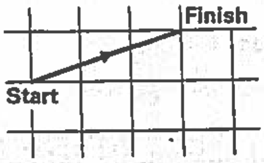
N5 Maths, Application (Part 1)

In this booklet:

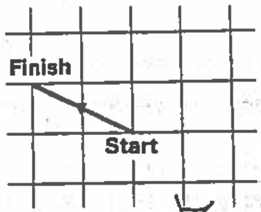
VECTORS

The lines show the distance and the direction of the translation.

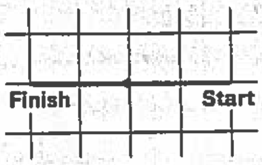
This diagram shows the translation $\begin{pmatrix}3\\1\end{pmatrix}$.

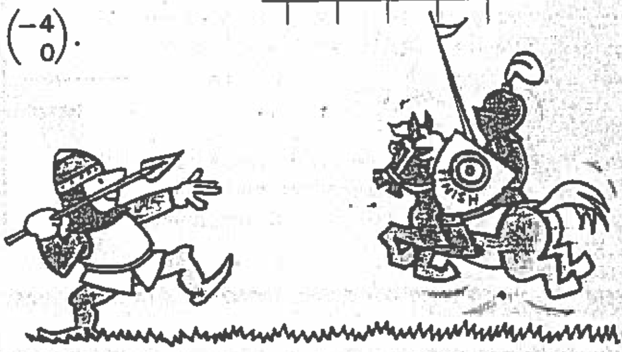


This diagram shows the translation $\begin{pmatrix}-2\\1\end{pmatrix}$.



This diagram shows the translation $\begin{pmatrix}-4\\0\end{pmatrix}$.

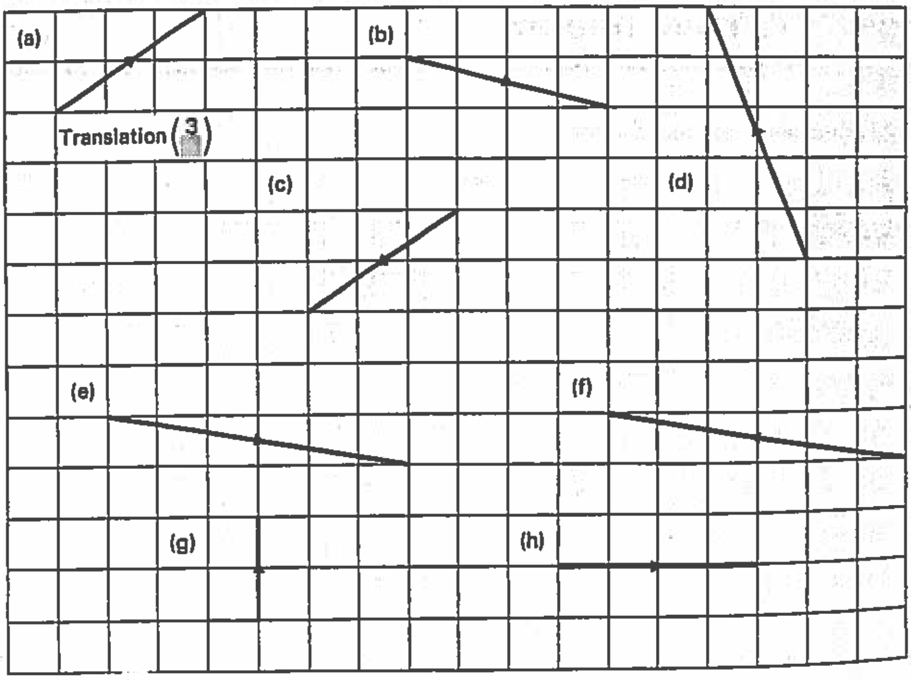




Note: Always make arrow point to Finish.

Exercise

1 Copy the following diagrams on 1 cm squared paper and write the components of each translation beside your drawing. Remember to show the arrow heads.



2 Draw diagrams on squared paper to show the following translations. Remember the arrow heads.

(a) $\begin{pmatrix}4\\3\end{pmatrix}$ (b) $\begin{pmatrix}2\\5\end{pmatrix}$ (c) $\begin{pmatrix}-1\\5\end{pmatrix}$ (d) $\begin{pmatrix}-3\\-3\end{pmatrix}$

(e) $\begin{pmatrix}0\\6\end{pmatrix}$ (f) $\begin{pmatrix}7\\0\end{pmatrix}$ (g) $\begin{pmatrix}-3\\2\end{pmatrix}$ (h) $\begin{pmatrix}4\\-1\end{pmatrix}$

The number pairs, like (a) to (h) above, which we have used to describe translations are called vectors.

Continue with Section I

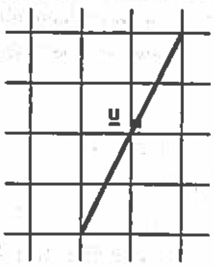
I Labelling vectors

Vectors can be used to describe other quantities in mathematics such as forces, velocities, accelerations.

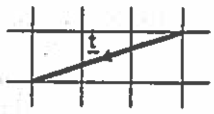
Sometimes we use underlined letters to stand for vectors. For example: $\underline{v}$, $\underline{a}$, $\underline{d}$, $\underline{w}$, $\underline{s}$, $\underline{t}$.

Suppose that this shows the vector $\underline{u}$.

We say that $\underline{u} = \begin{pmatrix}2\\4\end{pmatrix}$.

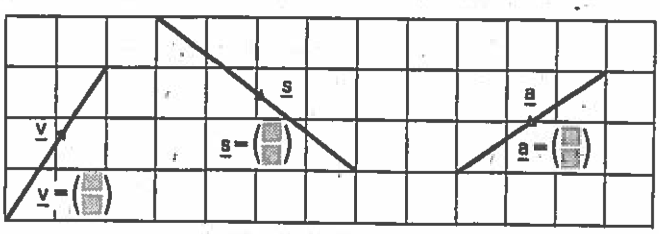


If this shows the vector $\underline{t}$, we say that $\underline{t} = \begin{pmatrix}-3\\-1\end{pmatrix}$.



Exercise

1 Copy on 1 centimetre squared paper the drawings showing these vectors and write down their components.



2 On 1 cm squared paper, draw lines to show the following vectors.
(Remember the arrow heads, and write the correct letter along each line.)

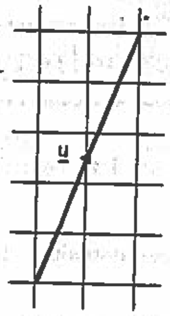
$\underline{u} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ $\underline{w} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$\underline{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -5 \\ -4 \end{pmatrix}$ $\underline{c} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

Continue with Section J

J Length of a vector—by measurement

The vector $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ is shown by a line like this. ⟶



The length of the line in centimetres is about 5.4 units (measure it!).

We say that the length of the vector u is about 5.4 units.

Exercise

1 Look back at the drawings of the boat and the plane on page 39, Section H. Measure the lengths of the straight lines showing how far the boat and the plane have moved.
If 1 unit stands for 1 km you should find that the boat has moved about 4.5 km and the plane has moved about 4.2 km.

2 Copy and complete the following by measuring the lines you drew to show the vectors in question 2 of Section I.

$\underline{u} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ Length of $\underline{u}$ = 5.4 units.

$\underline{w} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ Length of $\underline{w}$ = □ units.

$\underline{v} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ Length of $\underline{v}$ = □ units.

$\underline{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ Length of $\underline{a}$ = □ units.

$\underline{b} = \begin{pmatrix} -5 \\ -4 \end{pmatrix}$ Length of $\underline{b}$ = □ units.

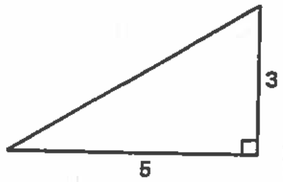
$\underline{c} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$ Length of $\underline{c}$ = □ units.

Continue with Section K

K Length of a vector—by calculation

We can find the length of a vector without measuring it if we know its components.

Using Pythagoras' Theorem in any right-angled triangle, we can find the length of the longest side by:



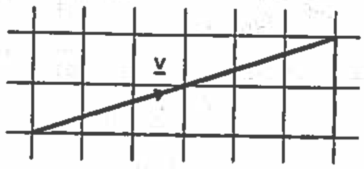
1 squaring the two shorter sides — $5^2, 3^2$

2. adding the squares together — $5^2 + 3^2$

3 taking the square root of the answer — $\sqrt{5^2 + 3^2} = \sqrt{25 + 9} = \sqrt{34}$ = 5.83 (from square root tables)

Example

This line shows the vector $\underline{v} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$



Its length is given by $\sqrt{6^2 + 2^2}$

$= \sqrt{36 + 4}$

$= \sqrt{40}$

$= 6.32$ (from square root tables)

We say that the length of $\underline{v}$ = 6.32 units

Exercise

Find the lengths of the vectors shown below by copying and completing the following:

1 Length of $\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \sqrt{3^2 + 4^2} = \sqrt{\square + \square} = \sqrt{\square} = \square$ units

2 Length of $\begin{pmatrix} 2 \\ 6 \end{pmatrix} = \sqrt{\square + \square} = \sqrt{\square + \square} = \sqrt{\square} = \square$ units

3 Length of $\begin{pmatrix} 1 \\ 7 \end{pmatrix} = \sqrt{\square + \square} = \sqrt{\square + \square} = \sqrt{\square} = \square$ units

4 Length of $\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \sqrt{(-2)^2 + 3^2} = \sqrt{4 + 9} = \sqrt{\square} = \square$ units

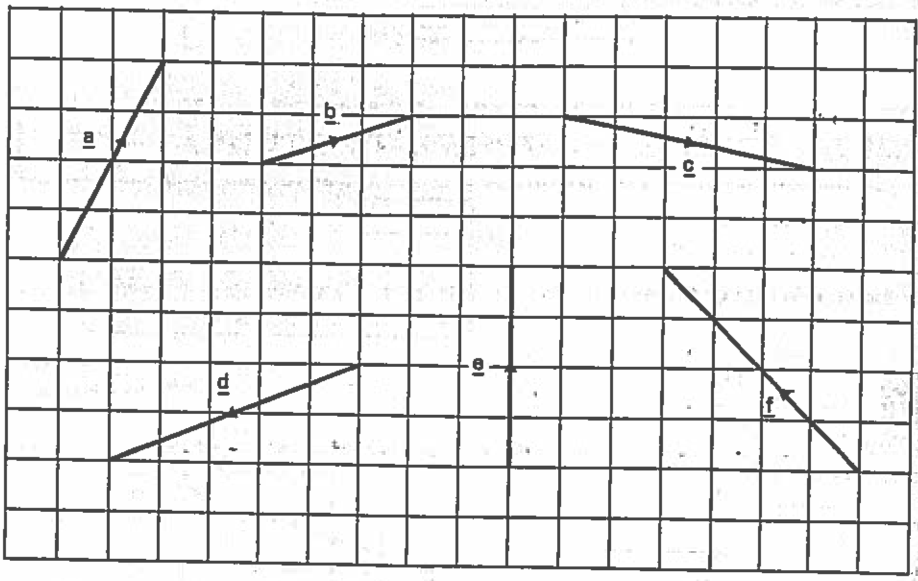
Remember that $(-2) \times (-2) = 4$.

4

5 Length of $\begin{pmatrix}4\\-3\end{pmatrix} = \sqrt{\square + \square} = \sqrt{\square + \square} = \sqrt{\square} = \square$ units

6 Length of $\begin{pmatrix}-3\\1\end{pmatrix} = \sqrt{\square + \square} = \sqrt{\square + \square} = \sqrt{\square} = \square$ units

In the following diagram the vectors $\underline{a}$, $\underline{b}$, $\underline{c}$, $\underline{d}$, $\underline{e}$, and $\underline{f}$ are shown.



Copy and complete the following:

7 $\underline{a} = \begin{pmatrix}2\\4\end{pmatrix}$ Length of $\underline{a} = \sqrt{\square + \square} = \sqrt{4 + \square} = \sqrt{\square} = \square$ units

8 $\underline{b} = \begin{pmatrix}\ \\ \ \end{pmatrix}$ Length of $\underline{b} = \sqrt{\square + \square} =$

9 $\underline{c} = \begin{pmatrix}\ \\ \ \end{pmatrix}$ Length of $\underline{c} =$

10 $\underline{d} = \begin{pmatrix}\ \\ \ \end{pmatrix}$ Length of $\underline{d} =$

11 $\underline{e} = \begin{pmatrix}\ \\ \ \end{pmatrix}$ Length of $\underline{e} =$

12 $\underline{f} = \begin{pmatrix}\ \\ \ \end{pmatrix}$ Length of $\underline{f} =$

Continue with Sheet M2/2 (reverse)

We call this method of combining moves 'addition' and we write

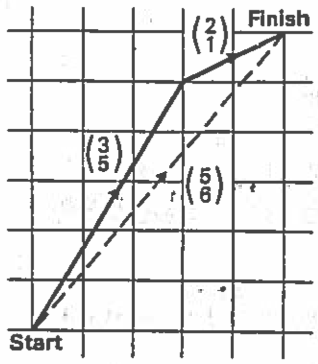
$$\begin{pmatrix}4\\1\end{pmatrix} + \begin{pmatrix}2\\2\end{pmatrix} = \begin{pmatrix}6\\3\end{pmatrix}$$

When we add two vectors we get another vector.

Example

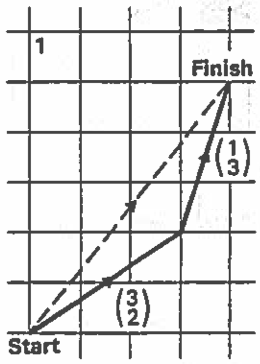
This diagram shows that

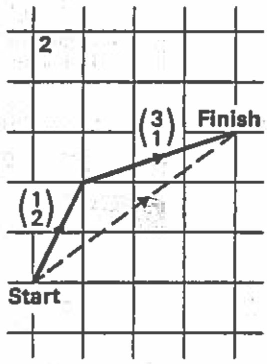
$$\begin{pmatrix}3\\5\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}5\\6\end{pmatrix}$$

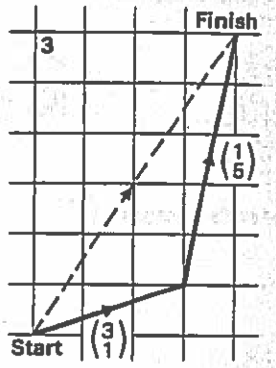


Notice when we add two vectors the lines go '*nose to tail*'.
We start the second line from the finish of the first one.

Exercise







Using the diagrams, copy and complete the additions below.

1 $\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}\square\\\square\end{pmatrix}$ **2** $\begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}\square\\\square\end{pmatrix}$ **3** $\begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}1\\5\end{pmatrix} = \begin{pmatrix}\square\\\square\end{pmatrix}$

You should have noticed that when you add two vectors you add the first components and the second components separately.

In question 1:

$$\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}3+1\\2+3\end{pmatrix} = \begin{pmatrix}4\\5\end{pmatrix}$$

Example

$$\text{If } \underline{s} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \text{ and } \underline{t} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ then } \underline{s} + \underline{t} = \begin{pmatrix} 3+2 \\ 5+1 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

Exercise

Copy and complete the additions below.

4 $\underline{v} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ $\quad \underline{w} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ $\quad \underline{v} + \underline{w} = \begin{pmatrix} \square + \square \\ \square + \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

5 $\underline{d} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ $\quad \underline{y} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\quad \underline{d} + \underline{y} = \begin{pmatrix} \square + \square \\ \square + \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

6 $\underline{k} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\quad \underline{s} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ $\quad \underline{k} + \underline{s} = \begin{pmatrix} \square + \square \\ \square + \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

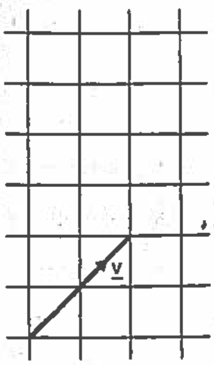
7 $\underline{x} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ $\quad \underline{y} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ $\quad \underline{x} + \underline{y} = \begin{pmatrix} \square + \square \\ \square + \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

8 $\underline{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ $\quad \underline{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\quad \underline{a} + \underline{b} = \begin{pmatrix} \square + \square \\ \square + \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

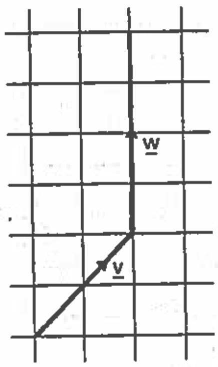
Example

On 1 cm squared paper show the addition in question 4 above.

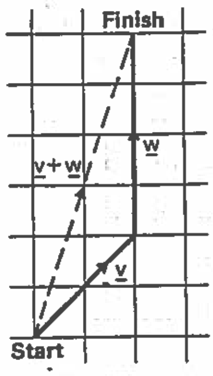
First draw $\underline{v}$.



Using the finish of $\underline{v}$ as the start of $\underline{w}$, draw $\underline{w}$.



Draw the direct route from start to finish and label it $\underline{v} + \underline{w}$.



Exercise

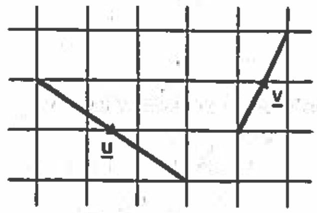
9 On 1 cm squared paper show the additions in questions 5 to 8 above.

Continue with Section M

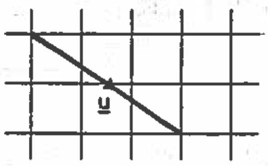
M The sum of two vectors

Example

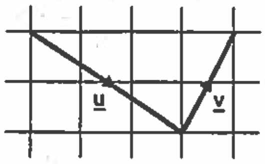
Add the vectors u and v.



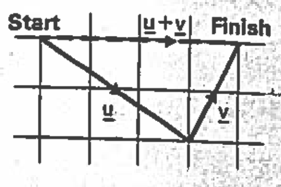
Copy the vector u.



Using the finish of u as the start of v, copy v.

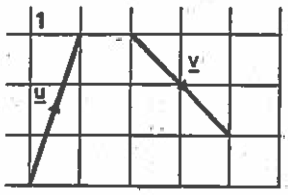


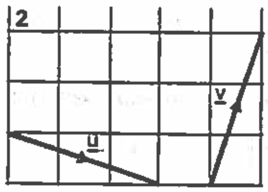
Draw the direct route from start to finish and label it u + v.

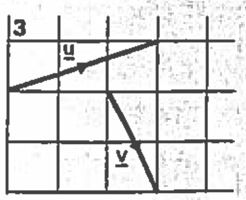


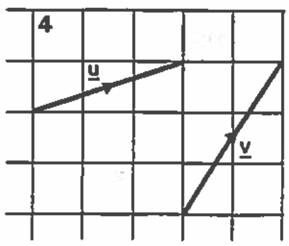
Exercise

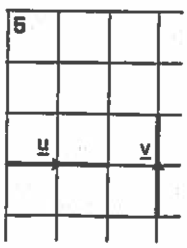
In the same way copy and add u and v in each of the following. Write down the components of u + v in each case.

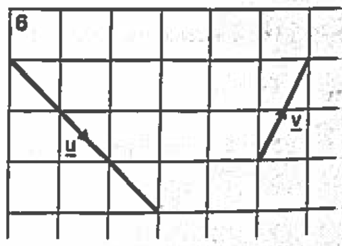












Example

For question 1 above find $\underline{u} + \underline{v}$ in component form.

$$\underline{u} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \qquad \underline{v} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} \qquad \underline{u} + \underline{v} = \begin{pmatrix} 1 + 2 \\ 3 + (-2) \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

which should check with the answer obtained in your diagram.

Exercise

Calculate $\underline{u} + \underline{v}$ as above for questions 2 to 6 and check with the answers obtained from your diagrams.

Continue with Section N

N Negative of a vector

Exercise

Copy and complete the following:

1 $\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ -3 \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

2 $\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} -4 \\ -2 \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

3 $\begin{pmatrix} -5 \\ -2 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

4 $\begin{pmatrix} -1 \\ -4 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

5 $\begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

6 $\begin{pmatrix} -4 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

> **Note:** In each case you should have found the answer was $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, the zero vector.

Copy and complete:

7 $\begin{pmatrix} 7 \\ 6 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

8 $\begin{pmatrix} 4 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

9 $\begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

When two vectors add together to give the zero vector like this:

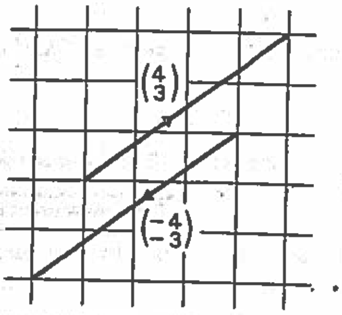
$$\begin{pmatrix}8\\2\end{pmatrix} + \begin{pmatrix}-8\\-2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

then each of them is said to be the negative of the other.

Example

$\begin{pmatrix}4\\3\end{pmatrix}$ is the negative of $\begin{pmatrix}-4\\-3\end{pmatrix}$ and

$\begin{pmatrix}-4\\-3\end{pmatrix}$ is the negative of $\begin{pmatrix}4\\3\end{pmatrix}$.



$\begin{pmatrix}-2\\5\end{pmatrix}$ is the negative of $\begin{pmatrix}2\\-5\end{pmatrix}$ $\begin{pmatrix}-2\\-1\end{pmatrix}$ is the negative of $\begin{pmatrix}2\\1\end{pmatrix}$

Exercise

Write down the negative of each of the following vectors.

10 $\begin{pmatrix}2\\5\end{pmatrix}$ 11 $\begin{pmatrix}1\\2\end{pmatrix}$ 12 $\begin{pmatrix}4\\-1\end{pmatrix}$

13 $\begin{pmatrix}-3\\2\end{pmatrix}$ 14 $\begin{pmatrix}-1\\-4\end{pmatrix}$ 15 $\begin{pmatrix}-2\\0\end{pmatrix}$

> **Note:** The negative of a vector $\underline{a}$ is the vector $-\underline{a}$,
> so if $\underline{a} = \begin{pmatrix}8\\-2\end{pmatrix}$ then $-\underline{a} = \begin{pmatrix}-8\\2\end{pmatrix}$.

Copy and complete the following:

16 $\underline{x} = \begin{pmatrix}3\\4\end{pmatrix}$, $-\underline{x} = \begin{pmatrix}\square\\\square\end{pmatrix}$ 17 $\underline{z} = \begin{pmatrix}7\\3\end{pmatrix}$, $-\underline{z} = \begin{pmatrix}\square\\\square\end{pmatrix}$

18 $\underline{w} = \begin{pmatrix}-2\\8\end{pmatrix}$, $-\underline{w} = \begin{pmatrix}\square\\\square\end{pmatrix}$ 19 $\underline{t} = \begin{pmatrix}4\\-1\end{pmatrix}$, $-\underline{t} = \begin{pmatrix}\square\\\square\end{pmatrix}$

20 For the vectors in questions 16 to 19, draw the following vectors on 1 cm squared paper:

$\underline{x}$ and $-\underline{x}$ $\underline{z}$ and $-\underline{z}$

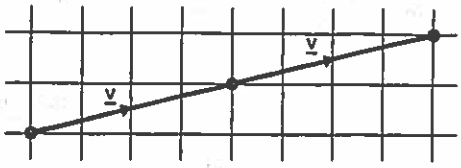
$\underline{w}$ and $-\underline{w}$ $\underline{t}$ and $-\underline{t}$

Continue with Section O

O Multiplication by a number

In this diagram the vector

$\underline{v} = \begin{pmatrix}4\\1\end{pmatrix}$ has been added to itself.



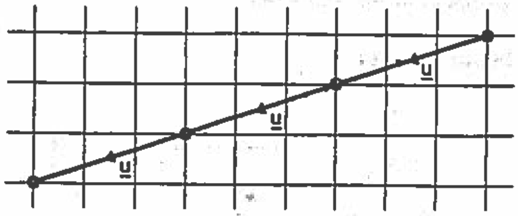
That is $\begin{pmatrix}4\\1\end{pmatrix} + \begin{pmatrix}4\\1\end{pmatrix} = \begin{pmatrix}8\\2\end{pmatrix}$.

$\underline{v} + \underline{v} = 2\underline{v}$

We will write $\underline{v} + \underline{v}$ as $2\underline{v}$.

Notice that both the x component and the y component of $\underline{v}$ are doubled so that $2\underline{v} = 2\begin{pmatrix}4\\1\end{pmatrix} = \begin{pmatrix}2\times4\\2\times1\end{pmatrix} = \begin{pmatrix}8\\2\end{pmatrix}$

In this diagram the vector $\underline{u} = \begin{pmatrix}-3\\-1\end{pmatrix}$ is added to itself and then $\underline{u}$ is added to the sum.



That is $\begin{pmatrix}-3\\-1\end{pmatrix} + \begin{pmatrix}-3\\-1\end{pmatrix} + \begin{pmatrix}-3\\-1\end{pmatrix} = \begin{pmatrix}-9\\-3\end{pmatrix}$

$\underline{u} + \underline{u} + \underline{u} = 3\underline{u}$

We will write $\underline{u} + \underline{u} + \underline{u}$ as $3\underline{u}$.

This time the x and y components are both multiplied by 3 so that

$$3\underline{u} = 3\begin{pmatrix}-3\\-1\end{pmatrix} = \begin{pmatrix}3\times(-3)\\3\times(-1)\end{pmatrix} = \begin{pmatrix}-9\\-3\end{pmatrix}$$

Exercise

1 On 1 cm squared paper draw diagrams to show the following:

(a) $\begin{pmatrix}3\\5\end{pmatrix} + \begin{pmatrix}3\\5\end{pmatrix}$ (b) $\begin{pmatrix}2\\-3\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix}$

2 Suppose that $\underline{s} = \begin{pmatrix}2\\3\end{pmatrix}$, $\underline{t} = \begin{pmatrix}-3\\4\end{pmatrix}$, and $\underline{a} = \begin{pmatrix}-1\\-5\end{pmatrix}$.

Draw diagrams on squared paper to show

$\underline{s}$, $2\underline{s}$, $4\underline{s}$, $\underline{t}$, $3\underline{t}$, $\underline{a}$, $3\underline{a}$

11

3 Copy and complete (a) and then do (b) to (f) in the same way:

(a) $3\begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}3\times4\\3\times3\end{pmatrix} = \begin{pmatrix}\square\\\square\end{pmatrix}$ (b) $5\begin{pmatrix}-2\\1\end{pmatrix}$

(c) $7\begin{pmatrix}-3\\-1\end{pmatrix}$ (d) $3\begin{pmatrix}5\\1\end{pmatrix}$ (e) $2\begin{pmatrix}0\\4\end{pmatrix}$ (f) $4\begin{pmatrix}3\\-2\end{pmatrix}$

Continue with Section P

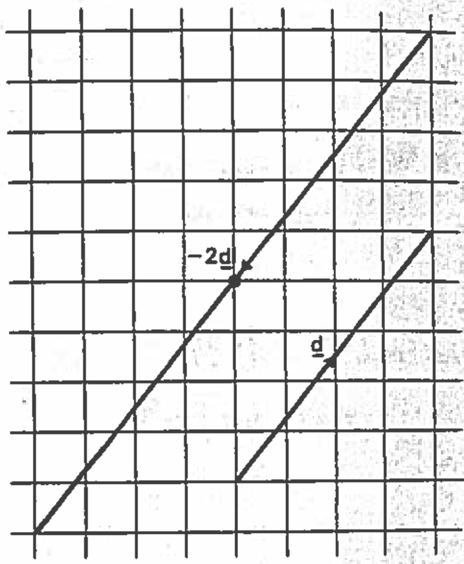
P Multiplication by a negative number

We can multiply vectors by negative numbers in the same way.

Suppose that $\underset{\sim}{d} = \begin{pmatrix}4\\5\end{pmatrix}$.

$-2\underset{\sim}{d} = -2\begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}-2\times4\\-2\times5\end{pmatrix} = \begin{pmatrix}-8\\-10\end{pmatrix}$

The diagram shows $\underset{\sim}{d}$ and $-2\underset{\sim}{d}$.



Notice the direction of the arrows.

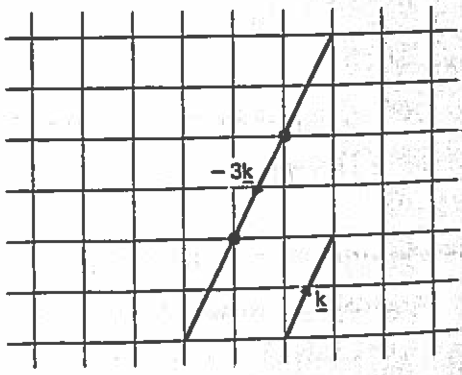
Example

This diagram shows the vectors

$\underset{\sim}{k} = \begin{pmatrix}1\\2\end{pmatrix}$ and $-3\underset{\sim}{k} = -3\begin{pmatrix}1\\2\end{pmatrix}$

$= \begin{pmatrix}-3\times1\\-3\times2\end{pmatrix}$

$= \begin{pmatrix}-3\\-6\end{pmatrix}$



Notice the direction of the arrows.

Exercise

1 Copy and complete (a) and then do (b) to (f) in the same way:

(a) $-5\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}-5\times 2\\-5\times 1\end{pmatrix} = \begin{pmatrix}\square\\\square\end{pmatrix}$ (b) $-4\begin{pmatrix}3\\1\end{pmatrix}$

(c) $-3\begin{pmatrix}-2\\1\end{pmatrix}$ (d) $-2\begin{pmatrix}4\\0\end{pmatrix}$ (e) $-5\begin{pmatrix}2\\-1\end{pmatrix}$ (f) $-3\begin{pmatrix}-3\\-1\end{pmatrix}$

2 If $\underline{m} = \begin{pmatrix}2\\-1\end{pmatrix}$ and $\underline{n} = \begin{pmatrix}-3\\2\end{pmatrix}$ draw on squared paper diagrams to show:

$\underline{m}$, $-3\underline{m}$, $-5\underline{m}$, $\underline{n}$, $-2\underline{n}$, $-3\underline{n}$

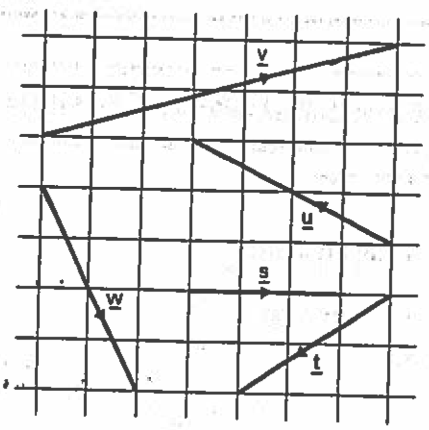
Continue with Section Q

Q Progress check

Exercise

1 What are the components of the following vectors?

Copy and complete:



$\underline{v} = \begin{pmatrix}\square\\\square\end{pmatrix}$

$\underline{u} = \begin{pmatrix}\square\\\square\end{pmatrix}$

$\underline{w} = \begin{pmatrix}\square\\\square\end{pmatrix}$

$\underline{s} = \begin{pmatrix}\square\\\square\end{pmatrix}$

$\underline{t} = \begin{pmatrix}\square\\\square\end{pmatrix}$

2 On 1 cm squared paper draw diagrams to show the following vectors:

$\underline{a} = \begin{pmatrix}3\\5\end{pmatrix}$ $\underline{b} = \begin{pmatrix}2\\-3\end{pmatrix}$ $\underline{c} = \begin{pmatrix}-4\\0\end{pmatrix}$ $\underline{d} = \begin{pmatrix}0\\3\end{pmatrix}$ $\underline{e} = \begin{pmatrix}-3\\2\end{pmatrix}$

Calculate the length of each of these vectors write your answer beside each drawing.

3 If $\underline{u} = \begin{pmatrix}4\\1\end{pmatrix}$, $\underline{v} = \begin{pmatrix}5\\3\end{pmatrix}$, and $\underline{w} = \begin{pmatrix}-2\\2\end{pmatrix}$, draw diagrams on squared paper to show the additions $\underline{u} + \underline{v}$ and $\underline{u} + \underline{w}$.

4 Copy and complete the following additions:

(a) $\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}2\\4\end{pmatrix}=\begin{pmatrix}\square\\\square\end{pmatrix}$ (b) $\begin{pmatrix}3\\1\end{pmatrix}+\begin{pmatrix}0\\4\end{pmatrix}=\begin{pmatrix}\square\\\square\end{pmatrix}$

(c) $\begin{pmatrix}2\\-1\end{pmatrix}+\begin{pmatrix}4\\3\end{pmatrix}=\begin{pmatrix}\square\\\square\end{pmatrix}$ (d) $\begin{pmatrix}6\\2\end{pmatrix}+\begin{pmatrix}-1\\-2\end{pmatrix}=\begin{pmatrix}\square\\\square\end{pmatrix}$

5 What is the negative of each of the following vectors?

$\underline{x}=\begin{pmatrix}2\\7\end{pmatrix}$ $\underline{y}=\begin{pmatrix}3\\-1\end{pmatrix}$ $\underline{v}=\begin{pmatrix}-4\\-3\end{pmatrix}$

Copy and complete: $-\underline{x}=\begin{pmatrix}\square\\\square\end{pmatrix}$, $-\underline{y}=\begin{pmatrix}\square\\\square\end{pmatrix}$, $-\underline{v}=\begin{pmatrix}\square\\\square\end{pmatrix}$

6 $\underline{u}=\begin{pmatrix}3\\4\end{pmatrix}$ and $\underline{a}=\begin{pmatrix}2\\4\end{pmatrix}$

Copy and complete: $2\underline{u}=\begin{pmatrix}\square\\\square\end{pmatrix}$, $-3\underline{a}=\begin{pmatrix}\square\\\square\end{pmatrix}$

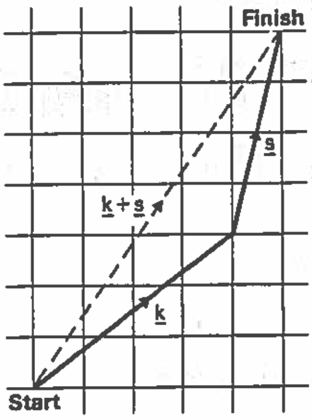
Draw $\underline{u}$, $2\underline{u}$, $\underline{a}$, and $-3\underline{a}$.

Ask your teacher what to do next

R Order of addition

In question 6 of the exercise in Section L, $\underline{k}=\begin{pmatrix}4\\3\end{pmatrix}$ and $\underline{s}=\begin{pmatrix}1\\4\end{pmatrix}$.

To find $\underline{k}+\underline{s}$, you started with $\underline{k}$ and then drew $\underline{s}$.

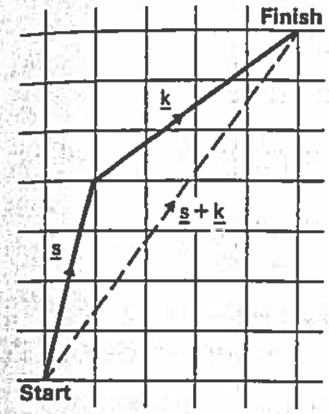


From the diagram,

$\underline{k}+\underline{s}=\begin{pmatrix}5\\7\end{pmatrix}$

What is the vector s + k?

This time we start by drawing s and then draw k.



From the diagram,

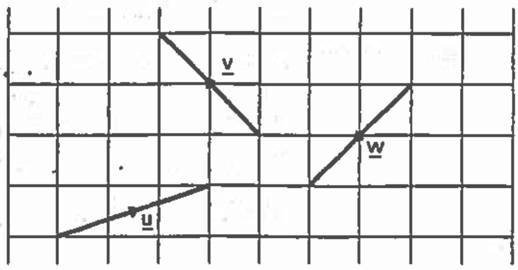
$$\underline{s} + \underline{k} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

So starting with k and adding s gives the same result as starting with s and adding k.

We have shown that the order in which we add two vectors is not important.

Continue with Sheet M2/3

S Adding three vectors

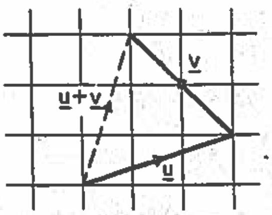


$\underline{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ and $\underline{w} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

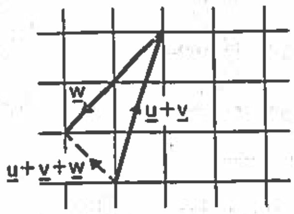
Let us find the sum of these three vectors.

Method 1

Start by adding u and v.



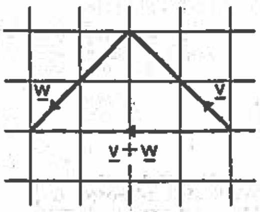
Next, to (u + v) add w.



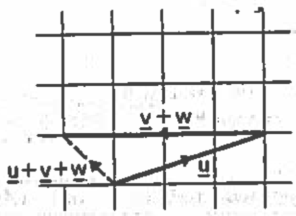
From the diagram u + v + w = $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Method 2

Suppose we start by finding v + w first.



Next, to u add (v + w).



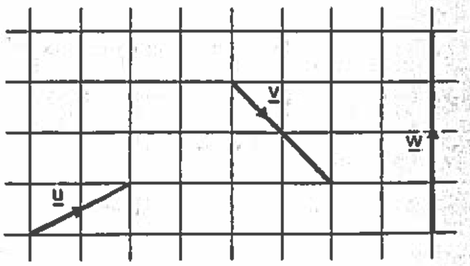
From the diagram u + v + w = $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

So adding (u + v) and w gives the same result as adding u and (v + w).

The order in which we add three vectors is not important.

Exercise

1 Vectors u, v, and w are shown. Draw diagrams to show the vectors u + v and u + v + w.



Copy and complete the following:

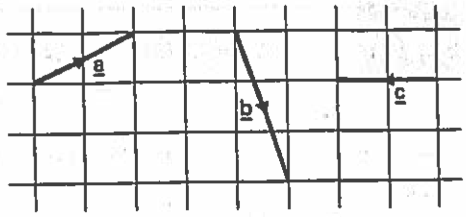
$\underline{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad \underline{v} = \begin{pmatrix} \square \\ \square \end{pmatrix}$ $\quad \underline{w} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

$\underline{u} + \underline{v} = \begin{pmatrix} \square + \square \\ \square + \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

$\underline{u} + \underline{v} + \underline{w} = \begin{pmatrix} 4 + \square \\ -1 + \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

Check that the vector u + v + w you have drawn has these components.

2 Vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ are shown. Draw diagrams to show the vectors $\underline{a} + \underline{b}$ and $\underline{a} + \underline{b} + \underline{c}$.



Copy and complete the following:

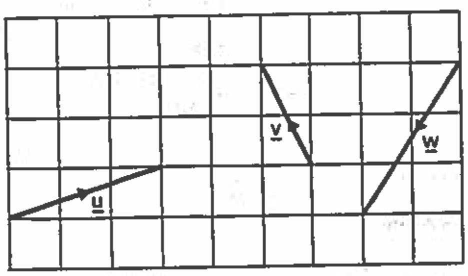
$\underline{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad \underline{b} = \begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix}$ $\quad \underline{c} = \begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix}$

$\underline{a} + \underline{b} = \begin{pmatrix} \blacksquare + \blacksquare \\ \blacksquare + \blacksquare \end{pmatrix} = \begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix}$

$\underline{a} + \underline{b} + \underline{c} = \begin{pmatrix} \blacksquare + \blacksquare \\ \blacksquare + \blacksquare \end{pmatrix} = \begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix}$

Check that the vector $\underline{a} + \underline{b} + \underline{c}$ you have drawn has these components.

3 Vectors $\underline{u}$, $\underline{v}$, and $\underline{w}$ are shown. Draw diagrams to show the vectors $\underline{u} + \underline{v}$ and $\underline{u} + \underline{v} + \underline{w}$.



Copy and complete the following:

$\underline{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\quad \underline{v} = \begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix}$ $\quad \underline{w} = \begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix}$

$\underline{u} + \underline{v} = \begin{pmatrix} \blacksquare + \blacksquare \\ \blacksquare + \blacksquare \end{pmatrix} = \begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix}$

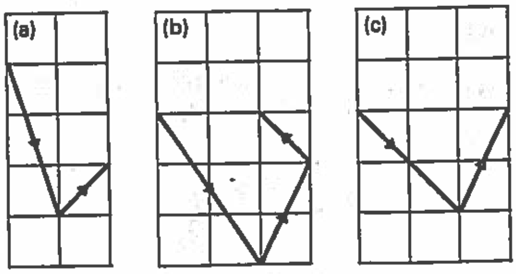
$\underline{u} + \underline{v} + \underline{w} = \begin{pmatrix} \blacksquare + \blacksquare \\ \blacksquare + \blacksquare \end{pmatrix} = \begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix}$

> **Note:** You should have found that the translation $\underline{u}$ followed by $\underline{v}$ followed by $\underline{w}$ brought you back to your starting point. That is, $\underline{u} + \underline{v} + \underline{w}$ has no effect. We say that $\underline{u} + \underline{v} + \underline{w}$ is the zero vector.
>
> In terms of components the zero vector is $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. It is also written as $\underline{0}$.

4 Copy diagram (a). On your diagram draw another vector so that the vectors in the diagram add up to $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Beside this last vector write its components.

Repeat this for diagrams (b) and (c).

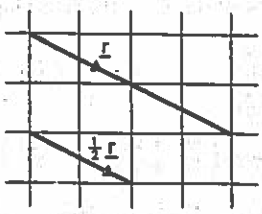


Continue with Section T

T Multiplication by a fraction

This diagram shows a vector $\underline{r}$, with components $\begin{pmatrix}4\\-2\end{pmatrix}$.

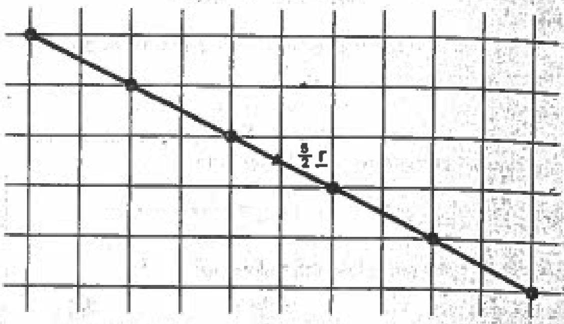
It also shows the vector $\frac{1}{2}\underline{r}$.



That is, $\frac{1}{2}\begin{pmatrix}4\\-2\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\times 4\\\frac{1}{2}\times -2\end{pmatrix} = \begin{pmatrix}2\\-1\end{pmatrix}$

This diagram shows $\frac{5}{2}\underline{r}$, which means $5 \times \frac{1}{2}\underline{r}$.

$$\frac{5}{2}\begin{pmatrix}4\\-2\end{pmatrix} = \begin{pmatrix}\frac{5}{2}\times 4\\\frac{5}{2}\times -2\end{pmatrix}$$
$$= \begin{pmatrix}5\times\frac{1}{2}\text{ of }4\\5\times\frac{1}{2}\text{ of }(-2)\end{pmatrix}$$
$$= \begin{pmatrix}5\times 2\\5\times(-1)\end{pmatrix}$$
$$= \begin{pmatrix}10\\-5\end{pmatrix}$$



Exercise

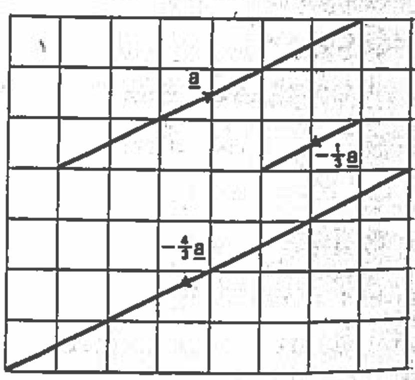
1 If $\underline{v} = \begin{pmatrix}6\\-4\end{pmatrix}$ and $\underline{w} = \begin{pmatrix}-3\\6\end{pmatrix}$ draw diagrams on 1 cm squared paper to show $\underline{v}$, $\frac{1}{2}\underline{v}$, $\frac{3}{2}\underline{v}$, $\underline{w}$, $\frac{1}{3}\underline{w}$, and $\frac{4}{3}\underline{w}$.

2 Find the value of each of the following:

(a) $\frac{1}{2}\begin{pmatrix}10\\-4\end{pmatrix}$ (b) $\frac{2}{3}\begin{pmatrix}-6\\9\end{pmatrix}$ (c) $\frac{3}{2}\begin{pmatrix}4\\2\end{pmatrix}$ (d) $\frac{5}{3}\begin{pmatrix}12\\-3\end{pmatrix}$

Example

This diagram shows the vector $\underline{a} = \begin{pmatrix}6\\3\end{pmatrix}$



and

$$-\tfrac{1}{3}\underline{a} = -\tfrac{1}{3}\begin{pmatrix}6\\3\end{pmatrix} = \begin{pmatrix}-\frac{1}{3}\times 6\\-\frac{1}{3}\times 3\end{pmatrix} = \begin{pmatrix}-2\\-1\end{pmatrix}$$

and

$$-\tfrac{4}{3}\underline{a} = -\tfrac{4}{3}\begin{pmatrix}6\\3\end{pmatrix} = \begin{pmatrix}-\frac{4}{3}\times 6\\-\frac{4}{3}\times 3\end{pmatrix}$$
$$= \begin{pmatrix}-4\times\frac{1}{3}\text{ of }6\\-4\times\frac{1}{3}\text{ of }3\end{pmatrix}$$
$$= \begin{pmatrix}-4\times 2\\-4\times 1\end{pmatrix}$$
$$= \begin{pmatrix}-8\\-4\end{pmatrix}$$

Exercise

3 If $\underline{b} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\underline{c} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$ draw diagrams on 1 cm squared paper to show: $\underline{b}$, $-\frac{1}{2}$, $-\frac{5}{2}\underline{b}$, $\underline{c}$, $-\frac{1}{2}\underline{c}$, $-\frac{3}{2}\underline{c}$.

4 Find the value of each of the following:

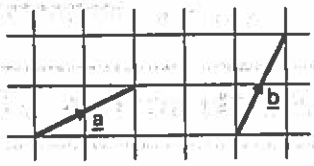
(a) $-\frac{1}{3}\begin{pmatrix} 3 \\ 6 \end{pmatrix}$ (b) $-\frac{1}{2}\begin{pmatrix} 10 \\ -6 \end{pmatrix}$ (c) $-\frac{3}{2}\begin{pmatrix} 4 \\ 8 \end{pmatrix}$ (d) $-\frac{5}{4}\begin{pmatrix} -8 \\ -12 \end{pmatrix}$

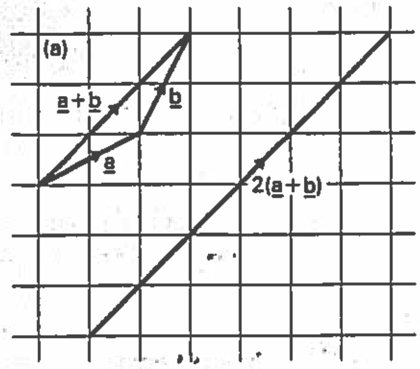
Continue with Section U

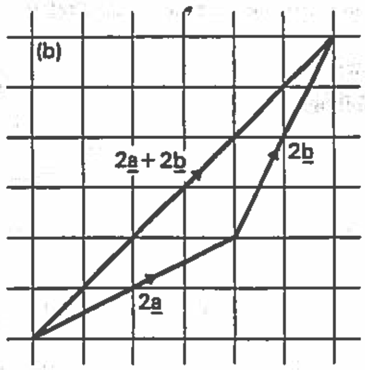
U Use of brackets

Example

On squared paper draw diagrams to show
(a) $\underline{a} + \underline{b}$ and $2(\underline{a} + \underline{b})$,
(b) $2\underline{a}$, $2\underline{b}$, and $2\underline{a} + 2\underline{b}$.







Notice that $2(\underline{a} + \underline{b}) = 2\underline{a} + 2\underline{b}$.

Exercise

1 Copy and complete the following:

If $\underline{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\underline{u} + \underline{v} = \begin{pmatrix} \square \\ \square \end{pmatrix}$ $2(\underline{u} + \underline{v}) = \begin{pmatrix} \square \\ \square \end{pmatrix}$

$2\underline{u} = \begin{pmatrix} \square \\ \square \end{pmatrix}$ $2\underline{v} = \begin{pmatrix} \square \\ \square \end{pmatrix}$ $2\underline{u} + 2\underline{v} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

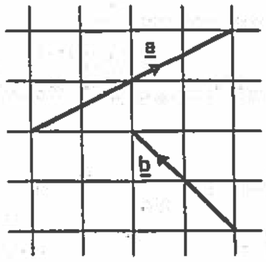
What do you notice?

2 On squared paper copy $\underline{a}$ and $\underline{b}$ and draw diagrams to show

(a) $\underline{a} + \underline{b}$ and $\frac{1}{2}(\underline{a} + \underline{b})$,

(b) $\frac{1}{2}\underline{a}$, $\frac{1}{2}\underline{b}$ and $\frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$.

You should find that $\frac{1}{2}(\underline{a} + \underline{b})$ is the same as $\frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$.



3 Copy and complete the following:

If $\underline{u} = \begin{pmatrix}6\\4\end{pmatrix}$ and $\underline{v} = \begin{pmatrix}4\\8\end{pmatrix}$

$\underline{u} + \underline{v} = \begin{pmatrix}\square\\\square\end{pmatrix}$ $\quad \frac{1}{2}(\underline{u} + \underline{v}) = \begin{pmatrix}\square\\\square\end{pmatrix}$

$\frac{1}{2}\underline{u} = \begin{pmatrix}\square\\\square\end{pmatrix}$ $\quad \frac{1}{2}\underline{v} = \begin{pmatrix}\square\\\square\end{pmatrix}$ $\quad \frac{1}{2}\underline{u} + \frac{1}{2}\underline{v} = \begin{pmatrix}\square\\\square\end{pmatrix}$

What do you notice?

Continue with Section V

V Subtraction of vectors

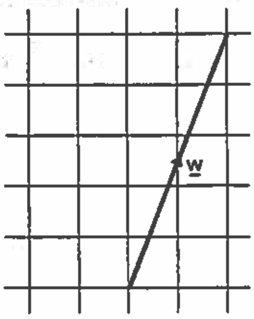
If $\underline{w}$ and $\underline{v}$ are two vectors we can find $\underline{w} - \underline{v}$ in the following way.

Example

Suppose $\underline{w} = \begin{pmatrix}2\\5\end{pmatrix}$ and $\underline{v} = \begin{pmatrix}4\\2\end{pmatrix}$.

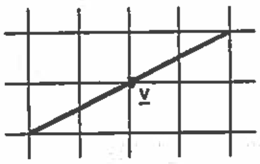
Step 1

Draw $\underline{w}$.



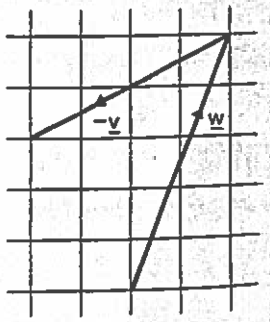
Step 2

Draw $\underline{v}$.



Step 3

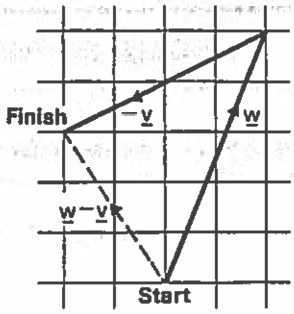
Using the Finish of $\underline{w}$ as the Start of $-\underline{v}$, draw $-\underline{v}$.



Remember: $-\underline{v}$ is the same length as $\underline{v}$ but in the opposite direction.

Step 4

Draw the direct route from Start to Finish. This is $\underline{w} + (-\underline{v})$, which we write as $\underline{w} - \underline{v}$ and label in the diagram.



Exercise

1 $\underline{w} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $\underline{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Draw a diagram on 1 cm squared paper to show the vector $\underline{w} - \underline{v}$ and write down the components of $\underline{w} - \underline{v}$.

2 $\underline{w} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$, $\underline{v} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Draw a diagram on 1 cm squared paper to show the vector $\underline{w} - \underline{v}$ and write down the components of $\underline{w} - \underline{v}$. Check that this answer can be obtained by subtracting the components, i.e. $\begin{pmatrix} 5 - 3 \\ 7 - 2 \end{pmatrix}$.

3 Repeat question 2 for $\underline{w} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$, $\underline{v} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

Example

$\underline{w} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, $\underline{v} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$. Calculate $\underline{w} - \underline{v}$ without drawing a diagram.

$$\underline{w} - \underline{v} = \begin{pmatrix} 4 - \quad 2 \\ -1 - (-4) \end{pmatrix} = \begin{pmatrix} 2 \\ -1 + 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

Exercise

In each of the following calculate $\underline{w} - \underline{v}$ without drawing a diagram.

4 $\underline{w} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

5 $\underline{w} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

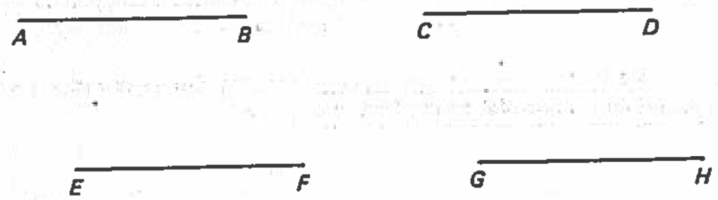
6 $\underline{w} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

7 $\underline{w} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$

Continue with Section W

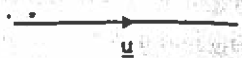
X Directed line segments

Point 1



In the diagram above, all the lines *AB*, *CD*, *EF*, and *GH* are of the same length and have the same direction (either they are parallel like *AB* and *EF*, or they are in line like *AB* and *CD*).

Each of these lines could be used as a representative of the vector $\underline{u}$.

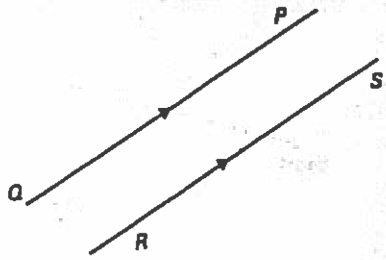


When we want to show that the line *AB* is a representative of a vector we write it as $\overrightarrow{AB}$ and call it a directed line segment. (Read $\overrightarrow{AB}$ as '*A* to *B*'.)

The fact that each of the lines above represents $\underline{u}$ would be shown by writing $\overrightarrow{AB} = \overrightarrow{CD} = \overrightarrow{EF} = \overrightarrow{GH} = \underline{u}$.

Point 2

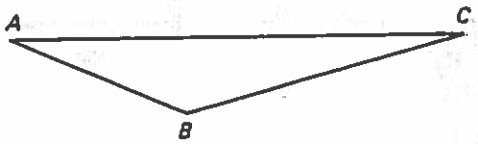
If $\overrightarrow{PQ} = \overrightarrow{RS}$ then the lines *PQ* and *RS* must be equal and parallel since they represent the same vector.



Point 3

If *ABC* is any triangle,
then $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$
but $AB + BC \neq AC$

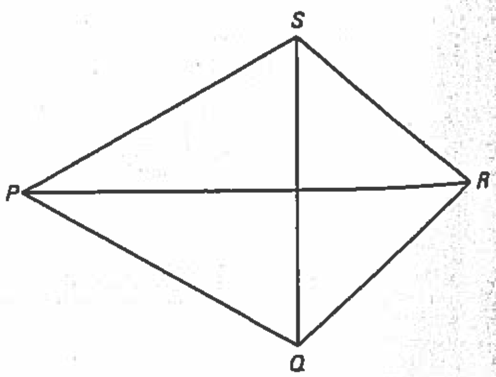
This means 'is not equal to'



Example

PQRS is a quadrilateral.

$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$
$\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RS} + \overrightarrow{SP} = \underline{0}$



Exercise

1 Copy and complete the following for the diagram above:

(a) $\overrightarrow{RS} + \overrightarrow{SP} = \square$ (b) $\overrightarrow{PQ} + \overrightarrow{QS} = \square$ (c) $\overrightarrow{PR} + \overrightarrow{RQ} = \square$

(d) $\overrightarrow{RQ} + \overrightarrow{QP} = \square$ (e) $\overrightarrow{RQ} + \overrightarrow{QS} + \overrightarrow{SP} = \square$ (f) $\overrightarrow{QP} + \overrightarrow{PS} = \square$

(g) $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RP} = \square$ (h) $\overrightarrow{QR} + \overrightarrow{RS} + \overrightarrow{SQ} = \square$

2 In the diagram opposite

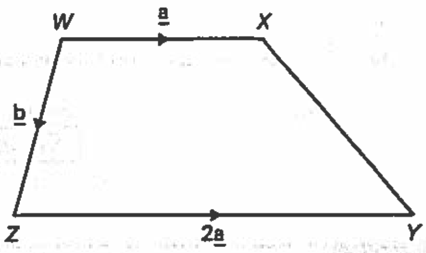
(a) What can you say about the direction of lines *WX* and *ZY*?

(b) What can you say about the lengths of lines *WX* and *ZY*?

(c) What is $\overrightarrow{XY}$ in terms of $\underline{a}$ and $\underline{b}$?

Copy and complete:

$\overrightarrow{XY} = \overrightarrow{XW} + \overrightarrow{WZ} + \square$
$= -\underline{a} + \square + \square$
$= \square$



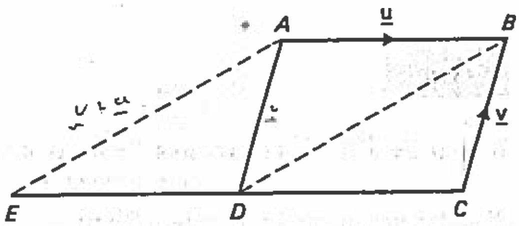
3 In the diagram, *ABCD* is a parallelogram and *CD* has been produced its own length to *E*. $\overrightarrow{AB}$ and $\overrightarrow{CB}$ are representatives of $\underline{u}$ and $\underline{v}$.

In terms of $\underline{u}$ and $\underline{v}$, find:

(a) $\overrightarrow{DC}$ (b) $\overrightarrow{DB}$

(c) $\overrightarrow{DA}$ (d) $\overrightarrow{ED}$ (e) $\overrightarrow{EA}$

By looking at the answers for (b) and (e), what can you say about *DB* and *EA*?

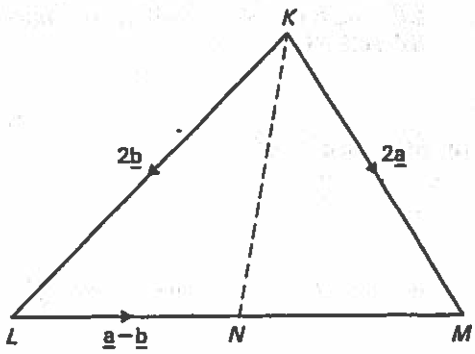


4 In the diagram, $\overrightarrow{KM}$ and $\overrightarrow{KL}$ represent vectors $2\underline{a}$ and $2\underline{b}$. *LN* represents the vector $\underline{a} - \underline{b}$.

In terms of $\underline{a}$ and $\underline{b}$, find:

(a) $\overrightarrow{KN}$
(b) $\overrightarrow{NM}$

What can you say about the position of *N* on the line *LM*?



5 In the diagram $ABCD$ and $AEFG$ are rhombuses with E and G the mid-points of AB and AD respectively.

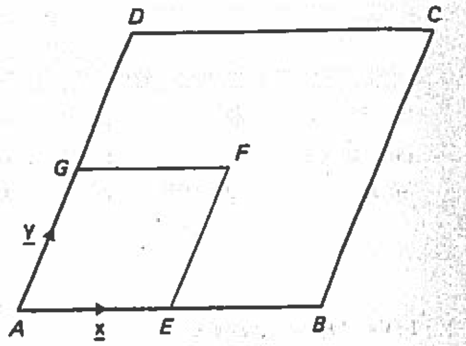
$\overrightarrow{AE}$ represents the vector $\underline{x}$ and $\overrightarrow{AG}$ represents the vector $\underline{y}$.

In terms of $\underline{x}$ and $\underline{y}$, find:

(a) $\overrightarrow{EF} =$ ▢ (b) $\overrightarrow{AF} =$ ▢

(c) $2\overrightarrow{AF} =$ ▢ (d) $\overrightarrow{AB} =$ ▢

(e) $\overrightarrow{BC} =$ ▢ (f) $\overrightarrow{AC} =$ ▢



Copy and complete:

So $\overrightarrow{AC} = 2$▢ and A, F, and C lie in a ▢ .

Continue with Section Y

Y Vectors applied to geometry

We can use vectors in geometry to establish some properties of figures.

Example

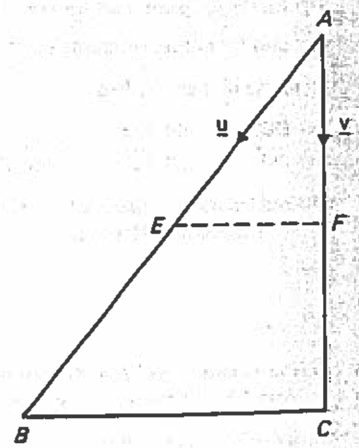
In triangle ABC, $\overrightarrow{AB}$ represents the vector $\underline{u}$ and $\overrightarrow{AC}$ represents the vector $\underline{v}$.

E and F are the mid-points of AB and AC respectively.

(a) Express $\overrightarrow{BC}$ in terms of $\underline{u}$ and $\underline{v}$.

(b) Express $\overrightarrow{EF}$ in terms of $\underline{u}$ and $\underline{v}$.

(c) By comparing the results for $\overrightarrow{BC}$ and $\overrightarrow{EF}$, make two statements about lines BC and EF.

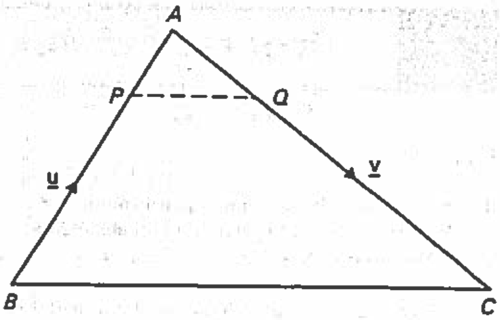


(a) $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$
$= -\underline{u} + \underline{v}$
$= \underline{v} - \underline{u}$

(b) $\overrightarrow{EF} = \overrightarrow{EA} + \overrightarrow{AF}$
$= -\tfrac{1}{2}\underline{u} + \tfrac{1}{2}\underline{v}$
$= \tfrac{1}{2}\underline{v} - \tfrac{1}{2}\underline{u}$
So $2\overrightarrow{EF} = \underline{v} - \underline{u}$

(c) Because $\overrightarrow{BC} = 2\overrightarrow{EF}$, BC is parallel to EF and BC is twice the length of EF.

Exercise



1 In triangle ABC, P and Q are points on AB and AC respectively such that $AP = \frac{1}{4}AB$ and $AQ = \frac{1}{4}AC$.

Show that PQ is parallel to BC and $PQ = \frac{1}{4}BC$. (In the figure $\overrightarrow{BA} = \underline{u}$ and $\overrightarrow{AC} = \underline{v}$.)

Copy and complete:

$\overrightarrow{BC} = \overrightarrow{BA} + \square$ $\qquad$ $\overrightarrow{PQ} = \overrightarrow{PA} + \square$

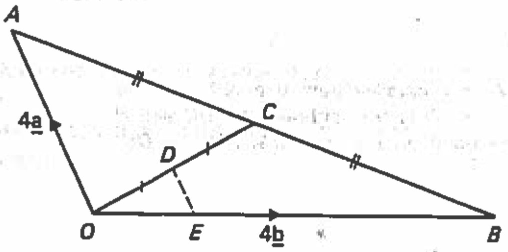
$\quad = \underline{u} + \square$ $\qquad$ $= \square + \square$

So $4\overrightarrow{PQ} = \square$

Because $\overrightarrow{BC} = 4\square$, PQ is parallel to $\square$ and PQ is a $\square$ of the length of BC.

2 In the figure, $\overrightarrow{OA} = 4\underline{a}$ and $\overrightarrow{OB} = 4\underline{b}$. C is the mid-point of AB and D is the mid-point of OC. Find, in terms of $\underline{a}$ and $\underline{b}$, (a) $\overrightarrow{BA}$; (b) $\overrightarrow{BC}$; (c) $\overrightarrow{OC}$; (d) $\overrightarrow{OD}$.

If E is a point on OB such that $OE = \frac{1}{4}OB$, show that ED is parallel to OA.



Copy and complete:

(a) $\overrightarrow{BA} = \overrightarrow{BO} + \square$
$= \square + 4\underline{a}$
$= 4\underline{a} - \square$

(b) $\overrightarrow{OC} = \overrightarrow{OB} + \square$
$= \square + \square$
$= \square$

(c) $\overrightarrow{BC} = \frac{1}{2}\overrightarrow{BA}$
$= \square$

(d) $\overrightarrow{OD} = \frac{1}{2}\,\square$
$= \square$

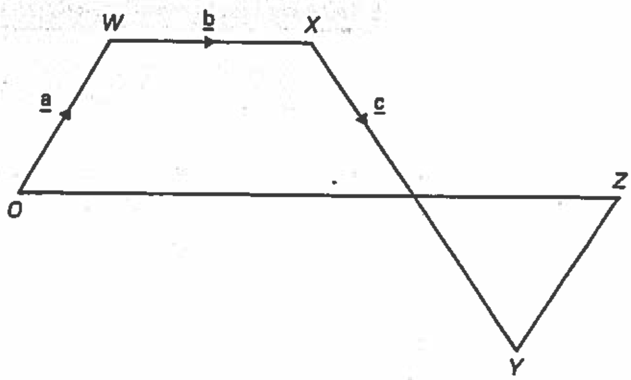
But $\overrightarrow{ED} = \overrightarrow{EO} + \square$
$= \square + \square$
$= \square$

So $\overrightarrow{ED} = \frac{1}{4}\overrightarrow{OA}$. So ED is parallel to $\square$.

3 $OWXYZ$ represents a path from O to Z with OW equal and parallel to YZ.

(a) If $\overrightarrow{OW}$, $\overrightarrow{WX}$, and $\overrightarrow{XY}$ represent vectors $\underline{a}$, $\underline{b}$, and $\underline{c}$ respectively, find the vector represented by $\overrightarrow{OZ}$ in terms of $\underline{a}$, $\underline{b}$, and $\underline{c}$.

(b) If OZ is parallel to WX and $OZ = 2WX$, show that $2\underline{a} = \underline{b} - \underline{c}$



Copy and complete:

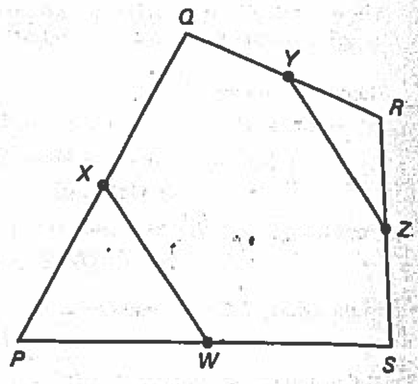
(c) $\overrightarrow{OZ} = \overrightarrow{OW} + \square + \square + \square$
$= \underline{a} + \square + \square + \square$
$= \square$

(d) But $\overrightarrow{OZ} = 2\overrightarrow{WX}$
So $\square + \square + \square = 2\underline{b}$
So . . . .

Continue with Section Z

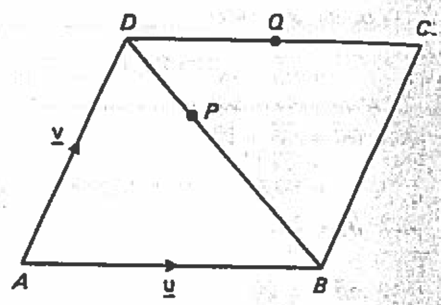
Z Progress check

Exercise



1 X, Y, Z, and W are the mid-points of the sides PQ, QR, RS, and SP of a quadrilateral $PQRS$. $\overrightarrow{QP} = \underline{a}$, $\overrightarrow{QR} = \underline{b}$, $\overrightarrow{RS} = \underline{c}$, and $\overrightarrow{PS} = \underline{d}$.

(a) Express $\overrightarrow{QS}$ and $\overrightarrow{XW}$ in terms of $\underline{a}$ and $\underline{d}$.
(b) Express $\overrightarrow{QS}$ and $\overrightarrow{YZ}$ in terms of $\underline{b}$ and $\underline{c}$.
(c) What can you say about XW and YZ?



2 $ABCD$ is a parallelogram with $\overrightarrow{AB} = \underline{u}$ and $\overrightarrow{AD} = \underline{v}$. Q is the mid-point of DC and P is the point on DB such that $DP = \frac{1}{3}\,DB$.

(a) Express $\overrightarrow{DQ}$ in terms of $\underline{u}$.
(b) Express $\overrightarrow{AQ}$ in terms of $\underline{u}$ and $\underline{v}$. What is $\frac{2}{3}\,\overrightarrow{AQ}$ in terms of $\underline{u}$ and $\underline{v}$?
(c) Express $\overrightarrow{DB}$, $\overrightarrow{DP}$, and $\overrightarrow{AP}$ in terms of $\underline{u}$ and $\underline{v}$.
(d) Using the results for $\overrightarrow{AQ}$ and $\overrightarrow{AP}$, what can you say about A, P, and Q?

Tell your teacher you have finished this Unit

26

RESULTANT FORCES

INFO: TO FIND THE RESULTANT FORCE JUST ADD THE VECTOR COMPONENTS

EXAMPLE IF $\underline{F_1} = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix}$ AND $\underline{F_2} = \begin{pmatrix} 2 \\ -5 \\ -1 \end{pmatrix}$ AND $\underline{F_3} = \begin{pmatrix} -4 \\ 5 \\ -2 \end{pmatrix}$

FIND THE RESULTANT FORCE

$$\underline{F_1} + \underline{F_2} + \underline{F_3} = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} + \begin{pmatrix} 2 \\ -5 \\ -1 \end{pmatrix} + \begin{pmatrix} -4 \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

EXERCISE FIND THE RESULTANT FORCE FOR EACH OF THE FOLLOWING:

1) $\underline{a_1} = \begin{pmatrix} -2 \\ -4 \\ 6 \end{pmatrix}$ $\underline{a_2} = \begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix}$ $\underline{a_3} = \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix}$

2) $\underline{f_1} = \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}$ $\underline{f_2} = \begin{pmatrix} -6 \\ -2 \\ 4 \end{pmatrix}$ $\underline{f_3} = \begin{pmatrix} 1 \\ -1 \\ -6 \end{pmatrix}$

3) $\underline{n_1} = \begin{pmatrix} -2 \\ -3 \\ -7 \end{pmatrix}$ $\underline{n_2} = \begin{pmatrix} 7 \\ 6 \\ 4 \end{pmatrix}$ $\underline{n_3} = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$ $\underline{n_4} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

4) $\underline{b_1} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$ $\underline{b_2} = \begin{pmatrix} -3 \\ -7 \\ 4 \end{pmatrix}$ $\underline{b_3} = \begin{pmatrix} -5 \\ 4 \\ -3 \end{pmatrix}$

5) $\underline{a} = \begin{pmatrix} -2 \\ -3 \\ -5 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$ $\underline{c} = \begin{pmatrix} -3 \\ -1 \\ -4 \end{pmatrix}$

MORE MAGNITUDE

EXAMPLE IF $\underline{a} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ AND $\underline{b} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, FIND $|2\underline{a} + 3\underline{b}|$

$$2\underline{a} + 3\underline{b}$$
$$= 2\begin{pmatrix} -3 \\ 4 \end{pmatrix} + 3\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$
$$= \begin{pmatrix} -6 \\ 8 \end{pmatrix} + \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$
$$= \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

so $|2\underline{a} + 3\underline{b}|$
$$= \sqrt{(-3)^2 + 2^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$ (or 3.61 to 2 dec. places)

EXERCISE

1. IF $\underline{p} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ AND $\underline{q} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ FIND $|\underline{p} + \underline{q}|$

2. IF $\underline{a} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ AND $\underline{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ FIND $|\underline{a} + 2\underline{b}|$

3. IF $\underline{m} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ AND $\underline{n} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ FIND $|2\underline{m} + \underline{n}|$

4. IF $\underline{x} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ AND $\underline{y} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ FIND $|2\underline{x} + 3\underline{y}|$

5. IF $\underline{e} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ AND $\underline{f} = \begin{pmatrix} -5 \\ -2 \end{pmatrix}$ FIND $|4\underline{e} - 3\underline{f}|$

6. IF $\underline{a} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ AND $\underline{b} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$ FIND $|2\underline{a} + \underline{b}|$

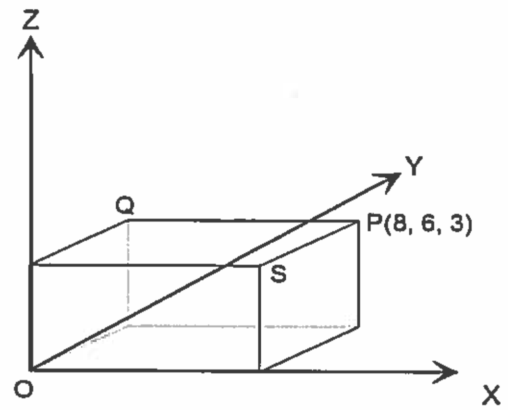
7. IF $\underline{p} = \begin{pmatrix} -3 \\ 4 \\ -5 \end{pmatrix}$ AND $\underline{q} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ FIND $|2\underline{p} + 3\underline{q}|$

3D Coordinates

1. The diagram shows a cuboid relative to the coordinate axes

P is the point (8, 6, 3).

Write down the coordinates of Q and S

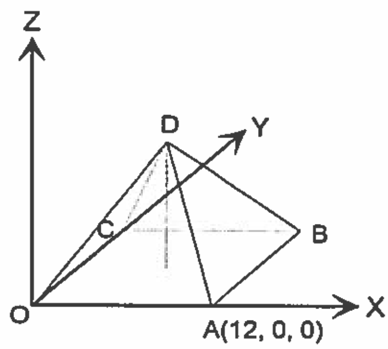


2. The diagram below shows a square based model of a glass pyramid of height 10cm. Square OABC has a side length of 12cm.

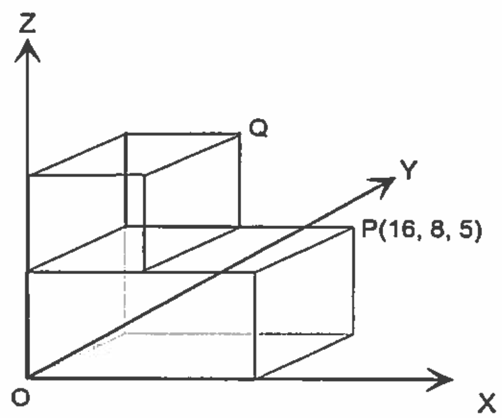
The coordinates of A are (12, 0, 0).

C lies on the y - axis.

Write down the coordinates of C and D.

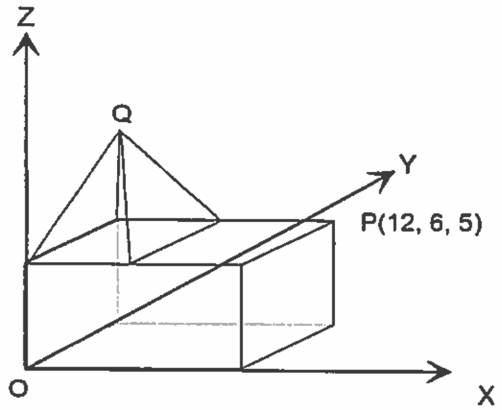


3. The diagram shows a cube placed on top of a cuboid, relative to the coordinate axes.



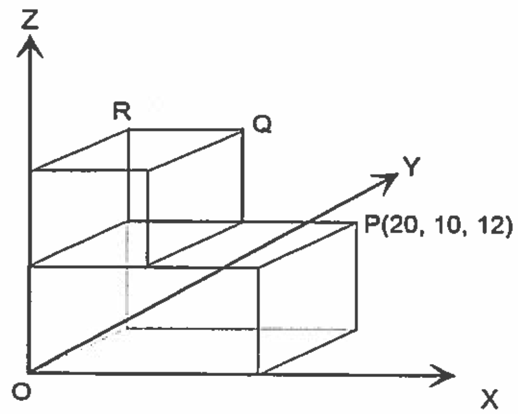
P is the point (16, 8, 5). Write down the coordinates of Q

4. The diagram shows a square-based pyramid of height 5cm placed on top of a cuboid, relative to the coordinate axes.



P is the point (12, 6, 5). Write down the coordinates of Q

5. The diagram shows a cube placed on top of a cuboid, relative to the coordinate axes.



P is the point (20, 10, 12). Write down the coordinates of Q and R

UNIT M2 Vectors

C 2 $\begin{pmatrix}5\\2\end{pmatrix}$ 3 $\begin{pmatrix}2\\1\end{pmatrix}$ 4 $\begin{pmatrix}6\\2\end{pmatrix}$ 5 $\begin{pmatrix}2\\1\end{pmatrix}$ 6 $\begin{pmatrix}3\\0\end{pmatrix}$

F 2 $\begin{pmatrix}-3\\2\end{pmatrix}$ 3 $\begin{pmatrix}-2\\-3\end{pmatrix}$ 4 $\begin{pmatrix}2\\-2\end{pmatrix}$ 5 $\begin{pmatrix}1\\-3\end{pmatrix}$ 6 $\begin{pmatrix}-1\\-3\end{pmatrix}$

H 1(a) $\begin{pmatrix}3\\2\end{pmatrix}$ (b) $\begin{pmatrix}4\\-1\end{pmatrix}$ (c) $\begin{pmatrix}-3\\-2\end{pmatrix}$ (d) $\begin{pmatrix}-2\\5\end{pmatrix}$ (e) $\begin{pmatrix}6\\-1\end{pmatrix}$ (f) $\begin{pmatrix}-6\\1\end{pmatrix}$ (g) $\begin{pmatrix}0\\2\end{pmatrix}$ (h) $\begin{pmatrix}4\\0\end{pmatrix}$

I 1 $\underline{v} = \begin{pmatrix}2\\3\end{pmatrix}$; $\underline{s} = \begin{pmatrix}4\\-3\end{pmatrix}$; $\underline{a} = \begin{pmatrix}4\\-2\end{pmatrix}$

J 2 length of $\underline{u}$ = 5.4 units; $\underline{w}$ = 4.1 units; $\underline{v}$ = 3.6 units; $\underline{a}$ = 5 units; $\underline{b}$ = 6.4 units; $\underline{c}$ = 2 units

K 1 $\sqrt{25}$ = 5 units 2 $\sqrt{40}$ = 6.32 units 3 $\sqrt{50}$ = 7.07 units
4 $\sqrt{13}$ = 3.61 units 5 $\sqrt{25}$ = 5 units 6 $\sqrt{10}$ = 3.16 units
7 $\sqrt{20}$ = 4.47 units 8 $\sqrt{10}$ = 3.16 units 9 $\sqrt{26}$ = 5.10 units
10 $\sqrt{29}$ = 5.39 units 11 $\sqrt{16}$ = 4 units 12 $\sqrt{32}$ = 5.66 units

L 1 $\begin{pmatrix}4\\5\end{pmatrix}$ 2 $\begin{pmatrix}4\\3\end{pmatrix}$ 3 $\begin{pmatrix}4\\6\end{pmatrix}$ 4 $\begin{pmatrix}2\\6\end{pmatrix}$ 5 $\begin{pmatrix}8\\5\end{pmatrix}$ 6 $\begin{pmatrix}5\\7\end{pmatrix}$ 7 $\begin{pmatrix}7\\7\end{pmatrix}$ 8 $\begin{pmatrix}5\\7\end{pmatrix}$

M 2 $\underline{u} = \begin{pmatrix}3\\-1\end{pmatrix}$; $\underline{v} = \begin{pmatrix}1\\3\end{pmatrix}$; $\underline{u} + \underline{v} = \begin{pmatrix}4\\2\end{pmatrix}$
3 $\underline{u} = \begin{pmatrix}3\\1\end{pmatrix}$; $\underline{v} = \begin{pmatrix}1\\-2\end{pmatrix}$; $\underline{u} + \underline{v} = \begin{pmatrix}4\\-1\end{pmatrix}$
4 $\underline{u} = \begin{pmatrix}3\\1\end{pmatrix}$; $\underline{v} = \begin{pmatrix}2\\3\end{pmatrix}$; $\underline{u} + \underline{v} = \begin{pmatrix}5\\4\end{pmatrix}$
5 $\underline{u} = \begin{pmatrix}2\\0\end{pmatrix}$; $\underline{v} = \begin{pmatrix}0\\2\end{pmatrix}$; $\underline{u} + \underline{v} = \begin{pmatrix}2\\2\end{pmatrix}$
6 $\underline{u} = \begin{pmatrix}3\\-3\end{pmatrix}$; $\underline{v} = \begin{pmatrix}1\\2\end{pmatrix}$; $\underline{u} + \underline{v} = \begin{pmatrix}4\\-1\end{pmatrix}$

N 7 $\begin{pmatrix}7\\6\end{pmatrix}$ 8 $\begin{pmatrix}4\\5\end{pmatrix}$ 9 $\begin{pmatrix}5\\2\end{pmatrix}$ 10 $\begin{pmatrix}-2\\-5\end{pmatrix}$ 11 $\begin{pmatrix}-1\\-2\end{pmatrix}$ 12 $\begin{pmatrix}-4\\1\end{pmatrix}$ 13 $\begin{pmatrix}3\\-2\end{pmatrix}$
14 $\begin{pmatrix}1\\4\end{pmatrix}$ 15 $\begin{pmatrix}2\\0\end{pmatrix}$ 16 $-\underline{x} = \begin{pmatrix}-3\\-4\end{pmatrix}$ 17 $-\underline{z} = \begin{pmatrix}-7\\-3\end{pmatrix}$ 18 $-\underline{w} = \begin{pmatrix}2\\-8\end{pmatrix}$
19 $-\underline{t} = \begin{pmatrix}-4\\1\end{pmatrix}$

O 3 (a) $\begin{pmatrix}12\\9\end{pmatrix}$ (b) $\begin{pmatrix}-10\\5\end{pmatrix}$ (c) $\begin{pmatrix}-21\\-7\end{pmatrix}$ (d) $\begin{pmatrix}15\\3\end{pmatrix}$ (e) $\begin{pmatrix}0\\8\end{pmatrix}$ $\begin{pmatrix}12\\-8\end{pmatrix}$

P 1 (a) $\begin{pmatrix}-10\\-5\end{pmatrix}$ (b) $\begin{pmatrix}-12\\-4\end{pmatrix}$ (c) $\begin{pmatrix}6\\-3\end{pmatrix}$ (d) $\begin{pmatrix}-8\\0\end{pmatrix}$ (e) $\begin{pmatrix}-10\\5\end{pmatrix}$ (f) $\begin{pmatrix}9\\3\end{pmatrix}$

Q Progress check

S 1 $\underline{u} = \begin{pmatrix}2\\1\end{pmatrix}$; $\underline{v} = \begin{pmatrix}2\\-2\end{pmatrix}$; $\underline{w} = \begin{pmatrix}0\\4\end{pmatrix}$; $\underline{u} + \underline{v} = \begin{pmatrix}4\\-1\end{pmatrix}$; $\underline{u} + \underline{v} + \underline{w} = \begin{pmatrix}4\\3\end{pmatrix}$
2 $\underline{a} = \begin{pmatrix}2\\1\end{pmatrix}$; $\underline{b} = \begin{pmatrix}1\\-3\end{pmatrix}$; $\underline{c} = \begin{pmatrix}-2\\0\end{pmatrix}$; $\underline{a} + \underline{b} = \begin{pmatrix}3\\-2\end{pmatrix}$; $\underline{a} + \underline{b} + \underline{c} = \begin{pmatrix}1\\-2\end{pmatrix}$
3 $\underline{u} = \begin{pmatrix}3\\1\end{pmatrix}$; $\underline{v} = \begin{pmatrix}-1\\2\end{pmatrix}$; $\underline{w} = \begin{pmatrix}-2\\-3\end{pmatrix}$; $\underline{u} + \underline{v} = \begin{pmatrix}2\\3\end{pmatrix}$; $\underline{u} + \underline{v} + \underline{w} = \begin{pmatrix}0\\0\end{pmatrix}$
4 (a) $\begin{pmatrix}-2\\2\end{pmatrix}$ (b) $\begin{pmatrix}-2\\0\end{pmatrix}$ (c) $\begin{pmatrix}-3\\0\end{pmatrix}$

T 2 (a) $\begin{pmatrix}5\\-2\end{pmatrix}$ (b) $\begin{pmatrix}-4\\6\end{pmatrix}$ (c) $\begin{pmatrix}6\\3\end{pmatrix}$ (d) $\begin{pmatrix}20\\-5\end{pmatrix}$ 4 (a) $\begin{pmatrix}-1\\-2\end{pmatrix}$ (b) $\begin{pmatrix}-5\\3\end{pmatrix}$
(c) $\begin{pmatrix}-6\\-12\end{pmatrix}$ (d) $\begin{pmatrix}10\\15\end{pmatrix}$

U 1 $\underline{u}+\underline{v}=\begin{pmatrix}4\\6\end{pmatrix}$; $2(\underline{u}+\underline{v})=\begin{pmatrix}8\\12\end{pmatrix}$; $2\underline{u}=\begin{pmatrix}6\\8\end{pmatrix}$; $2\underline{v}=\begin{pmatrix}2\\4\end{pmatrix}$;
$2\underline{u}+2\underline{v}=\begin{pmatrix}8\\12\end{pmatrix}$; $2(\underline{u}+\underline{v})=2\underline{u}+2\underline{v}$ 3 $\underline{u}+\underline{v}=\begin{pmatrix}10\\12\end{pmatrix}$;
$\frac{1}{2}(\underline{u}+\underline{v})=\begin{pmatrix}5\\6\end{pmatrix}$ $\frac{1}{2}\underline{u}=\begin{pmatrix}3\\2\end{pmatrix}$; $\frac{1}{2}\underline{v}=\begin{pmatrix}2\\4\end{pmatrix}$; $\frac{1}{2}\underline{u}+\frac{1}{2}\underline{v}=\begin{pmatrix}5\\6\end{pmatrix}$;
$\frac{1}{2}(\underline{u}+\underline{v})=\frac{1}{2}\underline{u}+\frac{1}{2}\underline{v}$

V 1 $\begin{pmatrix}2\\1\end{pmatrix}$ 2 $\begin{pmatrix}2\\5\end{pmatrix}$ 3 $\begin{pmatrix}2\\5\end{pmatrix}$ 4 $\begin{pmatrix}2\\2\end{pmatrix}$ 5 $\begin{pmatrix}-6\\-1\end{pmatrix}$ 6 $\begin{pmatrix}-3\\-3\end{pmatrix}$ 7 $\begin{pmatrix}-3\\-4\end{pmatrix}$

X 1 (a) $\overrightarrow{RP}$ (b) $\overrightarrow{PS}$ (c) $\overrightarrow{PQ}$ (d) $\overrightarrow{RP}$ (e) $\overrightarrow{RP}$ (f) $\overrightarrow{QS}$ (g) O (h) O
2 (a) WX is parallel to ZY (b) $WX=\frac{1}{2}ZY$ (c) $XY=\underline{a}+\underline{b}$
3 (a) $\underline{u}$ (b) $\underline{u}+\underline{v}$ (c) $\underline{v}$ (d) $\underline{u}$ (e) $\underline{u}+\underline{v}$. Lines EA and DB and parallel and equal in length.
4 (a) $\underline{a}+\underline{b}$ (b) $\underline{a}+\underline{b}$. N is the mid-point of LM.
5 (a) $\underline{y}$ (b) $\underline{x}+\underline{y}$ (c) $2\underline{x}+\underline{y}$ (d) $2\underline{x}$ (e) $2\underline{y}$ (f) $2\underline{x}+2\underline{y}$. So $\overrightarrow{AC}=2\overrightarrow{AF}$, so A, F, and C lie in a straight line.

Y 1 $\overrightarrow{BC}=\overrightarrow{BA}+\overrightarrow{AC}=\underline{u}+\underline{v}$; $\overrightarrow{PQ}+\overrightarrow{PA}+\overrightarrow{AQ}=\frac{1}{4}\underline{u}+\frac{1}{4}\underline{v}$. So $4\overrightarrow{PQ}=\underline{u}+\underline{v}$. Because $\overrightarrow{BC}=4\overrightarrow{PQ}$, PQ is parallel to BC and is a quarter the length of BC.
2 (a) $4\underline{a}-4\underline{b}$ (b) $2\underline{a}+2\underline{b}$ (c) $2\underline{a}+2\underline{b}$ (d) $\underline{a}+\underline{b}$. But $\overrightarrow{ED}=\overrightarrow{EO}+\overrightarrow{OD}=-\underline{b}+\underline{a}+\underline{b}=\underline{a}$. So $\overrightarrow{ED}=\frac{1}{4}\overrightarrow{OA}$, so ED is parallel to OA. 3 (a) $2\underline{a}+\underline{b}+\underline{c}$ (b) $2\underline{a}=\underline{b}-\underline{c}$.

Z Progress check

UNIT M3 Simple equations and inequations

A 1 $x=3$ 2 $x=4$ 3 $x=3$ 4 $x=4$ 5 $x=1$ 6 $x=2$ 7 $x=4$ 8 $x=0$

C 1 12 2 5 3 7 4 6 5 6 6 40 7 8 8 12 9 12 10 19
11 20 12 4 13 −3 14 −3 15 0 16 −7 17 −8 18 −2
19 11 20 −6 21 −4 22 $10-a$ 23 $9-b$ 24 $20-p$ 25 $-15-q$
26 $1-t$ 27 $-2-b$ 28 $10+b$ 29 $4+r$ 30 $9+c$ 31 $7+a$
32 $11+c$ 33 $q+p$ 34 $t-r$ 35 $a+b$ 36 $s-q$ 37 $z+y$ 38 $d-a$
39 $r-d$ 40 $v+t$

D 1 7 2 7 3 4 4 21 5 12 6 3 7 13 8 9 9 −4 10 9 11 −7
12 −4 13 −7 14 5 15 −8 16 6 17 4.5 18 2.5 19 −3.2
20 −2.5 21 −0.5 22 2.5 23 −2.8 24 1.2
25 $\frac{5}{p}$ 26 $-\frac{8}{r}$ 27 $\frac{p}{5}$ 28 $-\frac{d}{7}$ 29 $\frac{b}{a}$ 30 $-\frac{k}{b}$ 31 $\frac{p}{a}$ 32 $\frac{10}{c}$

E 1 28 2 35 3 24 4 21 5 16 6 6 7 90 8 22 9 −48 10 6
11 10 12 −44 13 −15 14 −56 15 40 16 −15 17 4
19 −21 20 8 21 −7 22 44 23 −4 24 −98 25 $9b$ 2
28 ap 29 $-ab$ 30 $-cd$

F 1 4 2 4 3 2 4 3 5 14.5 6 1 7 5.5 8 4.2 9 1
12 3.1 13 3 14 −5 15 −2 16 −2 17 2 18

ANSWERS TO PROGRESS CHECKS

Teachers may want to cut these pages from the book.

Unit M1

H 1 Fig. 2 2 $k = 5$; length = 29.5 cm; breadth = 18.0 cm 3 Length = 9.6 cm; width = 2.8 cm; height = 2.4 cm 5 (a) Reduction; $k = 0.5$; $PQ = 8$; $RP = 3.5$ (b) Enlargement; $k = 2$; $XY = 10$; $ZX = 8$

M 1 $k = 0.8$ 2 $PQ = 4.5$ cm 3 $OB = 7.2$ cm; $OD = 10$ cm 4 1.28 m^2 5 $k = 2$; 4 litres 6 2.4 m

Unit M2

Q 1 $\underline{v} = \begin{pmatrix}7\\2\end{pmatrix}$; $\underline{u} = \begin{pmatrix}-4\\2\end{pmatrix}$; $\underline{w} = \begin{pmatrix}2\\-4\end{pmatrix}$; $\underline{s} = \begin{pmatrix}3\\0\end{pmatrix}$; $\underline{t} = \begin{pmatrix}-3\\-2\end{pmatrix}$

Length of $\underline{a}$ is 5.83 units; $\underline{b}$ is 3.61 units; $\underline{c}$ is 4 units; $\underline{d}$ is 3 units; $\underline{e}$ is 3.61 units.

4 (a) $\begin{pmatrix}6\\6\end{pmatrix}$ (b) $\begin{pmatrix}3\\5\end{pmatrix}$ (c) $\begin{pmatrix}6\\2\end{pmatrix}$ (d) $\begin{pmatrix}5\\0\end{pmatrix}$ 5 $-\underline{x} = \begin{pmatrix}-2\\-7\end{pmatrix}$; $-\underline{y} = \begin{pmatrix}-3\\1\end{pmatrix}$; $-\underline{v} = \begin{pmatrix}4\\3\end{pmatrix}$

6 $2\underline{u} = \begin{pmatrix}6\\8\end{pmatrix}$; $-3\underline{a} = \begin{pmatrix}-6\\-12\end{pmatrix}$

Z 1 (a) $\overrightarrow{QS} = \underline{a} + \underline{d}$; $\overrightarrow{XD} = \frac{1}{2}\underline{a} + \frac{1}{2}\underline{d}$ (b) $\overrightarrow{QS} = \underline{b} + \underline{c}$; $\overrightarrow{YZ} = \frac{1}{2}\underline{b} + \frac{1}{2}\underline{c}$ (c) XW is equal and parallel to YZ. 2 (a) $\overrightarrow{DQ} = \frac{1}{2}\underline{u}$ (b) $\overrightarrow{AQ} = \underline{v} + \frac{1}{2}\underline{u}$; $\frac{2}{3}\overrightarrow{AQ} = \frac{2}{3}\underline{v}$; $\overrightarrow{AP} = \frac{1}{3}\underline{u} + \frac{2}{3}\underline{v}$ (d) A, P, and Q lie on a straight line.

Unit M3

I 1 (a) 1 (b) 0 (c) 2 (d) 4 2 (a) 18 (b) −8 (c) 4 (d) −28 3 (a) 20 − a (b) $p + q$ (c) $\frac{8}{r}$ (d) rs 4 (a) 4 (b) 2.5 5 (a) $3x - 12$ (b) $-2y + 6$ (c) $-15 + 6p$ 6 (a) 5 (b) 3 7 (a) $(d - c)/c$ (b) $(q + cp)/c$

R 1 (a) $x \geq 2$ (b) $x \geq -2$ (c) $x > -5$ 2 (a) $y \leq -8$ (b) $x > 3$ (c) $x > -8.4$ (d) $x < 2$ 3 (a) $x = 13$ (b) $x < \frac{11}{14}$ 4 $3x - 2(x + 5) = 8$; 18, 23

5 A pencil costs less than 9p.

6 (a) $R = \dfrac{100I}{PT}$ (b) $r = \sqrt{\dfrac{A}{4\pi}}$ (c) (i) $c = \dfrac{b^2 - D^2}{4a}$ (ii) $b = \sqrt{D^2 + 4ac}$

Unit M4

R 1 (a) 0.602 (b) 0.812 (c) 0.423 (d) 0.692 (e) 0.625 (f) 0.719 (g) 1.376 (h) 2.032
2 (a) 51.0 (b) 62.5 or 62.6 (c) 24.0 (d) 65.5 (e) 59.5 (f) 63.6 (g) 59.0 (h) 67.8
3 21.8 4 48.2 5 9.06 6 32.0 7 15.3 8 9.19

W 1 10.0 m 2, 8.77 m 3 6.53 m 4 7.16 m 5 7.71 m 6 10.8 m 7 18.7 cm 8 20.4 cm 9 2.80 m 10 739 m 11 4.79 m 12 8.09 m 13 10.2 m 14 KL = 13.8 m, alt = 5.79 m, area = 39.9 m^2 15 16.3 km 166 15.7 km.

ANSWERS : RESULTANT FORCES

(1) $\begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix}$ (2) $\begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix}$ (3) $\begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix}$ (4) $\begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix}$ (5) $\begin{pmatrix} -1 \\ -2 \\ -6 \end{pmatrix}$

ANSWERS : MORE MAGNITUDE

(1) $\sqrt{65} = 8.06$ (2) $\sqrt{29} = 5.39$ (3) 1 (4) 7

(5) $\sqrt{37} = 6.08$ (6) $\sqrt{18} = 3\sqrt{2} = 4.24$ (7) $\sqrt{14} = 3.74$

ANSWERS : 3D COORDINATES

(1) Q (0, 6, 3) S (8, 0, 3)

(2) C (0, 12, 0) D (6, 6, 10)

(3) Q (8, 8, 13)

(4) Q (3, 3, 10)

(5) Q (10, 10, 22) R (0, 10, 22)

EVALUATE THE FOLLOWING, EXPRESSING YOUR ANSWER IN ITS SIMPLEST FORM:-

(36) $\frac{1}{2}+\frac{3}{4}-\frac{5}{8}$ (37) $\frac{2}{3}+\frac{1}{4}-\frac{1}{6}$ (38) $\frac{5}{8}-\frac{1}{6}+\frac{2}{3}$

(39) $\frac{1}{3}-\frac{1}{6}+\frac{5}{12}$ (40) $\frac{4}{5}-\frac{1}{3}+\frac{1}{2}$ (41) $\frac{7}{8}-\frac{3}{4}+\frac{1}{5}$

(42) $\frac{3}{5}-\frac{2}{3}+\frac{5}{6}$ (43) $\frac{1}{4}-\frac{3}{5}+\frac{2}{3}$ (44) $\frac{1}{2}-\frac{1}{3}+\frac{1}{4}$

(45) $3\frac{2}{3}+1\frac{1}{12}-2\frac{1}{2}$ (46) $4\frac{1}{2}-2\frac{2}{3}+1\frac{5}{6}$ (47) $5\frac{1}{4}-3\frac{1}{2}+2\frac{3}{8}$

(48) $3\frac{4}{5}+2\frac{3}{10}-1\frac{2}{5}$ (49) $1\frac{2}{3}+3\frac{5}{6}-2\frac{1}{3}$ (50) $2\frac{1}{4}-1\frac{3}{8}+3\frac{3}{16}$

7. MULTIPLICATION OF FRACTIONS.

MULTIPLYING A FRACTION BY A WHOLE NUMBER

(1) WRITE THE WHOLE NUMBER AS A FRACTION.

(2) LOOK FOR SOMETHING TO CANCEL.

(3) MULTIPLY WHAT IS LEFT ON THE TOP AND WHAT IS LEFT ON THE BOTTOM.

EXAMPLES

(i) $\frac{1}{3}\times 4$

$=\frac{1}{3}\times\frac{4}{1}$

$=\frac{1\times 4}{3\times 1}$

$=\frac{4}{3}$

$=1\frac{1}{3}$

(ii) $\frac{3}{16}\times 24$

$=\frac{3}{16}\times\frac{24}{1}$

$=\frac{3\times \cancel{24}^{3}}{\cancel{16}_{2}\times 1}$

(AS 8 DIVIDES INTO 16 AND 24)

$=\frac{3\times 3}{2\times 1}$

$=\frac{3}{2}$

$=1\frac{1}{2}$

(iii) $\frac{9}{16}\times 20$

$=\frac{9}{16}\times\frac{20}{1}$

$=\frac{9\times \cancel{20}^{5}}{\cancel{16}_{4}\times 1}$

(AS 4 DIVIDES INTO 16 AND 20)

$\frac{9\times 5}{4\times 1}$

$=\frac{45}{4}$

$=11\frac{1}{4}$

26

EXERCISE 11.

EVALUATE THE FOLLOWING, EXPRESSING YOUR ANSWER IN ITS SIMPLEST FORM:—

(1) $\frac{3}{4} \times 15$ (2) $\frac{5}{8} \times 8$ (3) $\frac{2}{3} \times 7$ (4) $\frac{5}{6} \times 10$ (5) $\frac{3}{8} \times 12$

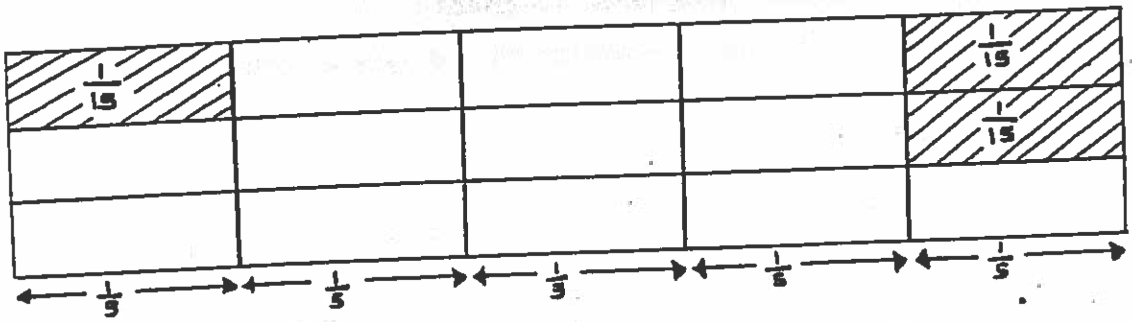
(6) $\frac{4}{9} \times 6$ (7) $\frac{4}{5} \times 20$ (8) $\frac{7}{8} \times 4$ (9) $\frac{5}{9} \times 12$ (10) $\frac{3}{7} \times 4$

(11) $\frac{2}{3} \times 15$ (12) $\frac{9}{15} \times 25$ (13) $24 \times \frac{1}{8}$ (14) $3 \times \frac{5}{9}$ (15) $12 \times \frac{7}{8}$

MULTIPLYING A FRACTION BY A FRACTION

Q (i) WHAT IS $\frac{1}{3}$ OF $\frac{1}{5}$? (ii) WHAT IS $\frac{2}{3}$ OF $\frac{1}{5}$?

LOOK AT THE DIAGRAM BELOW:—



(i) $\frac{1}{3}$ OF $\frac{1}{5}$

$= \frac{1}{3} \times \frac{1}{5}$

$= \frac{1 \times 1}{3 \times 5}$

$= \frac{1}{15}$

'OF' MEANS MULTIPLY

(ii) $\frac{2}{3}$ OF $\frac{1}{5}$

$= \frac{2}{3} \times \frac{1}{5}$

$= \frac{2 \times 1}{3 \times 5}$

$= \frac{2}{15}$

EXERCISE 12.

EVALUATE THE FOLLOWING, EXPRESSING YOUR ANSWER IN ITS SIMPLEST FORM:–

(1) $\frac{1}{3}$ OF $\frac{1}{2}$ (2) $\frac{2}{3}$ OF $\frac{1}{3}$ (3) $\frac{2}{5}$ OF $\frac{2}{3}$ (4) $\frac{3}{4}$ OF $\frac{1}{2}$

(5) $\frac{1}{2}$ OF $\frac{5}{6}$ (6) $\frac{1}{2}$ OF $\frac{6}{7}$ (7) $\frac{1}{4}$ OF $\frac{4}{5}$ (8) $\frac{3}{4}$ OF $\frac{1}{6}$

(9) $\frac{2}{7}$ OF $\frac{3}{5}$ (10) $\frac{4}{9}$ OF $\frac{1}{7}$ (11) $\frac{3}{8}$ OF $\frac{3}{4}$ (12) $\frac{2}{3}$ OF $\frac{3}{4}$

EXAMPLES

(i) $\frac{3}{8} \times \frac{5}{6}$

$= \frac{\overset{1}{\cancel{3}} \times 5}{8 \times \underset{2}{\cancel{6}}}$

(AS 3 DIVIDES INTO 3 AND 6)

$= \frac{1 \times 5}{8 \times 2}$

$= \frac{5}{16}$

(ii) $\frac{1}{3} \times \frac{4}{5} \times \frac{3}{8}$

$= \frac{1 \times \overset{1}{\cancel{4}} \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \times 5 \times \underset{2}{\cancel{8}}}$

(AS 3 DIVIDES INTO 3, AND 4 DIVIDES INTO 4 AND 8)

$= \frac{1 \times 1 \times 1}{1 \times 5 \times 2}$

$= \frac{1}{10}$

EVALUATE THE FOLLOWING, EXPRESSING YOUR ANSWER IN ITS SIMPLEST FORM:–

(13) $\frac{2}{3} \times \frac{3}{4}$ (14) $\frac{4}{5} \times \frac{7}{8}$ (15) $\frac{3}{4} \times \frac{8}{9}$ (16) $\frac{3}{5} \times \frac{5}{12}$ (17) $\frac{5}{6} \times \frac{9}{10}$

(18) $\frac{2}{3}$ OF $\frac{3}{10}$ (19) $\frac{2}{9}$ OF $\frac{3}{16}$ (20) $\frac{6}{7} \times \frac{7}{8}$ (21) $\frac{9}{11} \times \frac{11}{10}$ (22) $\frac{5}{6} \times \frac{3}{9}$

(23) $\frac{1}{9} \times \frac{5}{6} \times \frac{9}{10}$ (24) $\frac{3}{5} \times \frac{4}{7} \times \frac{5}{6}$ (25) $\frac{2}{3} \times \frac{3}{8} \times \frac{4}{5}$

(26) $\frac{3}{4} \times \frac{8}{9} \times \frac{1}{2}$ (27) $\frac{6}{7} \times \frac{14}{15} \times \frac{7}{8}$ (28) $\frac{9}{11} \times \frac{11}{15} \times \frac{5}{16}$

(29) $\frac{4}{5} \times \frac{7}{8} \times \frac{10}{21}$ (30) $\frac{9}{16} \times \frac{5}{6} \times \frac{4}{15}$ (31) $\frac{6}{7} \times \frac{8}{15} \times \frac{5}{16}$

(32) $\frac{7}{12} \times \frac{9}{10} \times \frac{5}{14}$.

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MULTIPLYING MIXED NUMBERS

WHEN MULTIPLYING, ALL MIXED NUMBERS **MUST BE CHANGED** INTO IMPROPER FRACTIONS.

EXAMPLES

(i) $2\frac{1}{7} \times 3\frac{1}{2}$

$= \frac{15}{7} \times \frac{7}{2}$

$= \frac{15 \times \cancel{7}^{1}}{{}_{1}\cancel{7} \times 2}$

$= \frac{15 \times 1}{1 \times 2}$

$= \frac{15}{2}$

$= \underline{\underline{7\frac{1}{2}}}$

(ii) $\frac{3}{10} \times 3\frac{3}{4} \times 1\frac{1}{3}$

$= \frac{3}{10} \times \frac{15}{4} \times \frac{4}{3}$

$= \frac{\cancel{3}^{1} \times \cancel{15}^{3} \times \cancel{4}^{1}}{{}_{2}\cancel{10} \times {}_{1}\cancel{4} \times \cancel{3}_{1}}$

$= \frac{1 \times 3 \times 1}{2 \times 1 \times 1}$

$= \frac{3}{2}$

$= \underline{\underline{1\frac{1}{2}}}$

EXERCISE 13.

EVALUATE THE FOLLOWING, EXPRESSING YOUR ANSWER IN ITS SIMPLEST FORM:-

(1) $2\frac{1}{6} \times \frac{12}{13}$ (2) $3\frac{3}{7} \times \frac{5}{13} \times 2\frac{1}{10}$ (3) $\frac{8}{9} \times 7\frac{1}{2}$

(4) $2\frac{1}{3} \times \frac{9}{14} \times 2\frac{2}{5}$ (5) $\frac{8}{9} \times 7\frac{1}{2} \times \frac{7}{10}$ (6) $2\frac{5}{8} \times 3\frac{3}{7} \times 1\frac{5}{6}$

(7) $2\frac{1}{6} \times 1\frac{4}{5} \times 1\frac{2}{13}$ (8) $2\frac{3}{4} \times 1\frac{5}{11} \times 1\frac{1}{2}$ (9) $1\frac{4}{7} \times 1\frac{5}{9} \times 1\frac{10}{11}$

(10) $1\frac{1}{3} \times \frac{3}{13} \times 2\frac{1}{6}$ (11) $6\frac{3}{4} \times \frac{7}{18} \times 1\frac{7}{9}$ (12) $5\frac{5}{8} \times \frac{4}{15} \times 1\frac{5}{9}$

(13) $2\frac{1}{7} \times \frac{5}{9} \times 3\frac{1}{2}$ (14) $1\frac{3}{5} \times 1\frac{7}{8} \times 2\frac{7}{10}$ (15) $\frac{5}{6} \times 9 \times 1\frac{2}{15}$

30.

TO DIVIDE A FRACTION BY 2, $\left(\frac{2}{1}\right)$. WE MULTIPLY IT BY $\frac{1}{2}$.

TO DIVIDE A FRACTION BY 3, $\left(\frac{3}{1}\right)$ WE MULTIPLY IT BY $\frac{1}{3}$.

TO DIVIDE A FRACTION BY 6, $\left(\frac{6}{1}\right)$ WE MULTIPLY IT BY $\frac{1}{6}$.

Q <u>DO YOU SEE WHAT IS HAPPENING</u>?

EXAMPLES

(i) $\frac{2}{3} \div 6$

$= \frac{2}{3} \div \frac{6}{1}$

$= \frac{2}{3} \times \frac{1}{6}$

$= \frac{{}^{1}\cancel{2} \times 1}{3 \times \cancel{6}_{3}}$

$= \frac{1 \times 1}{3 \times 3}$

$= \frac{1}{9}$

(ii) $\frac{15}{16} \div 10$

$= \frac{15}{16} \div \frac{10}{1}$

$= \frac{15}{16} \times \frac{1}{10}$

$= \frac{{}^{3}\cancel{15} \times 1}{16 \times \cancel{10}_{2}}$

$= \frac{3 \times 1}{16 \times 2}$

$= \frac{3}{32}$

(iii) $\frac{13}{16} \div 26$.

$= \frac{13}{16} \div \frac{26}{1}$

$= \frac{13}{16} \times \frac{1}{26}$

$= \frac{{}^{1}\cancel{13} \times 1}{16 \times \cancel{26}_{2}}$

$= \frac{1 \times 1}{16 \times 2}$

$= \frac{1}{32}$

EXERCISE 14.

<u>EVALUATE THE FOLLOWING, EXPRESSING YOUR ANSWER IN ITS SIMPLEST FORM:-</u>

① $\frac{1}{2} \div 3$ ② $\frac{2}{3} \div 4$ ③ $\frac{2}{3} \div 6$ ④ $\frac{3}{4} \div 6$ ⑤ $\frac{3}{4} \div 12$

⑥ $\frac{2}{5} \div 4$ ⑦ $\frac{3}{5} \div 9$ ⑧ $\frac{4}{5} \div 8$ ⑨ $\frac{1}{6} \div 2$ ⑩ $\frac{5}{6} \div 10$

⑪ $\frac{3}{8} \div 6$ ⑫ $\frac{5}{9} \div 15$ ⑬ $\frac{7}{8} \div 14$ ⑭ $\frac{7}{12} \div 21$ ⑮ $\frac{5}{6} \div 15$

EXERCISE 15.

EVALUATE THE FOLLOWING, EXPRESSING YOUR ANSWER IN ITS SIMPLEST FORM:-

(1) $\frac{1}{2} \div \frac{1}{4}$	(2) $\frac{1}{3} \div \frac{1}{2}$	(3) $\frac{2}{3} \div \frac{2}{5}$	(4) $\frac{1}{2} \div \frac{3}{4}$
(5) $\frac{3}{4} \div \frac{1}{2}$	(6) $\frac{2}{3} \div \frac{4}{5}$	(7) $\frac{3}{5} \div \frac{9}{10}$	(8) $\frac{3}{4} \div \frac{5}{6}$
(9) $\frac{2}{3} \div \frac{4}{9}$	(10) $\frac{4}{5} \div \frac{3}{10}$	(11) $\frac{3}{8} \div \frac{7}{12}$	(12) $\frac{5}{6} \div \frac{5}{9}$
(13) $\frac{1}{10} \div \frac{7}{30}$	(14) $\frac{4}{21} \div \frac{3}{14}$	(15) $\frac{12}{13} \div \frac{8}{39}$	(16) $\frac{14}{27} \div \frac{7}{18}$
(17) $\frac{8}{15} \div \frac{4}{25}$	(18) $\frac{15}{32} \div \frac{9}{16}$	(19) $\frac{18}{25} \div \frac{9}{20}$	(20) $\frac{5}{18} \div \frac{10}{27}$
(21) $\frac{14}{17} \div \frac{8}{51}$	(22) $\frac{25}{36} \div \frac{5}{18}$	(23) $\frac{13}{30} \div \frac{13}{30}$	(24) $\frac{14}{25} \div \frac{7}{15}$

DIVISION INVOLVING MIXED NUMBERS

WHEN DIVIDING, ALL MIXED NUMBERS **MUST BE CHANGED** INTO IMPROPER FRACTIONS.

EXAMPLES

(i) $5\frac{1}{3} \div 1\frac{3}{5}$

$= \frac{16}{3} \div \frac{8}{5}$

$= \frac{16}{3} \times \frac{5}{8}$

$= \frac{\overset{2}{\cancel{16}} \times 5}{3 \times \underset{1}{\cancel{8}}}$

$= \frac{2 \times 5}{3 \times 1}$

$= \frac{10}{3}$

$= 3\frac{1}{3}$

(ii) $2\frac{3}{4} \div 1\frac{1}{11}$

$= \frac{11}{4} \div \frac{12}{11}$ — DO NOT TRY TO CANCEL HERE

$= \frac{11}{4} \times \frac{11}{12}$ — NOTHING CANCELS

$= \frac{11 \times 11}{4 \times 12}$

$= \frac{121}{48}$

$= 2\frac{25}{48}$

EXERCISE 16

EVALUATE THE FOLLOWING, EXPRESSING YOUR ANSWER IN ITS SIMPLEST FORM:—

(1) $3\frac{3}{4} \div 1\frac{1}{8}$ (2) $3\frac{3}{5} \div 2\frac{7}{10}$ (3) $5\frac{2}{3} \div 1\frac{4}{7}$

(4) $2\frac{1}{10} \div \frac{7}{15}$ (5) $5\frac{1}{3} \div 2\frac{2}{9}$ (6) $5\frac{1}{10} \div 3\frac{2}{5}$

(7) $4\frac{1}{2} \div 1\frac{7}{8}$ (8) $6\frac{2}{3} \div 1\frac{4}{7}$ (9) $1\frac{5}{9} \div 1\frac{1}{6}$

(10) $1\frac{2}{9} \div \frac{22}{27}$ (11) $1\frac{11}{12} \div 2\frac{7}{8}$ (12) $7\frac{1}{4} \div 2\frac{5}{12}$

(13) $3\frac{2}{7} \div 4\frac{5}{6}$ (14) $3\frac{2}{5} \div 5\frac{1}{10}$ (15) $3\frac{3}{4} \div \frac{3}{4}$

ADDITIONAL MISCELLANEOUS EXERCISE

EVALUATE THE FOLLOWING, EXPRESSING YOUR ANSWER IN ITS SIMPLEST FORM:—

(1) (a) $3\frac{1}{7} \times 1\frac{3}{11}$ (b) $2\frac{1}{4} + 1\frac{1}{2} - 1\frac{2}{3}$ (c) $9 \times 5\frac{2}{9}$

(d) $5\frac{2}{3} \div 3\frac{1}{2}$ (e) $15 \div 2\frac{1}{7}$

(2) (a) $\frac{7}{8} + \frac{2}{3} - \frac{5}{6}$ (b) $2\frac{1}{5} \times 1\frac{1}{11}$ (c) $\frac{3}{7} \div \frac{4}{5}$

(d) $\frac{2}{5} + \frac{1}{3} - \frac{1}{2}$ (e) $1\frac{3}{8} - \frac{4}{5}$

(3) (a) $\frac{2}{3} + \frac{1}{2} - \frac{3}{4}$ (b) $3\frac{1}{2} \times 1\frac{2}{7}$ (c) $\frac{12}{25} \div \frac{6}{7}$

(d) $\frac{1}{5} - 1\frac{1}{9} + 1\frac{3}{4}$ (e) $\frac{7}{8} \div 1\frac{5}{16}$

34.

④ (a) $1\frac{3}{4} - \frac{7}{8}$ (b) $3\frac{3}{4} \times \frac{4}{5}$ (c) $2\frac{2}{11} \div 2\frac{2}{3}$

(d) $1\frac{3}{4} - 2 + 1\frac{1}{3}$ (e) $2\frac{1}{2} \div 1\frac{3}{8}$

⑤ (a) $2\frac{2}{3} - 1\frac{3}{4}$ (b) $\frac{3}{7}$ OF $3\frac{1}{2}$ (c) $\left(\frac{2}{5} + \frac{3}{10}\right) \times \frac{2}{7}$

(d) $\frac{5}{6} \div 2\frac{2}{5}$ (e) $5\frac{3}{5} - 2\frac{2}{5} \times \frac{1}{6}$

⑥ (a) $\frac{3}{5} + 1\frac{7}{10} + \frac{1}{2}$ (b) $\frac{2}{3} \times 4\frac{1}{2}$ (c) $5\frac{1}{4} \div 4\frac{2}{3}$

(d) $\left(\frac{3}{4} - \frac{1}{2}\right) + \frac{1}{4}$ (e) $\dfrac{\frac{1}{2} \text{ OF } \frac{4}{5}}{2}$

⑦ (a) $1\frac{2}{3} - 1\frac{5}{6} + \frac{1}{2}$ (b) $1\frac{3}{4} \times 3\frac{1}{7}$ (c) $1\frac{2}{3} \div \frac{5}{6}$

(d) $\frac{6}{7} + \frac{1}{2} - \frac{2}{3}$ (e) $2\frac{5}{6} - 1\frac{3}{4}$

⑧ (a) $1\frac{1}{7} \times 3\frac{1}{2}$ (b) $2\frac{3}{4} + 1\frac{1}{2} - 1\frac{2}{3}$ (c) $5 \times 2\frac{2}{5}$

(d) $7\frac{1}{3} \div 1\frac{5}{6}$ (e) $\frac{1}{2} + \frac{2}{3} - \frac{3}{4}$

⑨ (a) $\frac{9}{16} + \frac{3}{4} - \frac{5}{8}$ (b) $2\frac{1}{4} \times 3\frac{1}{2}$ (c) $\frac{6}{7} \div \frac{3}{5}$

(d) $2\frac{1}{4} - 1\frac{1}{2}$ (e) $6\frac{1}{2} - 8 + 3\frac{2}{3}$

⑩ (a) $3\frac{2}{3} - 2\frac{7}{8}$ (b) $\frac{2}{7} \times 2\frac{4}{5} \times \frac{1}{4}$ (c) $\dfrac{\frac{3}{8}}{\frac{1}{2} + 1\frac{1}{4}}$

(d) $3\frac{1}{3}$ OF $4\frac{1}{2}$ (e) $16\frac{1}{3} \div 1\frac{1}{6}$

FRACTIONS ANSWERS

P23. EX10. (contd.)

(33) $3\frac{11}{12}$ (34) $\frac{7}{10}$ (35) $1\frac{13}{20}$ (36) $\frac{5}{8}$ (37) $\frac{3}{4}$ (38) $1\frac{1}{8}$ (39) $\frac{7}{12}$ (40) $\frac{29}{30}$
(41) $\frac{13}{40}$ (42) $\frac{23}{30}$ (43) $\frac{19}{60}$ (44) $\frac{5}{12}$ (45) $2\frac{1}{4}$ (46) $3\frac{2}{3}$ (47) $4\frac{1}{8}$ (48) $4\frac{7}{10}$
(49) $3\frac{1}{6}$ (50) $4\frac{3}{16}$

PG 26. EX 11.

(1) $11\frac{1}{4}$ (2) 5 (3) $4\frac{2}{3}$ (4) $8\frac{1}{3}$ (5) $4\frac{1}{2}$ (6) $2\frac{2}{3}$ (7) 16 (8) $3\frac{1}{2}$
(9) $6\frac{2}{3}$ (10) $1\frac{5}{7}$ (11) 10 (12) 15 (13) 3 (14) $1\frac{2}{3}$ (15) $10\frac{1}{2}$

PG 27. EX12.

(1) $\frac{1}{6}$ (2) $\frac{2}{9}$ (3) $\frac{4}{15}$ (4) $\frac{3}{8}$ (5) $\frac{5}{12}$ (6) $\frac{3}{7}$ (7) $\frac{1}{5}$ (8) $\frac{1}{8}$
(9) $\frac{6}{35}$ (10) $\frac{4}{63}$ (11) $\frac{9}{32}$ (12) $\frac{1}{2}$ (13) $\frac{1}{2}$ (14) $\frac{7}{10}$ (15) $\frac{2}{3}$ (16) $\frac{1}{4}$
(17) $\frac{3}{4}$ (18) $\frac{1}{5}$ (19) $\frac{1}{24}$ (20) $\frac{3}{4}$ (21) $\frac{9}{10}$ (22) $\frac{20}{27}$ (23) $\frac{1}{12}$ (24) $\frac{2}{7}$
(25) $\frac{1}{5}$ (26) $\frac{1}{3}$ (27) $\frac{7}{10}$ (28) $\frac{3}{16}$ (29) $\frac{1}{3}$ (30) $\frac{1}{8}$ (31) $\frac{1}{7}$ (32) $\frac{3}{16}$

PG 28. EX13.

(1) 2 (2) 3 (3) $6\frac{2}{3}$ (4) $3\frac{3}{5}$ (5) $4\frac{2}{3}$ (6) $16\frac{1}{2}$ (7) $4\frac{1}{2}$ (8) 6
(9) $4\frac{2}{3}$ (10) $1\frac{1}{2}$ (11) $4\frac{2}{3}$ (12) $2\frac{1}{3}$ (13) $6\frac{2}{3}$ (14) $8\frac{1}{10}$ (15) $8\frac{1}{2}$

PG 30. EX 14.

(1) $\frac{1}{6}$ (2) $\frac{1}{6}$ (3) $\frac{1}{9}$ (4) $\frac{1}{8}$ (5) $\frac{1}{16}$ (6) $\frac{1}{10}$ (7) $\frac{1}{15}$ (8) $\frac{1}{10}$
(9) $\frac{1}{12}$ (10) $\frac{1}{12}$ (11) $\frac{1}{16}$ (12) $\frac{1}{24}$ (13) $\frac{1}{16}$ (14) $\frac{1}{36}$ (15) $\frac{1}{18}$

PG 32. EX 15.

(1) 2 (2) $\frac{2}{3}$ (3) $1\frac{2}{3}$ (4) $\frac{2}{3}$ (5) $1\frac{1}{2}$ (6) $\frac{5}{6}$ (7) $\frac{2}{3}$ (8) $\frac{9}{10}$
(9) $1\frac{1}{2}$ (10) $2\frac{2}{3}$ (11) $\frac{9}{14}$ (12) $1\frac{1}{2}$ (13) $\frac{3}{7}$ (14) $\frac{8}{9}$ (15) $4\frac{1}{2}$ (16) $1\frac{1}{3}$
(17) $3\frac{1}{3}$ (18) $\frac{5}{6}$ (19) $1\frac{3}{5}$ (20) $\frac{3}{4}$ (21) $5\frac{1}{4}$ (22) $2\frac{1}{2}$ (23) 1 (24) $1\frac{1}{5}$

PG 33. EX 16.

(1) $3\frac{1}{3}$ (2) $1\frac{1}{3}$ (3) $3\frac{20}{23}$ (4) $4\frac{1}{2}$ (5) $2\frac{2}{5}$ (6) $1\frac{1}{2}$ (7) $2\frac{2}{5}$ (8) $4\frac{8}{33}$
(9) $1\frac{1}{3}$ (10) $1\frac{1}{2}$ (11) $\frac{2}{3}$ (12) 3 (13) $\frac{2}{3}$ (14) $\frac{2}{3}$ (15) 5

4

ADDITIONAL MISCELLANEOUS EXERCISE

① (a) 2	(b) $2\frac{1}{12}$	(c) 47	(d) $1\frac{13}{21}$	(e) 7
② (a) $\frac{17}{24}$	(b) $2\frac{2}{5}$	(c) $\frac{15}{28}$	(d) $\frac{7}{30}$	(e) $\frac{23}{40}$
③ (a) $\frac{5}{12}$	(b) $4\frac{1}{2}$	(c) $\frac{14}{25}$	(d) $\frac{33}{40}$	(e) $\frac{2}{3}$
④ (a) $\frac{7}{8}$	(b) 3	(c) $\frac{9}{11}$	(d) $1\frac{1}{12}$	(e) $1\frac{9}{11}$
⑤ (a) $\frac{11}{12}$	(b) $1\frac{1}{2}$	(c) $\frac{1}{5}$	(d) $\frac{25}{72}$	(e) $5\frac{1}{5}$
⑥ (a) $2\frac{4}{5}$	(b) 3	(c) $2\frac{1}{4}$	(d) $\frac{1}{2}$	(e) $\frac{1}{5}$
⑦ (a) $\frac{1}{3}$	(b) $5\frac{1}{2}$	(c) 2	(d) $\frac{29}{42}$	(e) $1\frac{1}{12}$
⑧ (a) 4	(b) $2\frac{7}{12}$	(c) 12	(d) 4	(e) $\frac{5}{12}$
⑨ (a) $\frac{11}{16}$	(b) $7\frac{7}{8}$	(c) $1\frac{3}{7}$	(d) $\frac{3}{4}$	(e) $2\frac{1}{6}$
⑩ (a) $\frac{19}{24}$	(b) $\frac{1}{5}$	(c) $\frac{3}{14}$	(d) 15	(e) 14

PG 36. EX 17.

① 120°	② 750	③ 255	④ 24	⑤ 48	⑥ 10	⑦ 42	⑧ 300
⑨ $\frac{1}{6}$	⑩ $\frac{4}{5}$	⑪ $\frac{1}{8}$	⑫ $\frac{2}{15}$	⑬ (a) 90g	(b) 250g	(c) 152g	⑭ $\frac{1}{2}$
⑮ $\frac{3}{4}$	⑯ $\frac{7}{45}$	⑰ $\frac{1}{3}$	⑱ £224, £140	⑲ 60000			

PG 38. EX 18.

① 0.5	② 0.25	③ 0.75	④ 0.125	⑤ 0.8	⑥ 0.4375	⑦ 0.04	⑧ 0.625
⑨ 0.02	⑩ 0.1	⑪ 0.67	⑫ 0.44	⑬ 0.55	⑭ 0.92	⑮ 0.36	⑯ 0.21
⑰ 0.17	⑱ 0.57	⑲ 0.78	⑳ 0.18				

PG 41. EX 19.

① $\frac{1}{2}$	② $\frac{4}{5}$	③ $\frac{1}{10}$	④ $\frac{1}{5}$	⑤ $\frac{7}{10}$	⑥ $\frac{1}{4}$	⑦ $\frac{1}{50}$	⑧ $\frac{3}{4}$
⑨ $\frac{57}{100}$	⑩ $\frac{2}{25}$	⑪ $\frac{1}{8}$	⑫ $\frac{27}{40}$	⑬ $\frac{7}{8}$	⑭ $\frac{1}{125}$	⑮ $\frac{1}{40}$	⑯ $\frac{61}{200}$
⑰ $\frac{1}{2000}$	⑱ $\frac{201}{2000}$	⑲ $\frac{1}{1000}$	⑳ $\frac{7051}{10\,000}$				

PG 41. REVISION EXERCISE

① (a) $\frac{1}{3}$	(b) $\frac{4}{5}$	(c) $\frac{4}{9}$	(d) $\frac{7}{16}$	(e) $\frac{5}{8}$	
② (a) $\frac{2}{5}$	(b) $\frac{3}{7}$				
③ (a) $\frac{6}{1}$	(b) $\frac{13}{1}$	(c) $\frac{2}{1}$	(d) $\frac{1}{1}$	(e) $\frac{407}{1}$	
④ (a) $\frac{1}{2}=\frac{3}{6}$	(b) $\frac{3}{7}=\frac{8}{28}$	(c) $\frac{5}{6}=\frac{15}{18}$	(d) $\frac{5}{12}=\frac{25}{60}$	(e) $\frac{3}{4}=\frac{9}{12}$	(f) $\frac{8}{9}=\frac{72}{81}$

CALCULATIONS INVOLVING PERCENTAGES

Revision of Basic Percentages

Exercise 1

1. Calculate:

(a) 50% of £25·50	(b) 75% of £28	(c) 25% of £4·40
(d) 10% of £6·80	(e) 20% of £45	(f) 30% of £160
(g) 40% of £18	(h) 60% of £8	(i) 70% of £5
(j) 80% of £9·50	(k) 90% of £2200	(l) 15% of £3
(m) 17·5% of £400	(n) 22·5% of £200	(o) 8·2% of £600
(p) $17^1/_2$% of £20	(q) $8^1/_2$% of £40	(r) $12^1/_2$% of £4

2. What is:

   (a) $33^1/_3$% of £90? (b) $66^2/_3$% of £120?

3. At a dance, only 28% of the 150 people were female.
   How many were: (i) female? (ii) male?

4. A bottle holds 500 millilitres of diluted juice. 96·5% of this is water.
   How many millilitres of water is this?

5. Mavis bought a 750 gram box of chocolates on Saturday afternoon.
   By evening only 15% of them were left.
   What weight of chocolates remained?

6. The village of Elderslie has 3800 residents. Only 2% of them attended a local meeting.

   (a) How many villagers attended the meeting?
   (b) How many did not bother to go?

7. A jet was flying at 32 000 feet when one of its engines failed.
   The jet dropped by 42% in height. By how many feet did it drop?

8. When David was 14 he was 140 cm tall. Over the next year he grew by 2·5%.
   What was his height when he reached 15 years?

9. At Stanford City Football Club, 95% of its home support are season ticket holders.
   The stadium has room for 44 200 home supporters.
   How many home supporters do not have a season ticket?

10. Mrs. Nicolson borrows £1200. She must pay back the loan plus interest at a rate of 9% per year.

Calculate the amount she has to pay if she wishes to pay back the loan (plus interest) in:

(a) 1 year (b) 6 months (c) 9 months (d) 4 months (e) 5 months.

11. Of the 40 guests at a party, 32 of them were women.
What percentage were women?

12. Of the 180 cars which took part in a rally, 45 of them were green.
What percentage of them were not green?

13. From my weekly pay of £280, I spend £84 in rent.
What percentage of my pay do I spend on rent?

14. 2000 people were stuck at the airport, due to flight delays.
The first flight to leave was to Orkney. It left carrying 72 of the people.
What percentage of the people already at the airport remained there?

A. Compound Interest

Exercise 2

1. The following people have opened up Investment Accounts and are leaving their money to grow with compound interest.

   For each, calculate the total amount in their account after the stated period.

   (a) Anna, deposits £1200 for 3 years at a rate of interest of 5% per annum.

   (b) Judy, deposits £650 for 2 years at a rate of interest of 4% per annum.

   (a) Anna, deposits £50 for 2 years at a rate of interest of 2% per annum.

2. Calculate the total compound interest earned on a deposit of £450 for 3 years at 4% p.a.
(The interest should only be calculated on complete pounds of principal).

3. Conrad James deposited £500 in his bank and left it there for 3 years, gaining interest each year. Unfortunately, the interest rate dropped each year – from 10% in the first year to 8% in the second year to 5% in the third year.

   When he withdrew all his money at the end of year three how much did he receive?

4. A businessman borrowed £8000 at a rate of interest of 5% per annum. He made payments at the end of each year based on the sum outstanding at the end of that year.

   At the end of the first year and again at the end of the second year he paid back £3000.

   How much had he to pay at the end of the third year to clear the debt ?

5. Mary Telfer deposited £250 in her bank and left it there for 3 years, gaining interest each year. The interest rate rose from 4% in the first year to 5% in the second year, but fell drastically to 1% in the third year.

   She took out all her money atthe end of year 3.

   How much did she withdraw ?

6. Mrs. Donaldson deposits £750 in a Building Society which pays 3% compound interest half yearly.

   Mrs. Edgar, her neighbour, puts her £750 into another Building Society where her investment gains 6% compound interest annually.

   (a) How much will each have in their Building Society after 1 year?

   (b) Is a rate of 3% compound interest paid half yearly equivalent to a rate of 6% compound interest paid annually? Explain!

7. Use the $\boxed{y^x}$ key on your calculator for this question.

   Calculate the compound interest on £3340 for 10 years at 6·5% per annum.

8. How many years would it take for £50 to (at least) double at a rate of 10% compound interest?

B. Appreciation and Depreciation

Exercise 3

1. Mr. and Mrs. Pollard bought a semi-detached house for £60000.
   In each of the following two years its value appreciated by 10%.

   How much was the house worth after the two years?

2. Newly weds Jack and Jane Jones bought a flat for £55000. It appreciated in value by 7·5% p.a. for the next two years until they sold it.
   How much did they get for their flat? (to the nearest £)

3. The Herald's bought a bungalow for £110000.

   It appreciated in value for the next three years by 8% in year 1, by 6·5% in year 2 and by 5% in year 3.

   How much was the bungalow worth after three years?
   (to the nearest £).

4. Miss Hamilton retired to a villa which she bought for £68 500.

   The value of the villa rose by 5·4% each year.

   How much was the villa worth after 2 years? (to the nearest £)

5. Bert, the garage owner, bought a second-hand breakdown truck for £5000.

The truck lost 40% of its value during the first year, 20% during the second year and 10% during the third year.

How much was the breakdown truck worth after these 3 years?

6. A contractor bought a digger for £75 000. It depreciated by 75% in year one, by 40% in year two and by 20% in year three.

What was the digger worth after 3 years?

7. The value of a photocopier in a school office depreciates by 42% annually.

How much will an £18 000 copier be worth at the end of two years?

8. A small conservatory was valued at £8 000 in 1997 and again a year later at £8 336.

Calculate how much it had increased in value, and express this as a percentage of its 1997 value.

9. Mr. Able owns a detached villa in Melrose.

In 1996 he had the house valued - £85 000.

By 1997 it had depreciated by 15%, and by 1998 it was worth 20% more than in 1997.

Calculate:

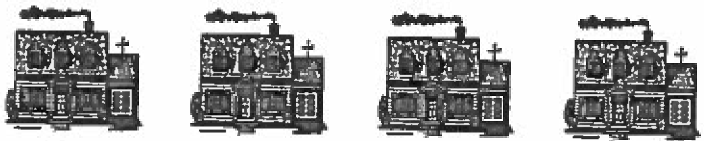
(a) its value in 1998.

(b) the percentage change in value from 1996 to 1998.

10. Calculate the percentage appreciation of the value of this detached villa:

(a) from 1996 to 1997.

(b) from 1996 to 1999.



1996	1997	1998	1999
£120 000	£126 000	£128 520	£129 600

11. Calculate the percentage depreciation of the value of this car:
   (a) from 1995 to 1996.
   (b) from 1997 to 1998.
   (c) from 1995 to 1999.

1995	1996	1997	1998	1999
£12 000	£4 800	£2 400	£1 920	£1 800

12. The value of an antique jug rose by 5% to £10 500.
    Work out its previous value. (<u>not</u> £9 975!)

C. Significant Figures

Exercise 4

1. Round the following numbers to one significant figure (1 sig. fig.).

(a) 4269	(b) 14 774	(c) 17	(d) 487
(e) 18 152	(f) 2085	(g) 7510	(h) 6551
(i) 42 670	(j) 451	(k) 14 308	(l) 24 859
(m) 6 890 000	(n) 55 847 155	(o) 38 749 886 541	(p) 25

2. Round the following numbers to two significant figures (2 sig. figs.).

(a) 5187	(b) 24 885	(c) 221	(d) 555
(e) 19 352	(f) 2065	(g) 7650	(h) 6549
(i) 42 501	(j) 448	(k) 78 209	(l) 29 899
(m) 6 890 000	(n) 55 847 155	(o) 38 749 886 541	(p) 351

3. Round the following numbers to three significant figures (3 sig. figs.).

(a) 8181	(b) 24882	(c) 2217	(d) 5554
(e) 19 551	(f) 2077	(g) 7682	(h) 6149
(i) 42 552	(j) 4499	(k) 78 209	(l) 29 897
(m) 6 893 000	(n) 55 847 155	(o) 38 749 886 541	(p) 35 150 001

4. Round <u>each</u> of the following decimals to:
   (i) 1 significant figure
   (ii) 2 significant figures
   (iii) 3 significant figures

(a) 8·33333	(b) 23·81558	(c) 1·53097	(d) 347·502

Exercise 5

In this exercise, round the answers to the required number of significant figures.

1. For each person, calculate the total amount in their account after the stated period.
   (a) Janice deposits £2000 for 3 years in her Investment Account at a compound interest rate of 5% per annum. (2 sig figs.)
   (b) Rob deposits £1500 for 2 years in his Investment Account at a compound interest rate of 4% per annum. (1 sig fig.)
   (c) Quasim deposits £3000 for 4 years in his Investment Account at a compound interest rate of 10% per annum. (3 sig figs.)

2. Sally James deposited £800 in her bank and left it there for 3 years, gaining interest each year. The interest rate was 10% in the first year, 5% in the second year and 3% in the third year.

   When she withdrew all her money at the end of year 3 how much did she receive? (answer to 2 sig figs.)

3. Calculate the compound interest on £6580 for 15 years at 3% per annum.
   Use the $\boxed{y^x}$ key on your calculator. (3 sig figs.)

4. Mr. and Mrs. Greig bought a detached house for £85 000.
   In each of the following two years its value appreciated by 8·5%.
   How much was the house worth after the two years? (2 sig fig.)

5. The Thomson's bought a seaside apartment for £32 500.
   It appreciated in value for the next three years by 10% in year one, by 4% in year two and by 3% in year three.
   How much was the apartment worth after three years? (2 sig figs.)

6. Ami bought a small aircraft with the money left to her by an old aunt. She paid £104 000.
   The plane lost 50% of its value during the first year, 35% during the second year, 20% during the third year and 12·5% during the fourth year.
   How much was the aircraft worth after these 4 years? (3 sig figs.)

cont'd ...

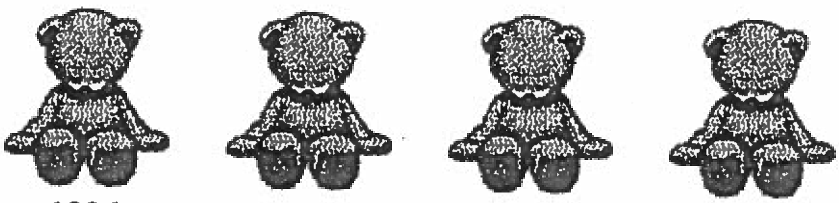
7. This table shows the value of a dishwasher, bought new in 1995, over a four year period.

Year	1995	1996	1997	1998	1999
Value	£600	£320	£240	£140	£50

Calculate the percentage depreciation of the value of the dishwasher:

(a) from 1995 to 1996. (2 sig figs.)
(b) from 1997 to 1998. (3 sig figs.)
(c) from 1995 to 1999. (1 sig fig.)

8. Calculate the percentage appreciation of the value of this precious teddy:
(a) from 1996 – 1997. (1 sig fig.)
(b) from 1997 – 1998. (2 sig figs.)
(c) from 1996 – 1999. (1 sig fig.)



1996	1997	1998	1999
£500	£542	£700	£978

Q Calculating the original amount

Example

This shirt has been increased in price by 30% and now costs £7.80. What was the price before the increase?

New price = original price + 30% of original price
= 100% of original price + 30% of original price
= 130% of original price

130% of original price = £7.80

So 1% of original price = $\frac{£7.80}{130}$ = 6p

100% of original price = 600p = £6

Original price = £6

Exercise

1 The cost of an article is increased by 5%. The new price is £2.10. What was the original cost?

Example

This time we have a *reduction* in price. We have to find the original price of the anoraks.

Think of the original price as 100% and consider increases or decreases relative to this starting point of 100%.

A *decrease* in price (as in this example) of 10% means that the new price is 100% minus 10%, that is 90% of the original price.

90% of the original price = £12.60

So 1% of the original price = $\frac{1260\text{p}}{90}$ = 14p

So 100% of the original price = 1400p = £14

Original price = £14

Exercise

2 In a sale prices are reduced by 10%. What was the original price of a coat now costing £14.40?

3 A man's wages are increased by 8%. If he now earns £54 per week what did he earn previously?

4 A man has 25% of his pay deducted to pay his income tax. If his net salary (that is, his salary *after* paying income tax) is £3750, what is his gross salary (that is, his salary *before* tax is deducted)?

Continue with Section R

Checkup for Calculations Involving Percentages

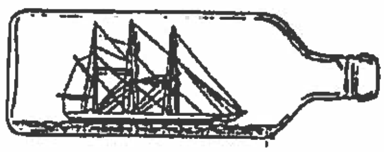
1. Calculate the total compound interest earned on a deposit of £200 for two years when the annual interest rate was 8%.

2. Frank Graham deposited £6000 in his bank and left it there for 3 years, gaining interest each year.
The interest rate fell from 7% in the first year to 5% in the second year, but rose to 10% in the third year.
He withdrew all his money at the end of year 3.
How much did he then receive? Give your answer correct to two significant figures.

3. A company director borrowed £20 000 and was charged a rate of interest of 3% per annum, calculated on the sum outstanding at the beginning of the year.
At the end of the first year and again at the end of the second year he paid back £10 000.
How much had he to pay at the end of the third year to clear the debt?
Give your answer correct to three significant figures.

4. Calculate the compound interest on £200 for 25 years at 5% per annum.
Give your answer correct to one significant figure.

5. Julie Rocks bought a flat in Peterhead for £20 000. It increased in value over the next three years at an annual rate of 6%.
What was the value of the flat at the end of these 3 years?
Give your answer correct to three significant figures.

6. 

This antique ship in a bottle appreciated in value over a four year period by consecutive rates of 10%, 20%, 50% and 100% per annum.
What was it worth after 4 years if its original price was £100.

7. A yacht was purchased new, at a cost of £250 000.
It fell by 15% of its value each year over the next three years and at the end of the fourth year it was found to be worth £100 000.
   (a) By how much money did the yacht depreciate during the fourth year?
   (b) Calculate the percentage depreciation over the first three years, giving your answer correct to two significant figures.

8. Mrs. Penny Black owns a treasured stamp which was valued, 40 years ago, at £300.
It is estimated that the stamp has grown in value by at least 10% per annum since then.
What is the estimated value of the stamp today?
Give your answer correct to three significant figures.

ANSWERS

Calculations Involving Percentages

Exercise 1

1. (a) £12·75 (b) £21 (c) £1·10 (d) 68p (e) £9
   (f) £48 (g) £7·20 (h) £4·80 (i) £3·50 (j) £7·60
   (k) £1980 (l) 45p (m) £70 (n) £45 (o) £49·20
   (p) £3·50 (q) £3·40 (r) 50p
2. (a) £30 (b) £80 3. (i) 42 (ii) 108 4. 482·5mm
5. 112·5g 6. (a) 76 (ii) 3724 7. 13440ft 8. 143·5cm 9. 2210
10. (a) £1308 (b) £1254 (c) £1281 (d) £1236 (e) £1245
11. 80% 12. 75% 13. 30% 14. 96·4%

Exercise 2

1. (a) £1389·15 (b) £703·04 (c) £52·02 2. £56·16
3. £623·70 4. £2803·50 5. £275·73
6. (a) Mrs. D £795·68 Mrs. E £795 (b) 3% per half year better as you get interest on the interest for rest of year.
7. £2929·64 8. 8 years

Exercise 3

1. £72600 2. £63559 3. £132848 4. £76098 5. £2160
6. £9000 7. £6055·20 8. 4·2% 9. (a) £86700 (b) 2%
10. (a) 5% (b) 8% 11. (a) 60% (b) 20% (c) 85% 12. £10000

Exercise 4

1. (a) 4000 (b) 10000 (c) 20 (d) 500 (e) 20000
   (f) 2000 (g) 8000 (h) 7000 (i) 40000 (j) 500
   (k) 10000 (l) 20000 (m) 7000000 (n) 60000000
   (o) 40000000000 (p) 30
2. (a) 5200 (b) 25000 (c) 220 (d) 560 (e) 19000
   (f) 2100 (g) 7700 (h) 6500 (i) 43000 (j) 450
   (k) 78000 (l) 30000 (m) 6900000 (n) 56000000
   (o) 39000000000 (p) 350
3. (a) 8180 (b) 24900 (c) 2220 (d) 5550 (e) 19600
   (f) 2080 (g) 7680 (h) 6150 (i) 42600 (j) 4500
   (k) 78200 (l) 29900 (m) 6890000 (n) 55800000
   (o) 38700000000 (p) 35200000
4. (a) 8, 8·3, 8·33 (b) 20, 24, 23·8 (c) 2, 1·5, 1·53 (d) 300, 350, 348

Exercise 5

1. (a) £2300 (b) £2000 (c) £4390
2. £950 3. £3670 4. £10000 5. £38000 6. £23700
7. (a) 47% (b) 41·7% (c) 90%
8. (a) 8% (b) 29% (c) 100%

P53 CALCULATING THE ORIGINAL AMOUNT

① £2 ② £16 ③ £50 ④ £5000

Checkup for Calculations Involving Percentages

1. £33·28 2. £7400 3. £946 4. £700 5. £23800 6. £396
7. (a) £53531·25 (b) 39%
8. £13600

Volumes of Solids

Exercise 1

1. (a) 80 cm^3 (b) 75 cm^3 (c) 232 cm^3 (d) 572 cm^3
   (e) 64·4 cm^3 (f) 69·3 cm^3
2. (a) 350 cm^3 (b) 84 cm^3 (c) 675 cm^3 (d) 2040 cm^3
   (e) 960 cm^3 (f) 1243·44 cm^3
3. (a) 2009·6 cm^3 (b) 268·47 cm^3 (c) 255·125 cm^3
   (d) 1148·0625 cm^3 (e) 314 cm^3
4. (a) 78·5 litres (b) 98·91 litres (c) 69·08 litres
5. 384·65 cm^3
6. (a) 180000 cm^3 (b) 4019·2 cm^3 (c) 44
7. (a) 4 x 6 = 24 (b) 3 (c) 72 (d) 10897·92 cm^3
8. 16956 cm^3
9. 15260·4 cm^3

Exercise 2

1. (a) 565·2 cm^3 (b) 512·9 cm^3 (c) 230·8 cm^3 (d) 6699 cm^3 (e) 384·6 cm^3
2. 94·2 cm^3
3. (a) 24 cm (b) 2512 cm^3
4. (a) 2616·7 cm^3 + 36000 cm^3 = 38616·7 cm^3
   (b) 10173·6 cm^3 + 2543·4 cm^3 = 12717 cm^3
5. (a) 904·32 cm^3 (b) 18 seconds

Exercise 3

1. (a) 5572·5 cm^3 (b) 1149·8 cm^3 (c) 3260·1 cm^3
   (d) 14130 cm^3 (e) 588·7 cm^3
2. 7234·6 cm^3
3. (a) 1285·6 cm^3 (b) 718·0 cm^3
4. (a) 16746·66... + 16746·.. + 75360 = 108853·3 cm^3 (b) 108·9 litres
5. 564·15.. + 718·01... = 1282·2 cm^3
6. 454·3 cm^3