$ (iii) $y=(x-3)^2+2$
   (b) (i) (1,−1) (ii) $x=1$ (iii) $y=(x-1)^2-1$
   (c) (i) (−2,1) (ii) $x=-2$ (iii) $y=(x+2)^2+1$
   (d) (i) (−3,1) (ii) $x=-3$ (iii) $y=(x+3)^2+1$
   (e) (i) (3,−2) (ii) $x=3$ (iii) $y=(x-3)^2-2$
   (f) (i) (−1,−3) (ii) $x=-1$ (iii) $y=(x+1)^2-3$
3. (a) (4, 1); $x=4$ (b) (2,7); $x=2$ (c) (8,3); $x=8$
   (d) (−1,2); $x=-1$ (e) (1,−3); $x=1$ (f) (−3,−7); $x=-3$
   (g) (5,0); $x=5$ (h) (−2,0); $x=-2$ (i) (0,3); $x=0$

4. (a) (2,6) (b) $x = 2$ (c) $y = -(x-2)^2 + 6$

5. (a) (i) (3,2) (ii) $x = 3$ (iii) $y = -(x-3)^2 + 2$
   (b) (i) (4,1) (ii) $x = 4$ (iii) $y = -(x-4)^2 + 1$
   (c) (i) (–1,4) (ii) $x = -1$ (iii) $y = -(x+1)^2 + 4$
   (d) (i) (–3,1) (ii) $x = -3$ (iii) $y = -(x+3)^2 + 1$
   (e) (i) (4,0) (ii) $x = 4$ (iii) $y = -(x-4)^2$
   (f) (i) (–3,–1) (ii) $x = -3$ (iii) $y = -(x+3)^2 - 1$

6. (a) (2,6) ; $x = 2$ (b) (5,1) ; $x = 5$ (c) (6,–2) ; $x = 6$
   (d) (–1,7) ; $x = -1$ (e) (–4,–5) ; $x = -4$ (f) (–3,0) ; $x = -3$
   (g) (1,7) ; $x = 1$ (h) (8,1) ; $x = 8$ (i) (–5,–2) ; $x = -5$

Exercise 3

1. (a) $x = -1, 3$ (b) $x = 1, 3$ (c) $x = 2, -2$
2. (a) 4, 0, –2, –2, 0, 4 (b) graph (c) $x = -1, 2$
3. (a) 0, 4 (b) 1, –2 (c) 2, 4
   (d) –3, 2 (e) 2, –2 (f) 5, –1

Exercise 4

1. (a) 0, 4 (b) 0, 10 (c) 0, 8
   (d) 0, –6 (e) 0, –1 (f) 0, 1
   (g) 0, 3 (h) 0, –3 (i) 0, $^3/_2$

2. (a) 2, –2 (b) 3, –3 (c) 5, –5
   (d) 4, –4 (e) 10, –10 (f) 7, –7
   (g) 9, –9 (h) 3, –3 (i) $^4/_5, -^4/_5$

3. (a) –1, –2 (b) 2, 3 (c) –1, –5
   (d) 5, 4 (e) –2, –5 (f) 3
   (g) 3, 4 (h) 1, 7 (i) 6, 7
   (j) –5, 2 (k) 4, –1 (l) –4, 2
   (m) 5, –4 (n) –4, 3 (o) –7, 5
   (p) –6, 2 (q) –6, 3 (r) –1, –20
   (s) 1, 8 (t) 12, –2 (u) –8, 3
   (v) 6, –4 (w) 24, –1 (x) 6, 9

4. (a) $-3, -^1/_2$ (b) $-1, -^3/_2$ (c) $-2, -^1/_3$
   (d) $3, ^3/_2$ (e) $-3, -^2/_3$ (f) $-2, -^1/_5$
   (g) $2, -^4/_3$ (h) $2, -^1/_3$ (i) $-1, ^1/_3$
   (j) $4, -^1/_2$ (k) $-3, ^2/_5$ (l) $-2, -^5/_2$

5. (a) 1, –3 (b) 5, –4 (c) 5, –2
   (d) 6, –1 (e) 7, –10 (f) 7, –8
   (g) 2, –5 (h) 5, –6 (i) 2, –4
   (j) 3, –3 (k) 5, –5 (l) 1, 4
   (m) –1, –3 (n) 2, –5 (o) 2, –4
   (p) $4, -^5/_2$ (q) $2, -^5/_3$ (r) $1, -^7/_5$
   (s) $3, -^{11}/_2$ (t) $2, -^7/_3$

Exercise 5

1. (a) –0·27, –3·73 (b) –0·38, –2·62 (c) –0·35, –5·65
   (d) –0·76, –9·24 (e) 3·41, 0·59 (f) 3·62, 1·38
   (g) –0·31, –3·19 (h) –0·61, –2·72 (i) 1·39, 0·36
2. (a) 0·732, –2·732 (b) 1·791, –2·791 (c) 1·372, –4·372
   (d) 3·449, –1·449 (e) 3·303, –0·303 (f) 0·243, –8·243
   (g) 0·851, –2·351 (h) 1·215, –0·549 (i) 0·588, –0·213
3. (a) 0·372, –5·37 (b) 3·30, –0·303 (c) 0·679, –3·68
   (d) 2·39, –1·39 (e) –0·807, –6·19 (f) 3·24, –1·24

Checkup for quadratic equations

1. (a) $y = x^2$ (b) $y = 3x^2$ (c) $y = -2x^2$
   (d) $y = (x-3)^2 + 2$ (e) $y = (x+2)^2 + 2$ (f) $y = (x+1)^2 - 4$
2. (a) minimum at (3, –1) ; $x = 3$ (b) minimum at (–1, 0) ; $x = -1$
   (c) maximum at (2, 1) ; $x = 2$
3. (a) minimum at (21, 5) ; $x = 21$ (b) minimum at (–2, –1) ; $x = -2$
   (c) maximum at (3,2) ; $x = 3$
4. (a) graph and $x = 4$ or $x = -2$ (b) graph and $x = 1$ or $x = -3$
5. (a) 0, 7 (b) 3, –3 (c) –2, –6 (d) 0, $-3/2$ (e) 5, –5 (f) 6, –5
   (g) $3/2$, $-3/2$ (h) 2, 5 (i) –5, $3/2$ (j) 2, –7 (k) 4, –5 (l) 6, –2
6. (a) –0·55, –5·45 (b) 3·73, 0·27 (c) –0·78, –2·55
   (d) 0·37, –5·37 (e) 6·61, –0·61 (f) 1·85, –1·35