



N5 - Maths - Expression and Formulae

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ALGEBRAIC OPERATIONS

By the end of this set of exercises, you should be able to

- (a) reduce an algebraic fraction to its simplest form
- (b) apply the four rules to algebraic fractions
- (c) change the subject of a formula
- (d) simplify surds
- (e) rationalise a surd denominator
- (f) simplify expressions using the laws of indices.

ALGEBRAIC OPERATIONS

A. Reducing algebraic fractions to their simplest form

Exercise 1

1. Simplify these fractions:

- | | | | |
|-------------------------|---------------------------|--------------------------------|--------------------------------|
| (a) $\frac{6}{10}$ | (b) $\frac{3}{9}$ | (c) $\frac{21}{28}$ | (d) $\frac{33}{44}$ |
| (e) $\frac{100}{400}$ | (f) $\frac{4a}{5a}$ | (g) $\frac{2p}{3p}$ | (h) $\frac{5v}{5w}$ |
| (i) $\frac{rs}{rt}$ | (j) $\frac{ab}{ac}$ | (k) $\frac{a}{ad}$ | (l) $\frac{m}{m^2}$ |
| (m) $\frac{a^2}{4a}$ | (n) $\frac{8z}{z^2}$ | (o) $\frac{x^2}{x}$ | (p) $\frac{d}{d^2}$ |
| (q) $\frac{5x^2}{6x}$ | (r) $\frac{3v}{6w}$ | (s) $\frac{8xy}{4x}$ | (t) $\frac{a^2}{3ab}$ |
| (u) $\frac{2pq}{6p}$ | (v) $\frac{5b}{5b^2}$ | (w) $\frac{xyz}{xz}$ | (x) $\frac{3ef}{7ef}$ |
| (y) $\frac{5pq}{2pq^2}$ | (z) $\frac{8x^2y}{4xy^2}$ | (aa) $\frac{(x+1)^2}{(x+1)^3}$ | (ab) $\frac{(x-5)^2}{(x-5)^5}$ |

2. Factorise either the numerator or the denominator, then simplify:

- | | | | |
|------------------------|-----------------------|-----------------------|------------------------|
| (a) $\frac{2x+8}{4}$ | (b) $\frac{3x+6}{9}$ | (c) $\frac{2x-8}{2}$ | (d) $\frac{5x-10}{25}$ |
| (e) $\frac{2}{2x+6}$ | (f) $\frac{4}{4x-12}$ | (g) $\frac{5}{5x+15}$ | (h) $\frac{8}{4x-10}$ |
| (i) $\frac{x^2-xy}{x}$ | (j) $\frac{pq+p}{p}$ | (k) $\frac{v^2+v}{v}$ | (l) $\frac{a-a^2}{a}$ |

3. Factorise the numerator and/or the denominator, then simplify:

- | | | | |
|------------------------------|-------------------------------|----------------------------------|-------------------------------------|
| (a) $\frac{2a+6}{a+3}$ | (b) $\frac{3c+9}{c+3}$ | (c) $\frac{d-2}{3d-6}$ | (d) $\frac{g^2+g}{g+1}$ |
| (e) $\frac{2x+2y}{5x+5y}$ | (f) $\frac{3p+3q}{7p+7q}$ | (g) $\frac{4-4w}{1-w}$ | (h) $\frac{x^2-xy}{8x-8y}$ |
| (i) $\frac{x^2-1}{x-1}$ | (j) $\frac{y^2-9}{y+3}$ | (k) $\frac{a^2-25}{a-5}$ | (l) $\frac{w+10}{w^2-100}$ |
| (m) $\frac{x^2-1}{x^2+2x+1}$ | (n) $\frac{3v^2-5v-2}{v^2-4}$ | (o) $\frac{2y^2+y-1}{2y^2+5y-3}$ | (p) $\frac{6x^2-13x+6}{3x^2+10x-8}$ |

B. Multiplying, dividing, adding and subtracting algebraic fractions

Exercise 2

1. Simplify these fractions by multiplying:

(a) $\frac{1}{3} \times \frac{3}{5}$	(b) $\frac{2}{3} \times \frac{1}{6}$	(c) $\frac{5}{6} \times \frac{3}{5}$	(d) $\frac{3}{10} \times \frac{20}{3}$
(e) $\frac{1}{6} \times 12$	(f) $\frac{9}{11} \times \frac{33}{9}$	(g) $\frac{7}{10} \times \frac{5}{1}$	(h) $8 \times \frac{2}{3}$
(i) $\frac{a}{b} \times \frac{c}{d}$	(j) $\frac{x}{y} \times \frac{v}{w}$	(k) $\frac{m}{n} \times \frac{m}{n}$	(l) $\frac{a}{b} \times \frac{b}{a}$
(m) $\frac{x}{2} \times \frac{x}{5}$	(n) $x \times \frac{x}{7}$	(o) $\frac{a}{5} \times \frac{a}{5}$	(p) $\frac{n}{3} \times \frac{3}{n}$
(q) $\frac{a}{6} \times \frac{6}{d}$	(r) $x^2 \times \frac{1}{x}$	(s) $\frac{a^2}{c} \times \frac{c}{a}$	(t) $\frac{a^3}{9} \times \frac{9}{a}$

2. Change these divisions to multiplications and simplify:

(a) $\frac{3}{7} \div \frac{6}{7}$	(b) $\frac{2}{3} \div \frac{8}{3}$	(c) $\frac{3}{8} \div \frac{9}{4}$	(d) $\frac{9}{10} \div \frac{18}{5}$
(e) $\frac{x}{2} \div \frac{x}{3}$	(f) $\frac{a}{8} \div \frac{a}{2}$	(g) $\frac{d}{3} \div \frac{d}{6}$	(h) $\frac{m}{10} \div \frac{m}{50}$
(i) $\frac{a^2}{3} \div \frac{a}{3}$	(j) $\frac{b^2}{6} \div \frac{b}{2}$	(k) $\frac{r^4}{6} \div \frac{r^2}{2}$	(l) $\frac{a}{b} \div \frac{a}{b}$
(m) $\frac{a^2}{d} \div \frac{a}{d^2}$	(n) $\frac{1}{w^2} \div \frac{5}{w}$	(o) $\frac{1}{a^3} \div \frac{1}{a^2}$	(p) $\frac{x^2}{y} \div \frac{2x}{d}$
(q) $\frac{a^2}{b} \div \frac{a}{b}$	(r) $\frac{2a^2}{5d^2} \div \frac{a^2}{d^2}$		

3. Do the following additions and subtractions:

(a) $\frac{1}{3} + \frac{1}{4}$	(b) $\frac{1}{4} + \frac{2}{3}$	(c) $\frac{3}{4} - \frac{1}{5}$	(d) $\frac{1}{5} + \frac{1}{3}$
(e) $\frac{1}{3} - \frac{1}{5}$	(f) $\frac{4}{7} - \frac{1}{2}$	(g) $\frac{1}{2} + \frac{1}{5}$	(h) $\frac{1}{2} - \frac{1}{5}$
(i) $\frac{5}{8} - \frac{1}{4}$	(j) $\frac{7}{10} + \frac{1}{5}$	(k) $\frac{x}{3} + \frac{a}{2}$	(l) $\frac{c}{5} + \frac{d}{2}$
(m) $\frac{e}{3} - \frac{h}{4}$	(n) $\frac{m}{4} - \frac{n}{8}$	(o) $\frac{2x}{3} + \frac{k}{2}$	(p) $\frac{u}{2} - \frac{2w}{5}$
(q) $\frac{4r}{5} + \frac{s}{2}$	(r) $\frac{a}{3} - \frac{2d}{5}$	(s) $\frac{2x}{3} + \frac{3y}{2}$	(t) $\frac{3x}{4} + \frac{2u}{5}$

4. By finding a common denominator with letters, work out these additions/subtractions:

(a) $\frac{2}{x} + \frac{3}{y}$	(b) $\frac{5}{a} - \frac{2}{b}$	(c) $\frac{4}{c} + \frac{1}{d}$	(d) $\frac{1}{p} - \frac{2}{q}$
(e) $\frac{2}{v} + \frac{2}{w}$	(f) $\frac{1}{g} - \frac{1}{h}$	(g) $\frac{7}{k} + \frac{1}{n}$	(h) $\frac{1}{x} - \frac{8}{y}$

5. Add or subtract these fractions:

(a) $\frac{x+1}{3} + \frac{x+1}{2}$	(b) $\frac{x+2}{4} + \frac{x-1}{5}$	(c) $\frac{x+3}{2} + \frac{x+1}{4}$	(d) $\frac{2x-3}{5} + \frac{x+1}{3}$
(e) $\frac{x+1}{2} - \frac{x+1}{3}$	(f) $\frac{x+2}{2} - \frac{x+1}{5}$	(g) $\frac{2x+1}{2} - \frac{x+1}{4}$	(h) $\frac{x+1}{2} - \frac{x-1}{5}$

C. Changing the subject of a formula

Exercise 3A

This exercise has a mixed selection of formulae.

Change the subject of each formula to the letter shown in the brackets.

ALL WORKING and ALL STEPS SHOULD BE SHOWN.

- | | | |
|------------------------|--------------------------|------------------------|
| 1. $x + 2 = c$ (x) | 2. $x - 4 = c$ (x) | 3. $x + p = q$ (x) |
| 4. $x - p = q$ (x) | 5. $x/2 = a$ (x) | 6. $x/7 = a$ (x) |
| 7. $x/y = a$ (x) | 8. $x/p = m$ (x) | 9. $x/r = s$ (x) |
| 10. $4x = 20$ (x) | 11. $4x = a$ (x) | 12. $gx = h$ (x) |
| 13. $nx = t$ (x) | 14. $2x + 1 = 5$ (x) | 15. $2x + 1 = b$ (x) |
| 16. $2x + c = b$ (x) | 17. $ax + c = b$ (x) | 18. $px + q = r$ (x) |
| 19. $vx - w = y$ (x) | 20. $D = S \times T$ (S) | 21. $C = \pi d$ (d) |
| 22. $x^2 = 16$ (x) | 23. $x^2 = y$ (x) | 24. $A = \pi r^2$ (r) |
| 25. $T = D/g$ (S) | 26. $A = y^2$ (y) | 27. $P = 3\pi r^2$ (π) |
| 28. $P = 5\pi r^2$ (r) | 29. $h - p = q$ (h) | 30. $h - p = q$ (p) |
| 31. $2h - 5p = q$ (h) | 32. $2h - 5p = q$ (p) | 33. $b - c = ax$ (x) |

Exercise 3B

1. Change the subject of each formula to h .

(a) $g = hf$	(b) $e = g + h$	(c) $k = h/f$ (d) $e = g - h$
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2. Change the subject of each formula to r .

(a) $Q = r^2$	(b) $N = \pi r^2$	(c) $M = 2\pi r^2$	(d) $P = \pi r^2 w$
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3. Change the subject of each formula to m .
- (a) $A = klm$ (b) $B = Km$ (c) $C = \pi mr^2$ (d) $D = \frac{1}{3pm}$
4. Change the subject to x .
- (a) $p = q + x$ (b) $r = s - x$ (c) $r = s - 5x$ (d) $r = 7x - 3$
 (e) $m = 2(x + 1)$ (f) $m = \frac{1}{2}(x - 5)$ (g) $n = \frac{1}{2}(x + 2)$ (h) $p = \frac{1}{2}(x + q)$
5. Change the subject of the formula to the letter in brackets.
- (a) $P/Q = R$ (P) (b) $t = 1/s$ (s) (c) $M = P/Q^2$ (Q) (d) $v = \sqrt{\frac{w}{z}}$ (w)
 (e) $d = \frac{e}{5f}$ (f) (f) $\frac{K}{mn} = T$ (n) (g) $R = \frac{7}{9s^2}$ (s) (h) $a^2 + b^2 = c^2$ (a)
6. Harder examples: Change the subject of the formula to the letter in brackets.
- (a) $A + d = V/T$ (T) (b) $px + qx = r$ (x) (c) $ax = bx + c$ (x)
 (d) $m = \frac{r-s}{s}$ (s) (e) $x = \frac{v-w}{v+w}$ (w) (f) $p = 2\sqrt{r-1}$ (r)

D. Simplifying surds

Exercise 4

1. Express each of the following in its simplest form:
- (a) $\sqrt{8}$ (b) $\sqrt{12}$ (c) $\sqrt{27}$ (d) $\sqrt{20}$ (e) $\sqrt{50}$ (f) $\sqrt{28}$
 (g) $\sqrt{18}$ (h) $\sqrt{24}$ (i) $\sqrt{200}$ (j) $\sqrt{75}$ (k) $\sqrt{45}$ (l) $\sqrt{72}$
 (m) $\sqrt{300}$ (n) $\sqrt{147}$ (o) $\sqrt{54}$ (p) $7\sqrt{8}$ (q) $5\sqrt{32}$ (r) $6\sqrt{40}$
2. Add or subtract the following:
- (a) $3\sqrt{2} + 5\sqrt{2}$ (b) $6\sqrt{5} - 5\sqrt{5}$ (c) $8\sqrt{10} + 5\sqrt{10}$ (d) $9\sqrt{20} - 9\sqrt{20}$
 (e) $\sqrt{6} - 3\sqrt{6}$ (f) $\sqrt{3} + \sqrt{3} - 3\sqrt{3}$ (g) $5\sqrt{7} - 8\sqrt{7} + 3\sqrt{7}$ (h) $10\sqrt{2} + 10\sqrt{3}$
3. Simplify:
- (a) $\sqrt{8} - \sqrt{2}$ (b) $\sqrt{18} - \sqrt{2}$ (c) $\sqrt{125} + 5\sqrt{5}$ (d) $\sqrt{48} + \sqrt{12}$
 (e) $\sqrt{45} + \sqrt{20}$ (f) $\sqrt{63} - \sqrt{28}$ (g) $\sqrt{50} + \sqrt{18}$ (h) $\sqrt{72} - \sqrt{32}$

Exercise 5

1. Simplify:
- (a) $\sqrt{3} \times \sqrt{3}$ (b) $\sqrt{5} \times \sqrt{5}$ (c) $\sqrt{6} \times \sqrt{6}$ (d) $\sqrt{1} \times \sqrt{1}$
 (e) $\sqrt{x} \times \sqrt{x}$ (f) $\sqrt{3} \times \sqrt{2}$ (g) $\sqrt{4} \times \sqrt{5}$ (h) $\sqrt{16} \times \sqrt{a}$
 (i) $\sqrt{2} \times \sqrt{c}$ (j) $\sqrt{x} \times \sqrt{y}$ (k) $\sqrt{2} \times \sqrt{8}$ (l) $\sqrt{2} \times \sqrt{32}$
 (m) $\sqrt{6} \times \sqrt{3}$ (n) $\sqrt{20} \times \sqrt{10}$ (o) $3\sqrt{2} \times \sqrt{2}$ (p) $3\sqrt{2} \times 2\sqrt{3}$

contd.

2. Multiply out the brackets:

- (a) $\sqrt{2}(1 + \sqrt{2})$ (b) $\sqrt{3}(\sqrt{2} + 1)$ (c) $\sqrt{5}(\sqrt{5} + 1)$
(d) $\sqrt{7}(1 + \sqrt{7})$ (e) $\sqrt{2}(5 - \sqrt{2})$ (f) $\sqrt{2}(5 - 4\sqrt{2})$
(g) $(\sqrt{3} + 2)(\sqrt{3} - 1)$ (h) $(\sqrt{2} - 2)(\sqrt{2} + 1)$ (i) $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$
(j) $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$ (k) $(\sqrt{3} + \sqrt{2})^2$ (l) $(\sqrt{5} - \sqrt{3})^2$

3. If $a = 1 + \sqrt{2}$ and $b = 1 - \sqrt{2}$, simplify:

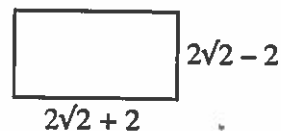
- (a) $3a + 3b$ (b) $2ab$ (c) $a^2 + b^2$

4. If $r = \sqrt{5} + \sqrt{3}$ and $s = \sqrt{5} - \sqrt{3}$, simplify:

- (a) $2r - 2s$ (b) $5rs$ (c) $r^2 - s^2$

5. A rectangle has sides of length $(2\sqrt{2} + 2)$ cm and $(2\sqrt{2} - 2)$ cm.
Calculate:

- (a) its area (b) the length of a diagonal



E. Rationalising a surd denominator

Exercise 6

1. Rationalise the denominators in the following and simplify where possible:

- (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{6}}$
(e) $\frac{1}{\sqrt{7}}$ (f) $\frac{10}{\sqrt{5}}$ (g) $\frac{2}{\sqrt{3}}$ (h) $\frac{3}{\sqrt{5}}$
(i) $\frac{20}{\sqrt{2}}$ (j) $\frac{6}{\sqrt{3}}$ (k) $\frac{12}{\sqrt{6}}$ (l) $\frac{3}{2\sqrt{5}}$
(m) $\frac{4}{5\sqrt{2}}$ (n) $\frac{1}{\sqrt{20}}$ (o) $\frac{1}{\sqrt{50}}$ (p) $\frac{4}{\sqrt{8}}$

2. Express each of the following in its simplest form with a rational denominator:

- (a) $\frac{\sqrt{4}}{\sqrt{3}}$ (b) $\frac{\sqrt{5}}{\sqrt{2}}$ (c) $\frac{\sqrt{4}}{\sqrt{10}}$ (d) $\frac{\sqrt{1}}{\sqrt{11}}$ (e) $\frac{\sqrt{3}}{\sqrt{5}}$ (f) $\frac{\sqrt{a}}{\sqrt{b}}$

3. Simplify the following by rationalising the denominator:

- (a) $\frac{1}{\sqrt{2} - 1}$ (b) $\frac{1}{\sqrt{7} - 1}$ (c) $\frac{1}{2 + \sqrt{2}}$ (d) $\frac{4}{\sqrt{5} + 1}$
(e) $\frac{3}{2 - \sqrt{3}}$ (f) $\frac{1}{\sqrt{3} - \sqrt{2}}$ (g) $\frac{2}{\sqrt{5} + \sqrt{3}}$ (h) $\frac{9}{\sqrt{5} - \sqrt{2}}$

F. Simplifying expressions using the laws of indices

Exercise 7

Rule 1 $a^m \times a^n = a^{m+n}$

1. Use Rule 1 to write down the simplest form of the products in the following:

- | | | | |
|----------------------|----------------------------|-----------------------|------------------------------|
| (a) $2^3 \times 2^4$ | (b) $3^5 \times 3^3$ | (c) $8^2 \times 8^5$ | (d) $10^{10} \times 10^{20}$ |
| (e) $a^3 \times a^4$ | (f) $b^5 \times b^3$ | (g) $c^2 \times c^6$ | (h) $d^5 \times d^5$ |
| (i) $v^3 \times v^8$ | (j) $x^2 \times x^{10}$ | (k) $w^3 \times w$ | (l) $z \times z^5$ |
| (m) $f^4 \times f^7$ | (n) $g^{10} \times g^{10}$ | (o) $k \times k^{11}$ | (p) $m^{100} \times m$ |

Rule 2 $a^m \div a^n = a^{m-n}$

2. Use Rule 2 to write down the simplest form of the quotients in the following:

- | | | | |
|--------------------------|--------------------------|---------------------|----------------------------|
| (a) $2^4 \div 2^3$ | (b) $3^5 \div 3^3$ | (c) $8^5 \div 8^2$ | (d) $10^{20} \div 10^{10}$ |
| (e) $a^7 \div a^6$ | (f) $b^5 \div b^3$ | (g) $c^9 \div c^4$ | (h) $d^5 \div d^5$ |
| (i) $v^{12} \div v^{11}$ | (j) $x^{10} \div x^2$ | (k) $w^3 \div w$ | (l) $z^2 \div z$ |
| (m) $f^7 \div f^4$ | (n) $g^{10} \div g^{10}$ | (o) $k^{11} \div k$ | (p) $m^{100} \div m$ |
| (q) $\frac{x^6}{x^2}$ | (r) $\frac{m^3}{m^2}$ | (s) $\frac{a^5}{a}$ | (t) $\frac{r^9}{r^8}$ |

Rule 3 $(a^m)^n = a^{mn}$

3. Simplify these. For example, $(a^3)^4 = a^{12}$.

- | | | | |
|---------------|---------------|---------------|---------------|
| (a) $(x^2)^3$ | (b) $(y^3)^5$ | (c) $(z^5)^2$ | (d) $(g^2)^8$ |
| (e) $(a^3)^7$ | (f) $(b^4)^4$ | (g) $(c^5)^6$ | (h) $(d^3)^7$ |

4. Express the following without brackets, writing answers in index form:

- | | | | |
|---------------|---------------|---------------|---------------|
| (a) $(2^5)^2$ | (b) $(7^3)^5$ | (c) $(6^5)^4$ | (d) $(8^3)^5$ |
| (e) $(2^7)^7$ | (f) $(3^4)^3$ | (g) $(9^5)^2$ | (h) $(2^5)^5$ |

5. Note $(ab)^m = a^m b^m$. Use this result to simplify:

- | | | | |
|--------------|--------------|-------------------|---------------|
| (a) $(ab)^3$ | (b) $(cd)^6$ | (c) $(x^2y)^{10}$ | (d) $(2pq)^2$ |
|--------------|--------------|-------------------|---------------|

6. Use the 3 rules learned so far to simplify the following:

- | | | | |
|-----------------------|-----------------------|----------------------------------|----------------------------------|
| (a) $y^3 \times y^2$ | (b) $t^4 \times t$ | (c) $x^5 \div x^2$ | (d) $v^7 \div v$ |
| (e) $3x^2 \times x^3$ | (f) $x^2 \times 4x^5$ | (g) $2x^2 \times 6x^2$ | (h) $8x^5 \div 2x^2$ |
| (i) $6x^3 + 2x$ | (j) $x^2(x^3 + x^4)$ | (k) $x^3(x^3 - 4)$ | (l) $(uv)^7$ |
| (m) $(5y)^2$ | (n) $(mn^2)^8$ | (o) $\frac{x^3 \times x^4}{x^2}$ | (p) $\frac{u^7 \times u^2}{u^3}$ |

7. Find m . For example: if $2^m = 8$, $m = 3$ since $2^3 = 8$.

(a) $2^m = 16$

(b) $2^m = 32$

(c) $4^m = 64$

(d) $5^m = 625$

Exercise 8

Rule 4 $a^0 = 1$

1. Write down the values of:

(a) 2^0

(b) 12^0

(c) x^0

(d) $(32546)^0$

Rule 5 $a^{-m} = 1/a^m$

2. Write the following with positive indices. For example: $2^{-5} = 1/2^5$.

(a) 3^{-2}

(b) 5^{-7}

(c) a^{-4}

(d) b^{-9}

(e) x^{-1}

(f) $3y^{-2}$

(g) xy^{-3}

(h) $\frac{1}{x^{-3}}$

(i) $\frac{1}{t^{-5}}$

(j) $\frac{6}{c^{-3}}$

(k) $\frac{1}{2y^{-2}}$

(l) $\frac{1}{7x^{-3}}$

3. Express these in a form without indices:

(a) 5^0

(b) 7^{-1}

(c) 3^{-3}

(d) 8^{-2}

(e) $\frac{1}{2^{-3}}$

(f) $\left(\frac{2}{3}\right)^{-2}$

4. Simplify the following. For example: $x^{-2} \times x^5 = x^{-2+5} = x^3$.

(a) $a^{-2} \times a^4$

(b) $b^6 \times b^{-4}$

(c) $c^{-1} \times c^{-1}$

(d) $d^{-6} \times d^6$

(e) $e^6 + e^{-3}$

(f) $g^{-4} + g^4$

(g) $w^{-3} + w^{-5}$

(h) $(x^{-2})^3$

(i) $(y^5)^{-1}$

(j) $(z^{-4})^{-4}$

(k) $(klm)^0$

5. Express with positive indices:

(a) $4^2 \times 4^{-5}$

(b) $6^{-2} \times 6^{-3}$

(c) $2^7 \times 2^{-2}$

(d) $4^{-3} \times 5^2$

(e) $3^4 \times 2^{-3}$

(f) $(9^0)^{-8}$

(g) $(w^{-3})^{-2}$

(h) $(x^{-2})^{-\frac{1}{2}}$

(i) $\frac{1}{2^{-2}}$

(j) $\frac{5}{2^{-3}}$

(k) $6h^{-2}$

(l) $9s^{-1}$

(m) $\frac{1}{2}k^{-1}$

(n) $\frac{3}{4}m^{-3}$

6. Multiply out the brackets:

(a) $x^5(x^2 + x^{-2})$

(b) $x^3(x - x^{-2})$

(c) $x^{-3}(x^4 + x)$

(d) $x^{-1}(x + x^2)$

(e) $x(x^2 - x^{-7})$

(f) $x^{-5}(2 - x^5)$

(g) $2x^{-2}(3 - x^3)$

(h) $3x^2(2x - x^{-2})$

7. (a) Find the value of: 3^3 , 3^0 , 3^{-2} , 3^1 , 3^{-3} .

(b) Write in the form 4^{-p} : $1/4$, $1/16$, $1/64$.

Exercise 9

Rule 6 $a^{m/n} = n\sqrt[n]{a^m}$

1. Write these in root form. For example: $x^{\frac{4}{5}} = \sqrt[5]{x^4}$

- (a) $x^{\frac{3}{4}}$ (b) $m^{\frac{3}{5}}$ (c) $r^{\frac{2}{3}}$ (d) $w^{\frac{1}{2}}$ (e) $n^{\frac{1}{3}}$ (f) $r^{-\frac{4}{3}}$

2. Write these in index form. For example: $\sqrt[5]{x^4} = x^{4/5}$

- (a) $\sqrt[3]{x^5}$ (b) $\sqrt[3]{b^4}$ (c) $\sqrt[3]{z^2}$ (d) $\sqrt[4]{w}$ (e) $\frac{1}{\sqrt[3]{x}}$ (f) \sqrt{u}

3. Evaluate the following. For example: $8^{\frac{2}{3}} = \sqrt[3]{8^2} = (2)^2 = 4$

- (a) $9^{\frac{1}{2}}$ (b) $64^{\frac{1}{2}}$ (c) $8^{\frac{1}{3}}$ (d) $64^{\frac{1}{3}}$ (e) $100^{\frac{1}{2}}$ (f) $27^{\frac{1}{3}}$
 (g) $9^{\frac{3}{2}}$ (h) $16^{\frac{3}{4}}$ (i) $49^{\frac{3}{2}}$ (j) $16^{-\frac{1}{2}}$ (k) $27^{-\frac{1}{3}}$ (l) $25^{-\frac{3}{2}}$
 (m) $81^{\frac{3}{4}}$ (n) $81^{-\frac{3}{4}}$ (o) $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$ (p) $\left(\frac{1}{8}\right)^{\frac{5}{3}}$

4. Simplify:

- (a) $(x^4)^{\frac{1}{2}}$ (b) $(m^9)^{\frac{1}{3}}$ (c) $(c^6)^{\frac{2}{3}}$ (d) $(n^{-15})^{\frac{1}{3}}$
 (e) $(n^{-2})^{\frac{1}{4}}$ (f) $\left(g^{-\frac{2}{3}}\right)^{-3}$ (g) $\left(b^{\frac{5}{2}}\right)^{-2}$ (h) $\left(z^{-\frac{3}{2}}\right)^{-2}$
 (i) $\left(x^{\frac{4}{5}}\right)^{\frac{1}{2}}$ (j) $\left(x^{\frac{3}{2}}\right)^{\frac{4}{3}}$ (k) $(5^2 + 12^2)^{\frac{1}{2}}$ (l) $(1^3 + 2^3 + 3^3)^{\frac{1}{2}}$

Exercise 10 *Miscellaneous examples*

1. Simplify:

- (a) $a^{\frac{3}{2}} \times a^{\frac{1}{2}}$ (b) $b^{\frac{5}{4}} \times b^{-\frac{1}{4}}$ (c) $c^{\frac{1}{3}} \times c^{-\frac{1}{3}}$ (d) $d^{\frac{5}{2}} + d^{\frac{1}{2}}$
 (e) $e^{\frac{3}{5}} + e^{-\frac{1}{5}}$ (f) $\left(z^{\frac{1}{2}}\right)^2$ (g) $\left(w^{-\frac{1}{2}}\right)^2$ (h) $\left(w^{-\frac{1}{2}}\right)^0$

2. Simplify:

- (a) $2a^{1/2} \times 2a^{-1/2}$ (b) $3b^{2/3} \times 4b^{1/3}$ (c) $6c^{1/2} \times c^{1/2}$ (d) $2d^{3/2} \times d^{-1/2}$
 (e) $4e^{3/2} + 2e^{1/2}$ (f) $8v^{3/4} + 2v^{-1/4}$ (g) $15z^{-1/3} + 15z^{-1/3}$

3. Multiply out the brackets:

(a) $x^{1/2}(x^{1/2} - x^{-1/2})$ (b) $x^{2/3}(x^{4/3} + x^{1/3})$ (c) $x^{-3/5}(x^{6/5} + x^{-1/5})$

4. Evaluate the following for $x = 16$ and $y = 27$:

(a) $3x^{1/2}$ (b) $4x^{3/4}$ (c) $5y^{2/3}$ (d) $8y^{-2/3}$ (e) $x^{-1/4} \times y^{1/3}$

5. Simplify the following, expressing your answers with positive indices:

(a) $p^6 \times p^{1/2}$ (b) $p^5 + p^{-1/2}$ (c) $(p^{2/3})^{-3/2}$ (d) $(p^{-3/4})^{12}$
 (e) $5p^{3/4} + 5p^{-3/4}$ (f) $7p^{1/2} \times 7p^{-3/2}$

6. Simplify:

(a) $\frac{x^4 \times x^{-3}}{x}$ (b) $\frac{x^{-1} \times x^5}{x^{-2}}$ (c) $\frac{x^{-3} \times x^3}{x^{-1}}$ (d) $\frac{x^{\frac{1}{4}} \times x^{-\frac{3}{4}}}{x^4}$
 (e) $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x}$ (f) $\frac{x^{\frac{3}{5}} \times x^{-\frac{3}{5}}}{x^{-1}}$

7. Multiply out the brackets:

(a) $(x^3 - 1)(x^{-3} - 1)$ (b) $(x^{-1} + 2)(x^{-1} - 2)$ (c) $(x^{1/2} + 4)(x^{-1/2} - 4)$

Exercise 11

Express the following with x in the numerator in index form:

For example :- $\frac{2}{5\sqrt{x}} = \frac{2}{5x^{\frac{1}{2}}} = \frac{2x^{-\frac{1}{2}}}{5} = \frac{2}{5}x^{-\frac{1}{2}}$

1. $\frac{1}{x}$ 2. $\frac{1}{x^2}$ 3. $\frac{7}{x^2}$ 4. $\frac{5}{x^3}$ 5. $\frac{1}{3x}$ 6. $\frac{4}{5x}$
 7. $\frac{3}{2x^2}$ 8. $\frac{1}{\sqrt{x}}$ 9. $\frac{5}{2\sqrt{x}}$ 10. $\frac{3}{\frac{5x^3}{m}}$ 11. $\frac{1}{\sqrt{(3x^6)}}$ 12. $\frac{2}{\sqrt{(5x^3)}}$
 $\frac{a^3 \times a^2}{a}$

$\frac{1}{4x^{-2}}$

$$\frac{x^{-1/4} \times x^{3/4}}{x^{-1/2}}$$

Checkup for algebraic operations

1. Factorise the numerator and/or denominator if possible, then simplify:

- (a) $\frac{7v}{14v}$ (b) $\frac{9a^2b}{3ab^2}$ (c) $\frac{6}{6x-18}$ (d) $\frac{v^2-vw}{v}$
 (e) $\frac{3x+3y}{6x+9y}$ (f) $\frac{a-8}{a^2-64}$ (g) $\frac{v^2+3v+2}{v^2-4}$

2. Simplify:

- (a) $\frac{b^3}{4} \times \frac{2}{b}$ (b) $\frac{a^2}{x} \times \frac{x}{a}$ (c) $\frac{z^2}{2} + \frac{z}{6}$ (d) $\frac{a^2}{b} + \frac{a}{b}$
 (e) $\frac{c}{4} + \frac{c}{8}$ (f) $\frac{3u}{4} - \frac{2v}{5}$ (g) $\frac{3}{k} + \frac{1}{m}$ (h) $\frac{x+1}{2} - \frac{x+2}{5}$

3. Change the subject of each formula to x .

- (a) $5+x=6$ (b) ~~$x-a=w$~~ (c) ~~$ax+m=p$~~ (d) ~~$x/z=P/w$~~
 (e) ~~$N=2\pi r^2$~~ (f) $T=4x+3$ (g) ~~$w=1/2(x+y)$~~ (h) ~~$M=\frac{5}{8x^2}$~~

4. Express each of these in its simplest form:

- (a) $\sqrt{32}$ (b) $\sqrt{1000}$ (c) $2\sqrt{45}$ (d) $\sqrt{45}-\sqrt{20}$
 (e) $\sqrt{50}-\sqrt{18}$ (f) $\sqrt{72}+\sqrt{50}$ (g) $\sqrt{\frac{9}{a^2}}$ (h) $2\sqrt{8}-\sqrt{2}$

5. If $x = 1 + \sqrt{3}$ and $y = 1 - \sqrt{3}$, simplify:

- (a) $2x+2y$ (b) $5xy$ (c) x^2+y^2

6. Rationalise the denominators in the following and simplify where possible:

- (a) $1/\sqrt{5}$ (b) $8/\sqrt{2}$ (c) $15/\sqrt{5}$ (d) $\sqrt{2}/\sqrt{6}$

7. Write in their simplest form:

- (a) $5^6 \times 5^8$ (b) $x^8 + x^6$ (c) (d) $(a^2)^3$
 (e) $(4a^3b)^2$ (f) (g) $(p^2)^0$ (h) $x^3(x^2-x)$

8. Write with positive indices:

- (a) 5^{-2} (b) ab^{-3} (c) $(y^{-3})^{-2}$ (d)

9. Write these in root form:

- (a) $b^{1/2}$ (b) $c^{-3/2}$

10. Write these in index form:

- (a) $\sqrt[3]{x^4}$ (b) $1/\sqrt{a^3}$

11. Evaluate:

- (a) $36^{3/2}$ (b) $32^{-2/5}$ (c) $(x^8)^{1/2}$ (d) $(y^{7/2})^{-2}$
 (e) $a^{-3/2}(a^{1/2}-a^{-5/2})$ (f) $2s^{1/2}+4s^{-1/2}$ (g)

ANSWERS

Algebraic operations

Exercise 1

1. (a) $\frac{3}{5}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{3}{4}$ (e) $\frac{1}{4}$ (f) $\frac{4}{5}$ (g) $\frac{2}{3}$ (h) $\frac{v}{w}$
(i) $\frac{s}{t}$ (j) $\frac{b}{c}$ (k) $\frac{1}{d}$ (l) $\frac{1}{m}$ (m) $\frac{a}{4}$ (n) $\frac{8}{z}$ (o) x (p) $\frac{1}{d}$
(q) $\frac{5x}{6}$ (r) $\frac{v}{2w}$ (s) $2y$ (t) $\frac{a}{3b}$ (u) $\frac{q}{3}$ (v) $\frac{1}{b}$ (w) y (x) $\frac{3}{7}$
(y) $\frac{5}{2q}$ (z) $\frac{2x}{y}$ (aa) $\frac{1}{(x+1)}$ (ab) $\frac{1}{(x-5)^3}$
2. (a) $\frac{x+4}{2}$ (b) $\frac{x+2}{3}$ (c) $x-4$ (d) $\frac{x-2}{5}$ (e) $\frac{1}{x+3}$
(f) $\frac{1}{x-3}$ (g) $\frac{1}{x+3}$ (h) $\frac{4}{2x-5}$ (i) $x-y$ (j) $q+1$
(k) $v+1$ (l) $1-a$
3. (a) 2 (b) 3 (c) $\frac{1}{3}$ (d) g (e) $\frac{2}{5}$
(f) $\frac{3}{7}$ (g) 4 (h) $\frac{x}{8}$ (i) $x+1$ (j) $y-3$
(k) $a+5$ (l) $\frac{1}{(w-10)}$ (m) $\frac{x-1}{x+1}$ (n) $\frac{3v+1}{v+2}$ (o) $\frac{y+1}{y+3}$
(p) $\frac{2x-3}{x+4}$

Exercise 2

1. (a) $\frac{1}{5}$ (b) $\frac{1}{9}$ (c) $\frac{1}{2}$ (d) 2 (e) 2 (f) 3 (g) $\frac{7}{2}$ (h) $\frac{16}{3}$
(i) $\frac{ac}{bd}$ (j) $\frac{xv}{yw}$ (k) $\frac{m^2}{n^2}$ (l) 1 (m) $\frac{x^2}{10}$ (n) $\frac{x^2}{7}$ (o) $\frac{a^2}{25}$ (p) 1
(q) $\frac{a}{d}$ (r) x (s) a (t) a^2
2. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{4}$ (e) $\frac{3}{2}$ (f) $\frac{1}{4}$ (g) 2 (h) 5
(i) a (j) $\frac{b}{3}$ (k) $\frac{r^2}{3}$ (l) 1 (m) ad (n) $\frac{1}{5w}$ (o) $\frac{1}{a}$ (p) $\frac{dx}{2y}$
(q) a (r) $\frac{2}{5}$
3. (a) $\frac{7}{12}$ (b) $\frac{11}{12}$ (c) $\frac{11}{20}$ (d) $\frac{8}{15}$ (e) $\frac{2}{15}$
(f) $\frac{1}{14}$ (g) $\frac{7}{10}$ (h) $\frac{3}{10}$ (i) $\frac{3}{8}$ (j) $\frac{9}{10}$
(k) $\frac{2x+3a}{6}$ (l) $\frac{2c+5d}{10}$ (m) $\frac{4e-3h}{12}$ (n) $\frac{2m-n}{8}$ (o) $\frac{4x+3k}{6}$
(p) $\frac{5u-4w}{10}$ (q) $\frac{8r+5s}{10}$ (r) $\frac{5a-6d}{15}$ (s) $\frac{4x+9y}{6}$ (t) $\frac{15x+8u}{20}$
4. (a) $\frac{2y+3x}{xy}$ (b) $\frac{5b-2a}{ab}$ (c) $\frac{4d+c}{cd}$ (d) $\frac{q-2p}{pq}$ (e) $\frac{2w+2v}{vw}$

(f) $\frac{h-g}{gh}$ (g) $\frac{7n+k}{kn}$ (h) $\frac{y-8x}{xy}$

5. (a) $\frac{5x+5}{6}$ (b) $\frac{9x+6}{20}$ (c) $\frac{3x+7}{4}$ (d) $\frac{11x-4}{15}$ (e) $\frac{x+1}{6}$

(f) $\frac{3x+8}{10}$ (g) $\frac{3x+1}{4}$ (h) $\frac{3x+7}{10}$

Exercise 3A

1. $x = c - 2$ 2. $x = c + 4$ 3. $x = q - p$ 4. $x = q + p$ 5. $x = 2a$

6. $x = 7a$ 7. $x = ya$ 8. $x = mp$ 9. $x = rs$ 10. $x = 5$

11. $x = a/4$ 12. $x = h/g$ 13. $x = t/n$ 14. $x = 2$ 15. $x = \frac{b-1}{2}$

16. $x = \frac{b-c}{2}$ 17. $x = \frac{b-c}{a}$ 18. $x = \frac{r-q}{p}$ 19. $x = \frac{y+w}{v}$ 20. $S = D/T$

21. $d = C/\pi$ 22. $x = 4$ or -4 23. $x = \sqrt{y}$ or $-\sqrt{y}$ 24. $r = \sqrt{(A/\pi)}$ 25. $S = D/T$

26. $y = \sqrt{A}$ (or $-\sqrt{A}$) 27. $\pi = P/3r^2$ 28. $r = \sqrt{(P/5\pi)}$ 29. $h = q + p$ 30. $p = h - q$

31. $h = \frac{q+5p}{2}$ 32. $p = \frac{2h-q}{5}$ 33. $x = \frac{b-c}{a}$

Exercise 3B

1. (a) $h = g/f$ (b) $h = e - g$ (c) $h = kf$ (d) $h = g - e$

2. (a) $r = \sqrt{Q}$ (b) $r = \sqrt{(N/\pi)}$ (c) $r = \sqrt{(M/2\pi)}$ (d) $r = \sqrt{(P/\pi w)}$

3. (a) $M = A/kl$ (b) $m = B/K$ (c) $m = c/\pi^2$ (d) $m = 1/3pD$

4. (a) $x = p - q$ (b) $x = s - r$ (c) $x = (s - r)/5$ (d) $x = (r + 3)/7$
 (e) $x = (m - 2)/2$ (f) $x = 2m + 5$ (g) $x = 2n - 2$ (h) $x = 2p - q$

5. (a) $P = QR$ (b) $s = 1/t$ (c) $Q = \sqrt{(P/M)}$ (d) $w = v^2z$
 (e) $f = e/5d$ (f) $n = K/mT$ (g) $s = \sqrt{(7/9r)}$ (h) $a = \sqrt{(c^2 - b^2)}$

6. (a) $T = V/(a+d)$ (b) $x = r/(p+q)$ (c) $x = c/(a-b)$ (d) $x = r/(m+1)$
 (e) $w = (v - vx)/(x + 1)$ (f) $r = \frac{(p+1)^2}{4}$

Exercise 4

1. (a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) $3\sqrt{3}$ (d) $2\sqrt{5}$ (e) $5\sqrt{2}$
 (f) $2\sqrt{7}$ (g) $3\sqrt{2}$ (h) $2\sqrt{6}$ (i) $10\sqrt{2}$ (j) $5\sqrt{3}$
 (k) $3\sqrt{5}$ (l) $6\sqrt{2}$ (m) $10\sqrt{3}$ (n) $7\sqrt{3}$ (o) $3\sqrt{6}$
 (p) $14\sqrt{2}$ (q) $20\sqrt{2}$ (r) $12\sqrt{10}$

2. (a) $8\sqrt{2}$ (b) $\sqrt{5}$ (c) $13\sqrt{10}$ (d) 0 (e) $-2\sqrt{6}$
 (f) $-\sqrt{3}$ (g) 0 (h) $10\sqrt{2} + 10\sqrt{3} !!$
3. (a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $10\sqrt{5}$ (d) $6\sqrt{3}$ (e) $5\sqrt{5}$
 (f) $\sqrt{7}$ (g) $8\sqrt{2}$ (h) $2\sqrt{2}$

Exercise 5

1. (a) 3 (b) 5 (c) 6 (d) 1 (e) x (f) $\sqrt{6}$ (g) $2\sqrt{5}$ (h) $4\sqrt{a}$
 (i) $\sqrt{2c}$ (j) \sqrt{xy} (k) 4 (l) 8 (m) $3\sqrt{2}$ (n) $10\sqrt{2}$ (o) 6 (p) $6\sqrt{6}$
2. (a) $\sqrt{2} + 2$ (b) $\sqrt{6} + \sqrt{3}$ (c) $5 + \sqrt{5}$ (d) $\sqrt{7} + 7$ (e) $5\sqrt{2} - 2$
 (f) $5\sqrt{2} - 8$ (g) $1 + \sqrt{3}$ (h) $-\sqrt{2}$ (i) 1 (j) 2
 (k) $5 + 2\sqrt{6}$ (l) $8 - 2\sqrt{15}$
3. (a) 6 (b) -2 (c) 6
4. (a) $4\sqrt{3}$ (b) 10 (c) $4\sqrt{15}$
5. (a) 4 cm^2 (b) $2\sqrt{6} \text{ cm}$

Exercise 6

1. (a) $\sqrt{2}/2$ (b) $\sqrt{3}/3$ (c) $\sqrt{5}/5$ (d) $\sqrt{6}/6$ (e) $\sqrt{7}/7$
 (f) $2\sqrt{5}$ (g) $2\sqrt{3}/3$ (h) $3\sqrt{5}/5$ (i) $10\sqrt{2}$ (j) $2\sqrt{3}$
 (k) $2\sqrt{6}$ (l) $3\sqrt{5}/10$ (m) $2\sqrt{2}/5$ (n) $\sqrt{5}/10$ (o) $\sqrt{2}/10$ (p) $\sqrt{2}$
2. (a) $2\sqrt{3}/3$ (b) $\sqrt{10}/2$ (c) $\sqrt{10}/5$ (d) $\sqrt{11}/11$ (e) $\sqrt{15}/5$ (f) $\frac{\sqrt{ab}}{b}$
3. (a) $\sqrt{2} + 1$ (b) $(\sqrt{7} + 1)/6$ (c) $(2 - \sqrt{2})/2$ (d) $\sqrt{5} - 1$ (e) $6 + 3\sqrt{3}$
 (f) $\sqrt{3} + \sqrt{2}$ (g) $\sqrt{5} - \sqrt{3}$ (h) $3\sqrt{5} + 3\sqrt{2}$

Exercise 7

1. (a) 2^7 (b) 3^8 (c) 8^7 (d) 10^{30} (e) a^7 (f) b^8 (g) c^8 (h) d^{10}
 (i) v^{11} (j) x^{12} (k) w^4 (l) z^6 (m) f^{11} (n) g^{20} (o) k^{12} (p) m^{101}
2. (a) 2 (b) 3^2 (c) 8^3 (d) 10^{10} (e) a (f) b^2 (g) c^5 (h) 1
 (i) v (j) x^8 (k) w^2 (l) z (m) f^3 (n) 1 (o) k^{10} (p) m^{99}
 (q) x^4 (r) m (s) a^4 (t) r
3. (a) x^6 (b) y^{15} (c) z^{10} (d) g^{16} (e) a^{21} (f) b^{16} (g) c^{30} (h) d^{21}
4. (a) 2^{10} (b) 7^{15} (c) 6^{20} (d) 8^{15} (e) 2^{49} (f) 3^{12} (g) 9^{10} (h) 2^{25}
5. (a) a^3b^3 (b) c^6d^6 (c) $x^{20}y^{10}$ (d) $4p^2q^2$
6. (a) y^5 (b) t^5 (c) x^3 (d) v^6 (e) $3x^5$ (f) $4x^7$ (g) $12x^4$
 (h) $4x^3$ (i) $3x^2$ (j) $x^5 + x^6$ (k) $x^6 - 4x^3$ (l) u^7v^7 (m) $25y^2$
 (n) m^8n^{16} (o) x^5 (p) u^6
7. (a) $m = 4$ (b) $m = 5$ (c) $m = 3$ (d) $m = 4$

Exercise 8

- (a) 1 (b) 1 (c) 1 (d) 1
- (a) $1/3^2$ (b) $1/5^7$ (c) $1/a^4$ (d) $1/b^9$ (e) $1/x$ (f) $3/y^2$ (g) x/y^3 (h) x^3
(i) t^5 (j) $6c^3$ (k) $1/2y^2$ (l) $1/7x^3$
- (a) 1 (b) $1/7$ (c) $1/27$ (d) $1/64$ (e) 8 (f) $9/4$
- (a) a^2 (b) b^2 (c) c^{-2} (d) 1 (e) e^9 (f) g^{-8} (g) w^2 (h) x^{-6}
(i) y^{-5} (j) z^{16} (k) 1
- (a) $1/4^3$ (b) $1/6^5$ (c) 2^5 (d) $25/4^3$ (e) $81/2^3$ (f) 1 (g) w^6 (h) x
(i) 4 (j) 40 (k) $6/h^2$ (l) $9/s$ (m) $1/2k$ (n) $3/4m^3$
- (a) $x^7 + x^3$ (b) $x^4 - x$ (c) $x + x^{-2}$ (d) $1 + x$ (e) $x^3 - x^{-6}$
(f) $2x^{-5} - 1$ (g) $6x^{-2} - 2x$ (h) $6x^3 - 3$
- (a) 27, 1, $1/9$, 3, $1/27$ (b) 4^{-1} , 4^{-2} , 4^{-3}

Exercise 9

- (a) $4\sqrt{x^3}$ (b) $5\sqrt{m^3}$ (c) $3\sqrt{r^2}$ (d) \sqrt{w} (e) $3\sqrt{n}$ (f) $1/(3\sqrt{r^4})$
- (a) $x^{5/3}$ (b) $b^{4/3}$ (c) $z^{2/3}$ (d) $w^{1/4}$ (e) $x^{-1/3}$ (f) $u^{1/2}$
- (a) 3 (b) 8 (c) 2 (d) 4 (e) 10 (f) 3 (g) 27 (h) 8
(i) 343 (j) $1/4$ (k) $1/3$ (l) $1/125$ (m) 27 (n) $1/27$ (o) 2 (p) $1/32$
- (a) x^2 (b) m^3 (c) c^4 (d) n^{-3} (e) $n^{-1/2}$ (f) g^2 (g) b^{-5}
(h) z^3 (i) $x^{2/5}$ (j) x^2 (k) 13 (l) 6

Exercise 10

- (a) a^2 (b) b (c) 1 (d) d^2 (e) $e^{4/5}$ (f) z (g) w^{-1} (h) 1
- (a) 4 (b) $12b$ (c) $6c$ (d) $2d$ (e) $2e$ (f) $4v$ (g) 1
- (a) $x - 1$ (b) $x^2 + x$ (c) $x^{3/5} + x^{-4/5}$
- (a) 12 (b) 32 (c) 45 (d) $8/9$ (e) $3/2$
- (a) $p^{13/2}$ (b) $p^{11/2}$ (c) $1/p$ (d) $1/p^9$ (e) $p^{3/2}$ (f) $49/p$
- (a) 1 (b) x^6 (c) x (d) $x^{-9/2}$ (e) 1 (f) x
- (a) $2 - x^3 - 1/x^3$ (b) $1/x^2 - 4$ (c) $-4x^{1/2} + 4/x^{1/2} - 15$

Exercise 11

- x^{-1} 2. x^{-2} 3. $7x^{-2}$ 4. $5x^{-3}$ 5. $1/3x^{-1}$ 6. $4/5x^{-1}$ 7. $3/2x^{-2}$
- $x^{-1/2}$ 9. $5/2x^{-1/2}$ 10. $3/5x^3$ 11. $\frac{x-3}{\sqrt{3}}$ 12. $\frac{2x-3/2}{\sqrt{5}}$

Checkup for algebraic operations

- (a) $1/2$ (b) $3a/b$ (c) $1/(x-3)$ (d) $v-w$ (e) $\frac{x+y}{2x+3y}$ (f) $1/(a+8)$
(g) $(v+1)/(v-2)$
- (a) $b^2/2$ (b) a (c) $3z$ (d) a (e) $3c/8$ (f) $(15u-8v)/20$
(g) $(3m+k)/km$ (h) $(3x+1)/10$
- (a) $x=1$ (b) $x=w+a$ (c) $x=(p-m)/a$ (d) $x=pz/w$
(e) $x=\sqrt{N/2\pi}$ (f) $x=(T-3)/4$ (g) $x=2w-y$ (h) $x=\sqrt{5/8M}$
- (a) $4\sqrt{2}$ (b) $10\sqrt{10}$ (c) $6\sqrt{5}$ (d) $\sqrt{5}$ (e) $2\sqrt{2}$ (f) $11\sqrt{2}$ (g) $3/a$ (h) $3\sqrt{2}$
- (a) 4 (b) -10 (c) 8 6. (a) $\sqrt{5/5}$ (b) $4\sqrt{2}$ (c) $3\sqrt{5}$ (d) $\sqrt{3/3}$
- (a) 5^{14} (b) x^2 (c) $6m^4$ (d) a^6 (e) $16a^6b^2$ (f) a^4 (g) 1 (h) x^5-x^4
- (a) $1/5^2$ (b) a/b^3 (c) y^6 (d) $(x^2)/4$
- (a) \sqrt{b} (b) $1/\sqrt{c^3}$
- (a) $x^{4/3}$ (b) $a^{-3/2}$
- (a) 216 (b) $1/4$ (c) x^4 (d) $1/y^7$ (e) $1/a-1/a^4$ (f) $s/2$ (g) x

Quadratic functions

Exercise 1

- (a) $y=x^2$ (b) $y=2x^2$ (c) $y=1/2x^2$
(d) $y=-2x^2$ (e) $y=-1/2x^2$ (f) $y=-5x^2$
- (a) $y=x^2+1$ (b) $y=x^2+3$ (c) $y=x^2-2$
(d) $y=-x^2+4$ (e) $y=-x^2+1$ (f) $y=-x^2-1$
- (a) $y=(x-1)^2+1$ (b) $y=(x-2)^2+3$ (c) $y=(x-3)^2$
(d) $y=(x+3)^2+2$ (e) $y=(x+4)^2$ (f) $y=(x+5)^2-3$
(g) $y=(x-3)^2+2$ (h) $y=(x-1)^2-4$ (i) $y=(x+3)^2-4$

Exercise 2

- (a) $(2,1)$ (b) $x=2$ (c) $y=(x-2)^2+1$
- (a) (i) $(3,2)$ (ii) $x=3$ (iii) $y=(x-3)^2+2$
(b) (i) $(1,-1)$ (ii) $x=1$ (iii) $y=(x-1)^2-1$
(c) (i) $(-2,1)$ (ii) $x=-2$ (iii) $y=(x+2)^2+1$
(d) (i) $(-3,1)$ (ii) $x=-3$ (iii) $y=(x+3)^2+1$
(e) (i) $(3,-2)$ (ii) $x=3$ (iii) $y=(x-3)^2-2$
(f) (i) $(-1,-3)$ (ii) $x=-1$ (iii) $y=(x+1)^2-3$
- (a) $(4,1); x=4$ (b) $(2,7); x=2$ (c) $(8,3); x=8$
(d) $(-1,2); x=-1$ (e) $(1,-3); x=1$ (f) $(-3,-7); x=-3$
(g) $(5,0); x=5$ (h) $(-2,0); x=-2$ (i) $(0,3); x=0$

ALGEBRAIC OPERATIONS

By the end of this set of exercises, you should be able to

- (a) multiply algebraic expressions involving brackets**
- (b) factorise algebraic expressions**
- (c) factorise trinomial expressions**

ALGEBRAIC OPERATIONS

A. Multiplying Algebraic Expressions Involving Brackets

Exercise 1

1. Write these without brackets:

- | | | | |
|------------------|-----------------|----------------|----------------|
| (a) $6(x + 2)$ | (b) $3(a + 1)$ | (c) $5(y - 4)$ | (d) $7(t - 1)$ |
| (e) $10(x - 10)$ | (f) $2(2 + x)$ | (g) $3(4 + y)$ | (h) $6(5 - w)$ |
| (i) $8(1 - c)$ | (j) $15(2 - h)$ | (k) $3(x + y)$ | (l) $9(a - c)$ |
| (m) $4(2 - x)$ | (n) $11(e - f)$ | (o) $1(1 - y)$ | (p) $1(y - 1)$ |

2. Remove the brackets:

- | | | | |
|-----------------|------------------|------------------|------------------|
| (a) $3(2x + 4)$ | (b) $2(4a + 3)$ | (c) $5(1 + 2y)$ | (d) $6(3 - 3x)$ |
| (e) $7(2w - 4)$ | (f) $c(x + 5)$ | (g) $d(v + 3)$ | (h) $g(h - 1)$ |
| (i) $s(r - 4)$ | (j) $m(n + 10)$ | (k) $x(v + w)$ | (l) $a(x + r)$ |
| (m) $x(a - y)$ | (n) $a(a + b)$ | (o) $r(r - s)$ | (p) $r(r - 1)$ |
| (q) $a(1 - a)$ | (r) $x(x - 8)$ | (s) $x(x + 3y)$ | (t) $w(3w - 1)$ |
| (u) $x(5x - 3)$ | (v) $a(7x - 5a)$ | (w) $m(4m + 8n)$ | (x) $v(27 - 2v)$ |

3. Multiply out the brackets:

- | | | |
|----------------------|-----------------------|----------------------|
| (a) $2(x + y + 4)$ | (b) $7(x + y + 1)$ | (c) $5(x - y - 6)$ |
| (d) $6(x + 2y + 5)$ | (e) $10(4x - y + z)$ | (f) $9(6a - 2b + 1)$ |
| (g) $x(3x + 5y + z)$ | (h) $2a(3a - 4b + c)$ | (i) $s(s^2 + 3)$ |
| (j) $x(x^2 + 1)$ | (k) $y(y^2 - 1)$ | (l) $c(c^2 - 6)$ |
| (m) $w(w^2 + w)$ | (n) $a(a^2 - a)$ | (o) $x(x^3 - 2x^2)$ |

Exercise 2A

1. Multiply out these brackets:

- | | | |
|----------------------|------------------------|------------------------|
| (a) $(x + 1)(x + 5)$ | (b) $(x + 2)(x + 3)$ | (c) $(x + 5)(x + 6)$ |
| (d) $(x + 3)(x + 7)$ | (e) $(x + 4)(x + 4)$ | (f) $(x + 1)(x + 1)$ |
| (g) $(a + 1)(a + 8)$ | (h) $(s + 11)(s + 10)$ | (i) $(w + 4)(w + 100)$ |

2. Multiply:

- | | | |
|------------------------|----------------------|----------------------|
| (a) $(x - 3)(x - 1)$ | (b) $(x - 4)(x - 2)$ | (c) $(x - 7)(x - 8)$ |
| (d) $(a - 2)(a - 5)$ | (e) $(b - 7)(b - 7)$ | (f) $(c - 3)(c - 2)$ |
| (g) $(v - 10)(v - 10)$ | (h) $(w - 6)(w - 3)$ | (i) $(z - 1)(z - 1)$ |

3. Multiply:

- | | | |
|----------------------|-----------------------|------------------------|
| (a) $(x + 5)(x + 1)$ | (b) $(c - 4)(c - 2)$ | (c) $(s - 6)(s + 3)$ |
| (d) $(a - 7)(a - 5)$ | (e) $(v + 9)(v + 9)$ | (f) $(q - 6)(q + 2)$ |
| (g) $(r + 6)(r - 2)$ | (h) $(w - 8)(w + 8)$ | (i) $(x + 1)(x - 1)$ |
| (j) $(d - 3)(d - 3)$ | (k) $(a - 6)(a + 11)$ | (l) $(z - 10)(z + 11)$ |

4. Multiply:

- | | | |
|------------------------|------------------------|--------------------------|
| (a) $(2x + 3)(2x - 3)$ | (b) $(5c - 1)(5c + 1)$ | (c) $(2s - 1)(2s + 3)$ |
| (d) $(2a - 3)(2a - 1)$ | (e) $(v + 1)(4v - 3)$ | (f) $(3q - 4)(2q + 3)$ |
| (g) $(4r - 2)(5r + 3)$ | (h) $(4w - 5)(2w + 5)$ | (i) $(10x + 1)(10x - 1)$ |
| (j) $(2 - d)(1 - d)$ | (k) $(4 - p)(3 + 2p)$ | (l) $(1 - 3p)(1 - 2p)$ |

5. Multiply out:

- | | | | |
|------------------|------------------|-------------------|-------------------|
| (a) $(x + 2)^2$ | (b) $(y + 4)^2$ | (c) $(z + 3)^2$ | (d) $(t + 10)^2$ |
| (e) $(x - 1)^2$ | (f) $(y - 6)^2$ | (g) $(z - 2)^2$ | (h) $(t - 8)^2$ |
| (i) $(a + b)^2$ | (j) $(g + h)^2$ | (k) $(r - s)^2$ | (l) $(e - f)^2$ |
| (m) $(3x + 1)^2$ | (n) $(4x - 3)^2$ | (o) $(x + 3y)^2$ | (p) $(a - 4b)^2$ |
| (q) $(4a + b)^2$ | (r) $(5c + d)^2$ | (s) $(5p + 2q)^2$ | (t) $(2x - 3y)^2$ |

Exercise 2B

Multiply out the brackets and simplify:

- | | |
|----------------------------|-------------------------------|
| 1. $(x + 1)(x^2 + 3x + 1)$ | 2. $(x + 2)(x^2 - 4x + 1)$ |
| 3. $(w - 3)(w^2 + w - 2)$ | 4. $(z - 1)(z^2 - 5z - 1)$ |
| 5. $(v + 2)(2v^2 + v + 5)$ | 6. $(a - 5)(5a^2 - 10a - 20)$ |
| 7. $(m + 2)^3$ | 8. $(n - 1)^3$ |
| 9. $(x + 1/x)^2$ | 10. $(x - 1/x)^2$ |

B. Factorising Algebraic Expressions - The Common Factor

Exercise 3

1. Factorise the following by taking out the common factors:

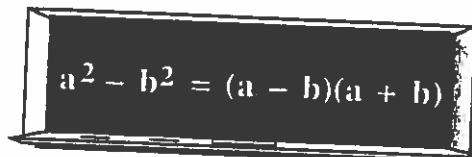
- | | | | |
|------------------|------------------|-----------------|-----------------|
| (a) $4a + 4b$ | (b) $7v + 7w$ | (c) $3x - 3y$ | (d) $6c - 6d$ |
| (e) $2r + 4s$ | (f) $9m - 12n$ | (g) $av + aw$ | (h) $pq - pr$ |
| (i) $bx + b$ | (j) $ax^2 + a$ | (k) $x^2 + dx$ | (l) $y^2 - yz$ |
| (m) $a^2 + a$ | (n) $t^2 - t$ | (o) $h^3 + h^2$ | (p) $m^3 - m^2$ |
| (q) $ab + bt$ | (r) $mn - nr$ | (s) $8x + 12y$ | (t) $35p - 21q$ |
| (u) $2a^2 + 8ab$ | (v) $12ab - 9ac$ | (w) $pqr + pqs$ | (x) $8c^2 - 2c$ |

2. Factorise:

- | | | | |
|----------------------|---------------------|-----------------------|--------------------------|
| (a) $am - bm$ | (b) $20 - 5w$ | (c) $d - d^2$ | (d) $yz + z$ |
| (e) $pr - pu$ | (f) $2mn + mp$ | (g) $6cd - 4ce$ | (h) $9pq - 12pr$ |
| (i) $8a^2 + 6a$ | (j) $15x^2 - 6xy$ | (k) $1/2x + 1/2y$ | (l) $pq + 1/2sq^2$ |
| (m) $10a^2b + 8ab^2$ | (n) $1/2 + 1/2x$ | (o) $1/2v - 3/2$ | (p) $2\pi rh + 2\pi r^2$ |
| (q) $6a + 3b - 12c$ | (r) $mn - mp + m^2$ | (s) $3x^2 - 2xy + 6x$ | (t) $25x^2 - 5x^2y$ |

C. Difference of Two Squares

Exercise 4


$$a^2 - b^2 = (a - b)(a + b)$$

1. Factorise:

- | | | | |
|-----------------|-----------------|-----------------|-------------------|
| (a) $x^2 - y^2$ | (b) $p^2 - q^2$ | (c) $d^2 - e^2$ | (d) $x^2 - 32$ |
| (e) $y^2 - 42$ | (f) $t^2 - 52$ | (g) $5^2 - t^2$ | (h) $9^2 - q^2$ |
| (i) $1 - v^2$ | (j) $x^2 - 4$ | (k) $k^2 - 25$ | (l) $n^2 - 36$ |
| (m) $d^2 - 100$ | (n) $e^2 - 121$ | (o) $144 - y^2$ | (p) $49 - x^2$ |
| (q) $x^2 - 1$ | (r) $1 - y^2$ | (s) $81 - a^2$ | (t) $10000 - b^2$ |

2. Factorise:

- | | | | |
|---------------------|-----------------------|--------------------|---------------------|
| (a) $9a^2 - 4$ | (b) $4b^2 - 25$ | (c) $16c^2 - 1$ | (d) $25d^2 - 36$ |
| (e) $9e^2 - 16$ | (f) $25f^2 - 81$ | (g) $4g^2 - h^2$ | (h) $j^2 - 25k^2$ |
| (i) $64m^2 - 49n^2$ | (j) $4p^2 - 9q^2$ | (k) $81r^2 - 1$ | (l) $1 - 64s^2$ |
| (m) $121 - 16t^2$ | (n) $100u^2 - 121v^2$ | (o) $10000w^2 - 1$ | (p) $25x^2 - 49y^2$ |

3. Factorise these, by taking out the common factor first:

- | | | | |
|-------------------|---------------------|---------------------|---------------------|
| (a) $2a^2 - 18$ | (b) $5b^2 - 5$ | (c) $6c^2 - 54$ | (d) $4d^2 - 16$ |
| (e) $7e^2 - 7g^2$ | (f) $6p^2 - 24q^2$ | (g) $10x^2 - 90y^2$ | (h) $12u^2 - 12v^2$ |
| (i) $am^2 - an^2$ | (j) $ka^2 - 25kb^2$ | (k) $nr^2 - 81nq^2$ | (l) $d^3 - 49d$ |
| (m) $64b - b^3$ | (n) $2u^3 - 32u$ | (o) $12w^3 - 27w$ | (p) $11x^5 - 11x^3$ |

D. Trinomial Expressions

Exercise 5

Factorise the expressions:

- | | | |
|----------------------|----------------------|----------------------|
| 1. $x^2 + 3x + 2$ | 2. $x^2 + 5x + 6$ | 3. $x^2 + 2x + 1$ |
| 4. $y^2 + 6y + 5$ | 5. $y^2 + 11y + 10$ | 6. $y^2 + 8y + 7$ |
| 7. $v^2 + 9v + 20$ | 8. $v^2 + 7v + 10$ | 9. $v^2 + 6v + 8$ |
| 10. $w^2 - 2w + 1$ | 11. $w^2 - 4w + 4$ | 12. $w^2 - 6w + 9$ |
| 13. $a^2 - 3a + 2$ | 14. $a^2 - 7a + 12$ | 15. $a^2 - 8a + 7$ |
| 16. $c^2 - 13c + 42$ | 17. $c^2 - 11c + 24$ | 18. $c^2 - 10c + 9$ |
| 19. $s^2 + 12s + 36$ | 20. $s^2 - 12s + 36$ | 21. $s^2 + 14s + 49$ |
| 22. $z^2 - 14z + 49$ | 23. $z^2 + 13z + 36$ | 24. $z^2 - 13z + 36$ |
| 25. $b^2 + 37b + 36$ | 26. $b^2 - 37b + 36$ | 27. $b^2 - 18b + 81$ |

- | | | |
|-----------------------|-----------------------|-----------------------|
| 28. $p^2 + 6p + 9$ | 29. $p^2 - 7p - 8$ | 30. $p^2 + 4p + 4$ |
| 31. $m^2 + 11m + 30$ | 32. $m^2 + m - 12$ | 33. $m^2 - m - 6$ |
| 34. $n^2 - 8n + 15$ | 35. $n^2 + 3n - 10$ | 36. $n^2 - 3n - 4$ |
| 37. $r^2 - 2r - 8$ | 38. $r^2 + 5r - 6$ | 39. $r^2 + 12r + 36$ |
| 40. $e^2 - 5e - 14$ | 41. $e^2 + 7e + 12$ | 42. $e^2 - e - 56$ |
| 43. $g^2 - 7g + 12$ | 44. $g^2 - g - 6$ | 45. $g^2 - g - 12$ |
| 46. $k^2 - 4k - 5$ | 47. $k^2 + k - 6$ | 48. $k^2 + 2k - 35$ |
| 49. $y^2 + 4y - 12$ | 50. $y^2 + 3y - 18$ | 51. $y^2 - 3y - 28$ |
| 52. $x^2 - 3x - 40$ | 53. $x^2 - 2x - 15$ | 54. $x^2 + 11x + 30$ |
| 55. $v^2 - 9v + 8$ | 56. $v^2 + 5v - 24$ | 57. $v^2 - 5v - 24$ |
| 58. $w^2 + 2w - 24$ | 59. $w^2 - 2w - 24$ | 60. $w^2 + 10w - 24$ |
| 61. $a^2 - 10a - 24$ | 62. $a^2 + 23a - 24$ | 63. $a^2 - 23a - 24$ |
| 64. $b^2 + 7b - 30$ | 65. $b^2 - 4b - 45$ | 66. $b^2 - 7b - 18$ |
| 67. $c^2 + 15c + 56$ | 68. $c^2 - 15c + 54$ | 69. $c^2 + 18c + 81$ |
| 70. $d^2 - 12d - 28$ | 71. $d^2 + 49d - 50$ | 72. $d^2 - 51d + 50$ |
| 73. $a^2 + 2ab + b^2$ | 74. $x^2 - 2xy + y^2$ | 75. $p^2 - pq - 2q^2$ |

Exercise 6

Factorise these expressions:

- | | | |
|----------------------|-----------------------|-------------------------|
| 1. $2x^2 + 7x + 3$ | 2. $2y^2 + 5y + 3$ | 3. $3w^2 + 7w + 2$ |
| 4. $10a^2 + 17a + 3$ | 5. $6b^2 + 7b + 2$ | 6. $6c^2 + 7c + 1$ |
| 7. $3d^2 + 14d + 15$ | 8. $10m^2 + 19m + 6$ | 9. $2p^2 - 7p + 3$ |
| 10. $12n^2 - 8n + 1$ | 11. $2q^2 - 5q + 3$ | 12. $6x^2 - 13x + 6$ |
| 13. $8s^2 - 14s + 5$ | 14. $9r^2 - 24r + 16$ | 15. $12g^2 - 23g + 10$ |
| 16. $3k^2 - 5k + 2$ | 17. $3y^2 - 2y - 8$ | 18. $3w^2 - 5w - 2$ |
| 19. $6u^2 - 5u - 6$ | 20. $5v^2 + 4v - 1$ | 21. $2x^2 + x - 1$ |
| 22. $3d^2 - 2d - 1$ | 23. $8a^2 + 2a - 3$ | 24. $12y^2 - 11y - 5$ |
| 25. $4p^2 - 11p + 6$ | 26. $15 - 7x - 2x^2$ | 27. $5 + 11x - 12x^2$ |
| 28. $1 - 8x + 16x^2$ | 29. $1 - 3x - 18x^2$ | 30. $4p^2 - 7pq - 2q^2$ |

Exercise 7 (Miscellaneous Examples on Factorisation)

Factorise FULLY:

- | | | |
|----------------------------------|----------------------|-----------------------|
| 1. $4x + 12y$ | 2. $a^2 - 81$ | 3. $w^2 + 10w + 25$ |
| 4. $y^2 - y$ | 5. $v^2 - v - 12$ | 6. $1 - b^2$ |
| 7. $u^2 + 12u + 36$ | 8. $ap - aq + ar$ | 9. $7x^2 - 28$ |
| 10. $w^2 - r^2$ | 11. $h^2 - 11h$ | 12. $x^2 - 2x + 1$ |
| 13. $t^2 - 1$ | 14. $t^2 - t$ | 15. $a^2 - 2a - 3$ |
| 16. $3c^2 - 48$ | 17. $5d^2 - 20d$ | 18. $a^4 - a^3$ |
| 19. $2s^2 + 3s - 5$ | 20. $x^2 - 12x + 36$ | 21. $16y^2 + 8y + 1$ |
| 22. $49 - g^2$ | 23. $36 - 4r^2$ | 24. $14z - 7z^2$ |
| 25. $25 - 9g^2$ | 26. $2b^2 - b - 1$ | 27. $6x^2 + 7x - 3$ |
| 28. $11u^2 - 44v^2$ | 29. $21u^2 + 28v^2$ | 30. $25p^2 - 10p + 1$ |
| 31. $3m^2n - 6mn^2$ | 32. $1 - 2n + n^2$ | 33. $27 - 6s - s^2$ |
| 34. $3a^3 - 48a$ | 35. $8n^2 + 8n - 6$ | 36. $8n^2 - 8n + 2$ |
| 37. $5r^2 + 5r - 10$ | 38. $4w^2 + 14w - 8$ | 39. $7x - 63x^3$ |
| 40. $9x + 27x^3$ | 41. $xy^2 - xz^2$ | 42. $2e^2 - 11e - 21$ |
| 43. $x^4 - 1$ | 44. $2 - 4q + 2q^2$ | 45. $g^2 + gh - 6h^2$ |
| 46. $2k^2 + 3\pi Rk + \pi^2 R^2$ | 47. $a^2 - a^6$ | 48. $k^4 + 2k^2 + 1$ |
| 49. $2a^4 - 2a^2 - 12$ | 50. $b^5 - 81b$ | 51. $3x^4 + 5x^2 - 2$ |
| 52. $9x^4 - 24x^2 + 16$ | 53. $2x^4 - x^2 - 3$ | 54. $1 - y^8$ |

Checkout for Algebraic Operations

1. Remove the brackets:

(a) $3(4x + 1)$

(b) $y(a - y)$

(c) $v(v - 1)$

(d) $7w(2w - 5)$

(e) $6(3x + 2y - 1)$

(f) $c(c^2 + c - 1)$

(g) $3d(4a + 3b)$

(h) $g(h^2 - g^2)$

(i) $6x(3x + 2y - 1)$

(j) $c^2(c^2 + c - 4)$

(k) $ab(3a + 4b)$

(l) $2pq(5 - q)$

2. Multiply out the brackets:

(a) $(x + 1)(x + 7)$

(b) $(x - 2)(x - 3)$

(c) $(x + 5)(x - 6)$

(d) $(x - 3)(x + 9)$

(e) $(x + 1)^2$

(f) $(x - 2)^2$

(g) $(5x - 1)(4x + 7)$

(h) $(2x - 1)(6x - 3)$

(i) $(2x - 4)(3x + 1)$

(j) $(2x - 3)^2$

(k) $(x - 2)(4x^2 - 3x + 2)$

(l) $(x - 3)^3$

3. Factorise fully:

(a) $9m - 9n$

(b) $6a - 15b$

(c) $y - y^2$

(d) $14p^2 + 6q$

(e) $3pr + pu$

(f) $4p^2 + 6pq - 2p$

(g) $6x + 30y - 15z$

(h) $9pq - 12pr$

(i) $r^2 - s^2$

(j) $81 - q^2$

(k) $16r^2 - 49$

(l) $2b^2 - 32$

(m) $20w^3 - 45w$

(n) $y^2 - 3y + 2$

(o) $a^2 - 7a - 30$

(p) $y^2 + y - 6$

(q) $24 + 10r - r^2$

(r) $x^2 - 14x + 49$

(s) $6p^2 - 17p + 12$

(t) $4x^2 + 4x + 1$

(u) $2q^2 - 2q - 144$

(v) $2x^2 + 3xy - 2y^2$

(w) $6a^4 + 2a^2 - 4$

(x) $5y^4 - 12y^2 - 9$

Algebraic Operations

Exercise 1

- | | | | |
|-----------------------|---------------------|-----------------------|------------------------|
| 1. (a) $6x + 12$ | (b) $3a + 3$ | (c) $5y - 20$ | (d) $7t - 7$ |
| (e) $10x - 100$ | (f) $4 + 2x$ | (g) $12 + 3y$ | (h) $30 - 6w$ |
| (i) $8 - 8c$ | (j) $30 - 15h$ | (k) $3x + 3y$ | (l) $9a - 9c$ |
| (m) $8 - 4x$ | (n) $11e - 11f$ | (o) $1 - y$ | (p) $y - 1$ |
| 2. (a) $6x + 12$ | (b) $8a + 6$ | (c) $5 + 10y$ | (d) $18 - 18x$ |
| (e) $14w - 28$ | (f) $cx + 5c$ | (g) $dv + 3d$ | (h) $gh - g$ |
| (i) $sr - 4s$ | (j) $mn + 10m$ | (k) $xv + xw$ | (l) $ax + ar$ |
| (m) $xa - xy$ | (n) $a^2 + ab$ | (o) $r^2 - rs$ | (p) $r^2 - r$ |
| (q) $a - a^2$ | (r) $x^2 - 8x$ | (s) $x^2 + 3xy$ | (t) $3w^2 - w$ |
| (u) $5x^2 - 3x$ | (v) $7ax - 5a^2$ | (w) $4m^2 + 8mn$ | (x) $27v - 2v^2$ |
| 3. (a) $2x + 2y + 8$ | (b) $7x + 7y + 7$ | (c) $5x - 5y - 30$ | (d) $6x + 12y + 30$ |
| (e) $40x - 10y + 10z$ | (f) $54a - 18b + 9$ | (g) $3x^2 + 5xy + xz$ | (h) $6a^2 - 8ab + 2ac$ |
| (i) $s^3 + 3s$ | (j) $x^3 + x$ | (k) $y^3 - y$ | (l) $c^3 - 6c$ |
| (m) $w^3 + w^2$ | (n) $a^3 - a^2$ | (o) $x^4 - 2x^3$ | |

Exercise 2A

- | | | | |
|-------------------------|--------------------------|---------------------------|--------------------------|
| 1. (a) $x^2 + 6x + 5$ | (b) $x^2 + 5x + 6$ | (c) $x^2 + 11x + 30$ | (d) $x^2 + 10x + 21$ |
| (e) $x^2 + 8x + 16$ | (f) $x^2 + 2x + 1$ | (g) $a^2 + 9a + 8$ | (h) $s^2 + 21s + 110$ |
| (i) $w^2 + 104w + 400$ | | | |
| 2. (a) $x^2 - 4x + 3$ | (b) $x^2 - 6x + 8$ | (c) $x^2 - 15x + 56$ | (d) $a^2 - 7a + 10$ |
| (e) $b^2 - 14b + 49$ | (f) $c^2 - 5c + 6$ | (g) $v^2 - 20v + 100$ | (h) $w^2 - 9w + 18$ |
| (i) $z^2 - 2z + 1$ | | | |
| 3. (a) $x^2 + 6x + 5$ | (b) $c^2 - 6c + 8$ | (c) $s^2 - 3s - 18$ | (d) $a^2 - 12a + 35$ |
| (e) $v^2 + 18v + 81$ | (f) $q^2 - 4q - 12$ | (g) $r^2 + 4r - 12$ | (h) $w^2 - 64$ |
| (i) $x^2 - 1$ | (j) $d^2 - 6d + 9$ | (k) $a^2 + 5a - 66$ | (l) $z^2 + z - 110$ |
| 4. (a) $4x^2 - 9$ | (b) $25c^2 - 1$ | (c) $4s^2 + 4s - 3$ | (d) $4a^2 - 8a + 3$ |
| (e) $4v^2 + v - 3$ | (f) $6q^2 + q - 12$ | (g) $20r^2 + 2r - 6$ | (h) $8w^2 + 10w - 25$ |
| (i) $100x^2 - 1$ | (j) $2 - 3d + d^2$ | (k) $12 + 5p - 2p^2$ | (l) $1 - 5p + 6p^2$ |
| 5. (a) $x^2 + 4x + 4$ | (b) $y^2 + 8y + 16$ | (c) $z^2 + 6z + 9$ | (d) $t^2 + 20t + 100$ |
| (e) $x^2 - 2x + 1$ | (f) $y^2 - 12y + 36$ | (g) $z^2 - 4z + 4$ | (h) $t^2 - 16t + 64$ |
| (i) $a^2 + 2ab + b^2$ | (j) $g^2 + 2gh + h^2$ | (k) $r^2 - 2rs + s^2$ | (l) $e^2 - 2ef + f^2$ |
| (m) $9x^2 + 6x + 1$ | (n) $16x^2 - 24x + 9$ | (o) $x^2 + 6xy + 9y^2$ | (p) $a^2 - 8ab + 16b^2$ |
| (q) $16a^2 + 8ab + b^2$ | (r) $25c^2 + 10cd + d^2$ | (s) $25p^2 + 20pq + 4q^2$ | (t) $4x^2 - 12xy + 9y^2$ |

Exercise 2B

- | | | |
|---------------------------|----------------------------|-------------------------------|
| 1. $x^3 + 4x^2 + 4x + 1$ | 2. $x^3 - 2x^2 - 7x + 2$ | 3. $w^3 - 2w^2 - 5w + 6$ |
| 4. $z^3 - 6z^2 + 4z + 1$ | 5. $2v^3 + 5v^2 + 7v + 10$ | 6. $5a^3 - 35a^2 + 30a + 100$ |
| 7. $m^3 + 6m^2 + 12m + 8$ | 8. $n^3 - 3n^2 + 3n - 1$ | 9. $x^2 + 2 + 1/x^2$ |
| 10. $x^2 - 2 + 1/x^2$ | | |

Exercise 3

- | | | | | |
|------------------|-------------------|-----------------|------------------|------------------|
| (a) $4(a + b)$ | (b) $7(v + w)$ | (c) $3(x - y)$ | (d) $6(c - d)$ | (e) $2(r + 2s)$ |
| (f) $3(3m - 4n)$ | (g) $a(v + w)$ | (h) $p(q - r)$ | (i) $b(x + 1)$ | (j) $a(x^2 + 1)$ |
| (k) $x(x + d)$ | (l) $y(y - z)$ | (m) $a(a + 1)$ | (n) $t(t - 1)$ | (o) $h^2(h + 1)$ |
| (p) $m^2(m - 1)$ | (q) $b(a + t)$ | (r) $n(m - r)$ | (s) $4(2x + 3y)$ | (t) $7(5p - 3q)$ |
| (u) $2a(a + 4b)$ | (v) $3a(4b - 3c)$ | (w) $pq(r + s)$ | (x) $2c(4c - 1)$ | |
- | | | | | |
|--------------------------|----------------------------|--------------------|--------------------------|--------------------------|
| (a) $m(a - b)$ | (b) $5(4 - w)$ | (c) $d(1 - d)$ | (d) $z(y + 1)$ | (e) $p(r - u)$ |
| (f) $m(2n + p)$ | (g) $2c(3d - 2e)$ | (h) $3p(3q - 4r)$ | (i) $2a(4a + 3)$ | (j) $3x(5x - 2y)$ |
| (k) $\frac{1}{2}(x + y)$ | (l) $q(p + \frac{1}{2}sq)$ | (m) $2ab(5a + 4b)$ | (n) $\frac{1}{2}(1 + x)$ | (o) $\frac{1}{2}(v - 3)$ |
| (p) $2\pi r(h + r)$ | (q) $3(2a + b - 4c)$ | (r) $m(n - p + m)$ | (s) $x(3x - 2y + 6)$ | (t) $5x^2(5 - y)$ |

Exercise 4

- | | | | |
|------------------------|------------------------|------------------------|--------------------------|
| (a) $(x - y)(x + y)$ | (b) $(p - q)(p + q)$ | (c) $(d - e)(d + e)$ | (d) $(x - 3)(x + 3)$ |
| (e) $(y - 4)(y + 4)$ | (f) $(t - 5)(t + 5)$ | (g) $(5 - t)(5 + t)$ | (h) $(9 - q)(9 + q)$ |
| (i) $(1 - v)(1 + v)$ | (j) $(x - 2)(x + 2)$ | (k) $(k - 5)(k + 5)$ | (l) $(n - 6)(n + 6)$ |
| (m) $(d - 10)(d + 10)$ | (n) $(e - 11)(e + 11)$ | (o) $(12 - y)(12 + y)$ | (p) $(7 - x)(7 + x)$ |
| (q) $(x - 1)(x + 1)$ | (r) $(1 - y)(1 + y)$ | (s) $(9 - a)(9 + a)$ | (t) $(100 - b)(100 + b)$ |
- | | | | |
|----------------------------|--------------------------|------------------------------|------------------------|
| (a) $(3a - 2)(3a + 2)$ | (b) $(2b - 5)(2b + 5)$ | (c) $(4c - 1)(4c + 1)$ | (d) $(5d - 6)(5d + 6)$ |
| (e) $(3e - 4)(3e + 4)$ | (f) $(5f - 9)(5f + 9)$ | (g) $(2g - h)(2g + h)$ | (h) $(j - 5k)(j + 5k)$ |
| (i) $(8m - 7n)(8m + 7n)$ | (j) $(2p - 3q)(2p + 3q)$ | (k) $(9r - 1)(9r + 1)$ | (l) $(1 - 8s)(1 + 8s)$ |
| (m) $(11 - 4t)(11 + 4t)$ | | (n) $(10u - 11v)(10u + 11v)$ | |
| (o) $(100w - 1)(100w + 1)$ | | (p) $(5x - 7y)(5x + 7y)$ | |
- | | | | |
|--------------------------|-------------------------|---------------------------|------------------------|
| (a) $2(a - 3)(a + 3)$ | (b) $5(b - 1)(b + 1)$ | (c) $6(c - 3)(c + 3)$ | (d) $4(d - 2)(d + 2)$ |
| (e) $7(e - g)(e + g)$ | (f) $6(p - 2q)(p + 2q)$ | (g) $10(x - 3y)(x + 3y)$ | (h) $12(u - v)(u + v)$ |
| (i) $a(m - n)(m + n)$ | (j) $k(a - 5b)(a + 5b)$ | (k) $n(r - 9q)(r + 9q)$ | (l) $d(d - 7)(d + 7)$ |
| (m) $b(8 - b)(8 + b)$ | | (n) $2u(u - 4)(u + 4)$ | |
| (o) $3w(2w - 3)(2w + 3)$ | | (p) $11x^3(x - 1)(x + 1)$ | |

Exercise 5

- | | | |
|-----------------------|-----------------------|-----------------------|
| 1. $(x + 2)(x + 1)$ | 2. $(x + 3)(x + 2)$ | 3. $(x + 1)(x + 1)$ |
| 4. $(y + 5)(y + 1)$ | 5. $(y + 10)(y + 1)$ | 6. $(y + 7)(y + 1)$ |
| 7. $(v + 4)(v + 5)$ | 8. $(v + 2)(v + 5)$ | 9. $(v + 4)(v + 2)$ |
| 10. $(w - 1)(w - 1)$ | 11. $(w - 2)(w - 2)$ | 12. $(w - 3)(w - 3)$ |
| 13. $(a - 2)(a - 1)$ | 14. $(a - 3)(a - 4)$ | 15. $(a - 7)(a - 1)$ |
| 16. $(c - 6)(c - 7)$ | 17. $(c - 8)(c - 3)$ | 18. $(c - 1)(c - 9)$ |
| 19. $(s + 6)(s + 6)$ | 20. $(s - 6)(s - 6)$ | 21. $(s + 7)(s + 7)$ |
| 22. $(z - 7)(z - 7)$ | 23. $(z + 4)(z + 9)$ | 24. $(z - 4)(z - 9)$ |
| 25. $(b + 36)(b + 1)$ | 26. $(b - 36)(b - 1)$ | 27. $(b - 9)(b - 9)$ |
| 28. $(p + 3)(p + 3)$ | 29. $(p - 8)(p + 1)$ | 30. $(p + 2)(p + 2)$ |
| 31. $(m + 5)(m + 6)$ | 32. $(m + 4)(m - 3)$ | 33. $(m + 2)(m - 3)$ |
| 34. $(n - 5)(n - 3)$ | 35. $(n - 2)(n + 5)$ | 36. $(n + 1)(n - 4)$ |
| 37. $(r - 4)(r + 2)$ | 38. $(r - 1)(r + 6)$ | 39. $(r + 6)(r + 6)$ |
| 40. $(e - 7)(e + 2)$ | 41. $(e + 3)(e + 4)$ | 42. $(e - 8)(e + 7)$ |
| 43. $(g - 4)(g - 3)$ | 44. $(g + 2)(g - 3)$ | 45. $(g - 4)(g + 3)$ |
| 46. $(k + 1)(k - 5)$ | 47. $(k + 3)(k - 2)$ | 48. $(k + 7)(k - 5)$ |
| 49. $(y + 6)(y - 2)$ | 50. $(y + 6)(y - 3)$ | 51. $(y + 4)(y - 7)$ |
| 52. $(x + 5)(x - 8)$ | 53. $(x + 3)(x - 5)$ | 54. $(x + 5)(x + 6)$ |
| 55. $(v - 1)(v - 8)$ | 56. $(v - 3)(v + 8)$ | 57. $(v + 3)(v - 8)$ |
| 58. $(w + 6)(w - 4)$ | 59. $(w - 6)(w + 4)$ | 60. $(w + 12)(w - 2)$ |
| 61. $(a + 2)(a - 12)$ | 62. $(a + 24)(a - 1)$ | 63. $(a + 1)(a - 24)$ |
| 64. $(b + 10)(b - 3)$ | 65. $(b + 5)(b - 9)$ | 66. $(b + 2)(b - 9)$ |

67. $(c + 7)(c + 8)$
 70. $(d + 2)(d - 14)$
 73. $(a + b)(a + b)$

68. $(c - 9)(c - 6)$
 71. $(d + 50)(d - 1)$
 74. $(x - y)(x - y)$

69. $(c + 9)(c + 9)$
 72. $(d - 1)(d - 50)$
 75. $(p - 2q)(p + q)$

Exercise 6

1. $(x + 3)(2x + 1)$
 4. $(2a + 3)(5a + 1)$
 7. $(3d + 5)(d + 3)$
 10. $(2n - 1)(6n - 1)$
 13. $(4s - 5)(2s - 1)$
 16. $(3k - 2)(k - 1)$
 19. $(2u - 3)(3u + 2)$
 22. $(3d + 1)(d - 1)$
 25. $(4p - 3)(p - 2)$
 28. $(1 - 4x)^2$

2. $(2y + 3)(y + 1)$
 5. $(2b + 1)(3b + 2)$
 8. $(2m + 3)(5m + 2)$
 11. $(2q - 1)(q - 3)$
 14. $(3r - 4)^2$
 17. $(3y + 4)(y - 2)$
 20. $(5v - 1)(v + 1)$
 23. $(4a + 3)(2a - 1)$
 26. $(3 - 2x)(5 + x)$
 29. $(1 - 6x)(1 + 3x)$

3. $(3w + 1)(w + 2)$
 6. $(6c + 1)(c + 1)$
 9. $(2p - 1)(p - 3)$
 12. $(2x - 3)(3x - 2)$
 15. $(3g - 2)(4g - 5)$
 18. $(3w + 1)(w - 2)$
 21. $(2x - 1)(x + 1)$
 24. $(4y - 5)(3y + 1)$
 27. $(5 - 4x)(1 + 3x)$
 30. $(4p + q)(p - 2q)$

Exercise 7

1. $4(x + 3y)$
 4. $y(y - 1)$
 7. $(u + 6)^2$
 10. $(w - r)(w + r)$
 13. $(t + 1)(t - 1)$
 16. $3(c - 4)(c + 4)$
 19. $(2s + 5)(s - 1)$
 22. $(7 - g)(7 + g)$
 25. $(5 - 3g)(5 + 3g)$
 28. $11(u - 2v)(u + 2v)$
 31. $3mn(m - 2n)$
 34. $3a(a - 4)(a + 4)$
 37. $5(r - 1)(r + 2)$
 40. $9x(1 + 3x^2)$
 43. $(x - 1)(x + 1)(x^2 + 1)$
 46. $(2k + \pi r)(k + \pi r)$
 49. $2(a^2 + 2)(a^2 - 3)$
 52. $(3x^2 - 4)^2$
 54. $(1 - y)(1 + y)(1 + y^2)(1 + y^4)$

2. $(a - 9)(a + 9)$
 5. $(v - 4)(v + 3)$
 8. $a(p - q + r)$
 11. $h(h - 11)$
 14. $t(t - 1)$
 17. $5d(d - 4)$
 20. $(x - 6)^2$
 23. $4(3 - r)(3 + r)$
 26. $(2b + 1)(b - 1)$
 29. $7(3u^2 + 4v^2)$
 32. $(1 - n^2)$
 35. $2(2n - 1)(2n + 3)$
 38. $2(2w - 1)(w + 4)$
 41. $x(y - z)(y + z)$
 44. $2(1 - q)^2$
 47. $a^2(1 - a)(1 + a)(1 + a^2)$
 50. $b(b - 3)(b + 3)(b^2 + 9)$
 53. $(2x^2 - 3)(x^2 + 1)$

3. $(w + 5)^2$
 6. $(1 - b)(1 + b)$
 9. $7(x - 2)(x + 2)$
 12. $(x - 1)^2$
 15. $(a - 3)(a + 1)$
 18. $a^3(a - 1)$
 21. $(4y + 1)^2$
 24. $7z(2 - z)$
 27. $(2x + 3)(3x - 1)$
 30. $(5p - 1)^2$
 33. $(3 - s)(9 + s)$
 36. $2(2n - 1)^2$
 39. $7x(1 - 3x)(1 + 3x)$
 42. $(2e + 3)(e - 7)$
 45. $(g + 3h)(g - 2h)$
 48. $(k^2 + 1)^2$
 51. $(3x^2 - 1)(x^2 + 2)$

Checkup for Algebraic Operations

1. (a) $12x + 3$ (b) $ya - y^2$ (c) $v^2 - v$ (d) $14w^2 - 35w$
 (e) $18x + 12y - 6$ (f) $c^3 + c^2 - c$ (g) $12da + 9db$ (h) $gh^2 - g^3$
 (i) $18x^2 + 12xy - 6x$ (j) $c^4 + c^3 - 4c^2$ (k) $3a^2b + 4ab^2$ (l) $10pq - 2pq^2$
2. (a) $x^2 + 8x + 7$ (b) $x^2 - 5x + 6$ (c) $x^2 - x - 30$ (d) $x^2 + 6x - 27$
 (e) $x^2 + 2x + 1$ (f) $x^2 - 4x + 4$ (g) $20x^2 + 31x - 7$ (h) $12x^2 - 12x + 3$
 (i) $6x^2 - 10x - 4$ (j) $4x^2 - 12x + 9$ (k) $4x^3 - 11x^2 + 8x - 4$
 (l) $x^3 - 9x^2 + 27x - 27$
3. (a) $9(m - n)$ (b) $3(2a - 5b)$ (c) $y(1 - y)$
 (d) $2(7p^2 + 3q)$ (e) $p(3r + u)$ (f) $2p(2p + 3q - 1)$
 (g) $3(2x + 10y - 5z)$ (h) $3p(3q - 4r)$ (i) $(r - s)(r + s)$
 (j) $(9 - q)(9 + q)$ (k) $(4r - 7)(4r + 7)$ (l) $2(b - 4)(b + 4)$

- (m) $5w(2w - 3)(2w + 3)$ (n) $(y - 2)(y - 1)$ (o) $(a - 10)(a + 3)$
 (p) $(y - 2)(y + 3)$ (q) $(12 - r)(2 + r)$ (r) $(x - 7)^2$
 (s) $(2p - 3)(3p - 4)$ (t) $(2x + 1)^2$ (u) $2(q + 8)(q - 9)$
 (v) $(2x - y)(x + 2y)$ (w) $2(a^2 + 1)(3a^2 - 2)$ (x) $(5y^2 + 3)(y^2 - 3)$

Properties of the Circle

Exercise 1

1. (i) 5.2 cm (ii) 9.42 cm (iii) 25.1 cm (iv) 14.0 cm (v) 47.1 cm (vi) 44.7 cm
 (vii) 42.4 cm
 2. 47.1 cm 3. 83.7 cm 4. 314 m 5. 45° 33.0 inches 6. 92°

Exercise 2

1. (a) 105 cm² (b) 177 cm² (c) 471 cm² (d) 236 cm²
 (e) 367 cm² (f) 377 cm²
 2. 2152 cm² 3. 1.64 m² 4. 2261 cm²
 5. (a) 112 cm² (b) 134 cm³ 6. (a) 179 cm² (b) 2144 cm³ 7. 90°

Exercise 3

1. $a = 38, b = 25, c = 30, d = 20, e = 125, f = 20, g = 130, h = 70, i = 24,$
 $j = 26, k = 51.$
 2. $w = 117$ 3. $x = 63$ 4. $v = 12, w = 7$
 5. $x = 30, y = 38.9, z = 66$ 6. (a) 100mm (b) $p = 48, q = 42, r = 42$

Exercise 4

1. $a = 59, b = 45, c = 15, d = 50, e = 34, f = 40, g = 59.$
 2. (a) 23° (b) 49° 3. (a) 12.2 (b) 12.0
 4. (a) $v = 100, w = 36.9$ (b) $v = 10.5, w = 58.2$ (c) $v = 10.2, w = 42.8$
 (d) $v = 96.6, w = 75$

Exercise 5

1. (a) 140° 20° 20° (b) 20° 20° 70° 70° (c) 10° 10° 80° 80° 90° 90° 90° 90°
 (d) 4 x 90°, 4 x 75°, 4 x 15°
 2. (a) 6 (b) 7 (c) 4.77 3. (a) 11.0 (b) 14.4
 4. $a = 7.8, b = 8.9, c = 41.4, d = 48.6.$
 5. 18 cm 6. 73.9 cm 7. 59.2 cm

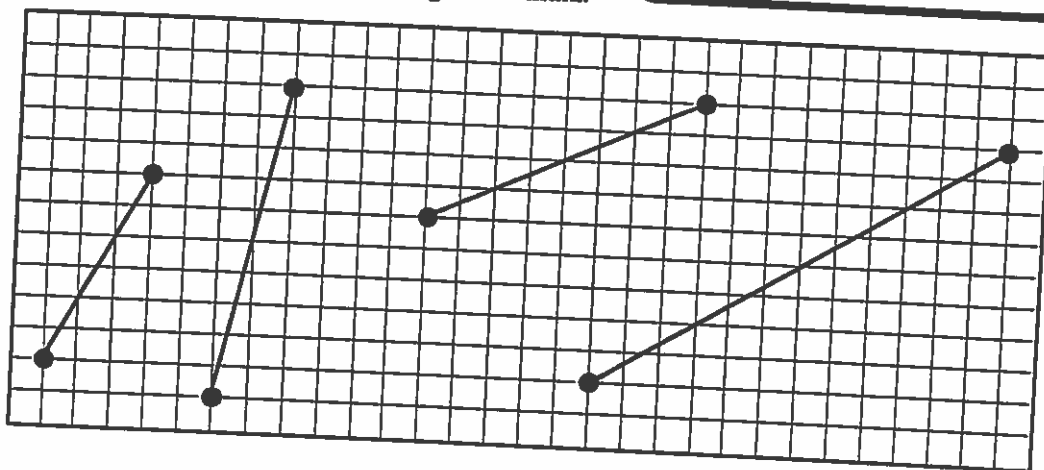
LINEAR RELATIONSHIPS

A. The Gradient of a Line

Exercise 1

1. Find the gradient of each line using the formula:

$$\text{gradient} = \frac{\text{vertical distance}}{\text{horizontal distance}}$$

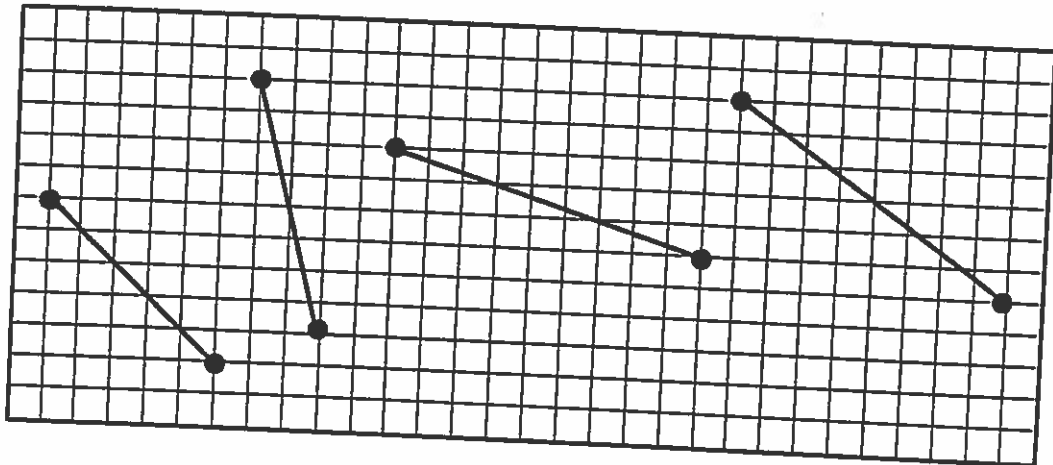


2. For each of the following pairs of points:
- draw a (small) coordinate diagram,
 - plot the two points and join them to form a straight line,
 - calculate the gradient of the line joining the two points.
- (a) P(1,1), Q(3,9) (b) A(3,0), B(5,6)
(c) R(-3,1), S(5,5) (d) L(-4,-1), M(2,3)
3. Calculate the gradients of the lines joining the following pairs of points:
- (a) C(1,5), D(7,7) (b) U(0,3), V(12,7)
(c) J(-1,-6), K(1,6) (d) O(0,0), T(5,15)

So far, all the lines you have met in this exercise have had gradients which were positive.

4. Describe how a line with a negative gradient differs in shape from that of a line with a positive gradient.

5. Calculate the gradient of each line.



6. Calculate the gradients of the lines joining the following points. (Some are positive, some negative).

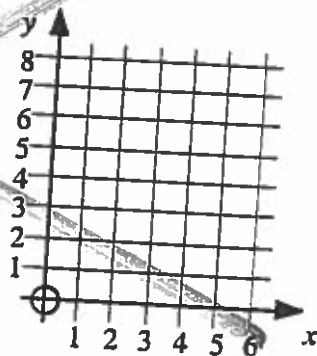
- | | | |
|-----------------------|------------------------|-----------------------|
| (a) A(1,6), B(6,1) | (b) D(0,7), E(2,3) | (c) G(-2,5), H(1,-4) |
| (d) J(-6,-3), K(3,0) | (e) M(-6,0), N(0,-4) | (f) P(1,-1), Q(3,1) |
| (g) S(-1,10), T(3,-2) | (h) V(-6,-10), W(2,-6) | (i) Y(-12, 5), Z(3,0) |

7. (a) On a small coordinate diagram plot the two points A(1,3) and B(6,3).
 (b) Find the gradient of the line joining A and B using your formula.
 (c) Comment on the connection between the shape (slope) of the line drawn in part (a) and the corresponding value of its gradient as calculated in part (b).

B. Sketching Lines in the form $y = ax + b$

Exercise 2

1. Drawing the line $y = 2x + 1$:
- Make a copy of this coordinate diagram.
 - Where does the line $y = 2x + 1$ cut the y -axis? (plot this point).
 - The gradient of the line is 2. From your first plotted point, move 1 box right and 2 boxes up. Plot this 2nd point.
 - Join your 2 points and extend the line.
 - Label the line $y = 2x + 1$.



Checkup for Volumes of Solids

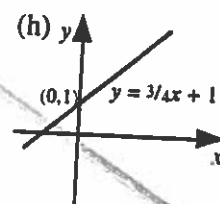
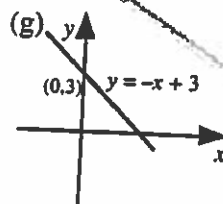
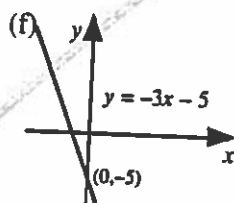
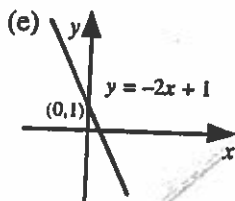
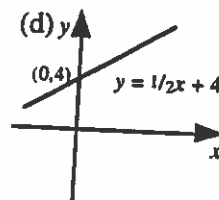
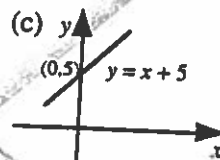
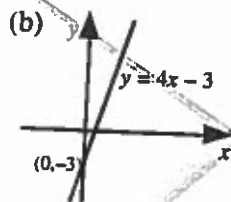
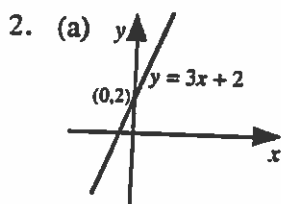
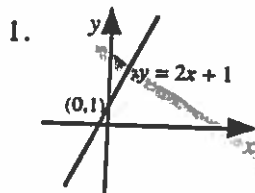
- (a) 112.5 cm^3 (b) 168 cm^3 (c) 185 cm^3
- (a) 459 cm^3 (b) 1001 cm^3 (c) 536.1 cm^3
- (a) 2797.7 cm^3 (b) 769.3 cm^3 (c) 588.7 cm^3
- $678.24 \text{ cm}^3 + 3391.2 \text{ cm}^3 + 452.16 \text{ cm}^3 = 4521.6 \text{ cm}^3$

Linear Relationships

Exercise 1

- 2, 5, $\frac{1}{2}$, $\frac{2}{3}$
- (a) 4 (b) 3 (c) $\frac{1}{2}$ (d) $\frac{2}{3}$
- (a) $\frac{1}{3}$ (b) $\frac{1}{3}$ (c) 6 (d) 3
- slopes downwards if gradient is negative as you move from left to right
- 1, -4, $-\frac{1}{3}$, $-\frac{3}{4}$
- (a) -1 (b) -2 (c) -3 (d) $\frac{1}{3}$ (e) $-\frac{2}{3}$
(f) 1 (g) -3 (h) $\frac{1}{2}$ (i) $-\frac{1}{3}$
- (a) sketch showing vertical line. (b) gradient doesn't exist (error)
(c) gradient of a vertical line does not exist.

Exercise 2



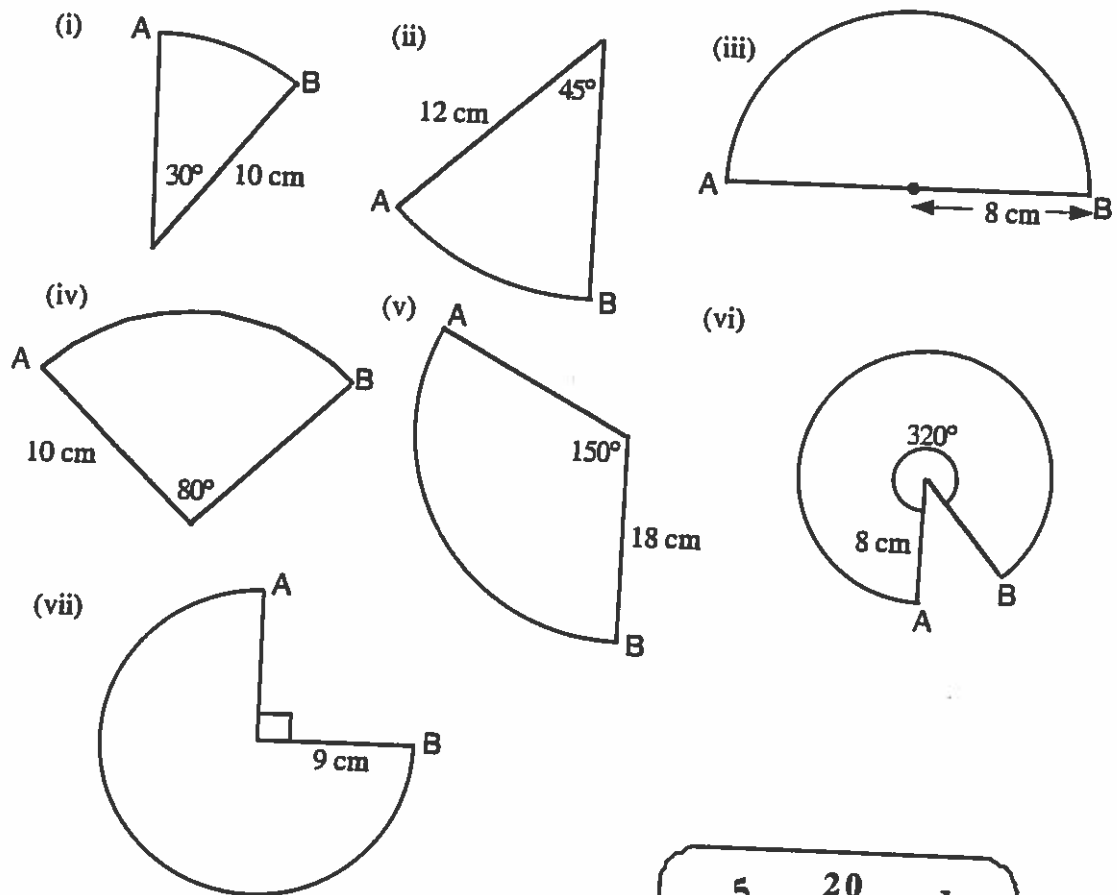
3. Line A - $a_6 = 2$ Line B - $a_5 = -1$ Line C - $a_1 = \frac{1}{2}$
Line D - $a_3 = -\frac{1}{2}$ Line E - $a_4 = 0$ Line F - $a_2 = -3$

PROPERTIES OF THE CIRCLE

A. Finding the length of an arc

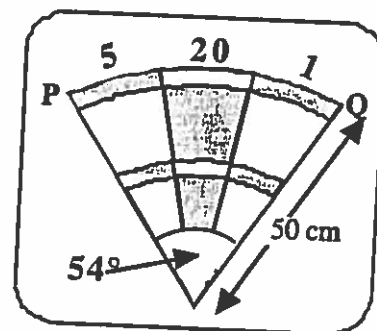
Exercise 1

1. In each diagram, calculate the length of the arc AB of the sector.

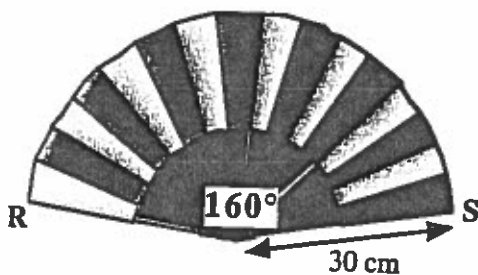


2. As the T.V. camera zoomed in on the dart board the sector involving the numbers 5, 20 and 1 was focused.

Calculate the length of the arc PQ.

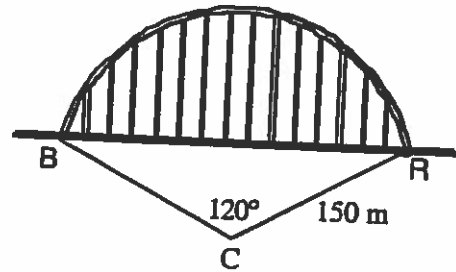


3.

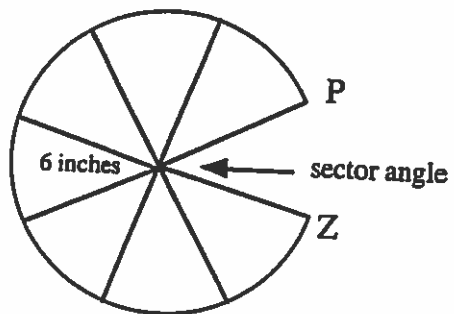


Calculate the length of the curved edge RS of the fan which is in the shape of a sector of a circle.

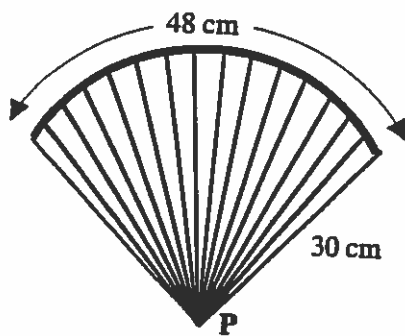
4. Calculate the length of the arch BR of the bridge which is the arc of a circle, centre C.



5. This circular pizza has been sliced into 8 pieces.
Calculate:
(a) the size of the sector angle of one piece.
(b) the length of the major arc PZ.



6.

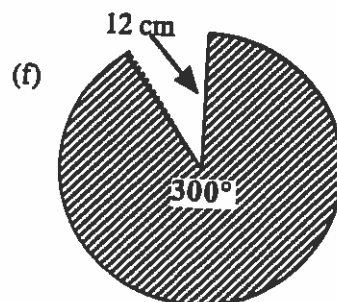
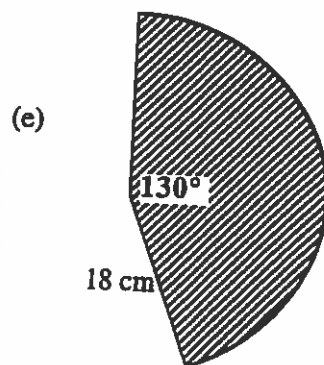
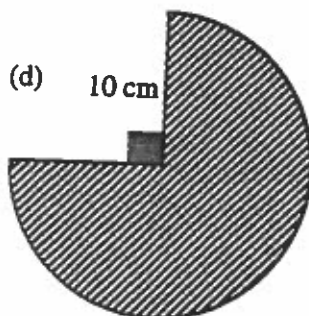
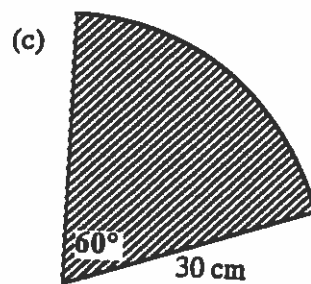
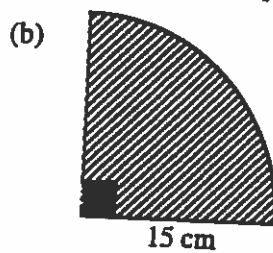
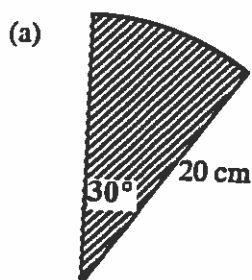


The lace edge of this fan is 48 cm long.
It is an arc of a circle, centre P.
Calculate the size of the angle at P.
(Answer to the nearest whole degree.)

B. Finding the area of a sector

Exercise 2

1. Calculate the area of each sector (to the nearest square centimetre):



2.

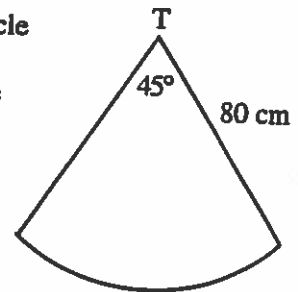


This house has an unusually shaped living room window.

It is in the shape of a sector of a circle with radius 80 cm.

Unfortunately there is a crack in the glass and a new pane is required.

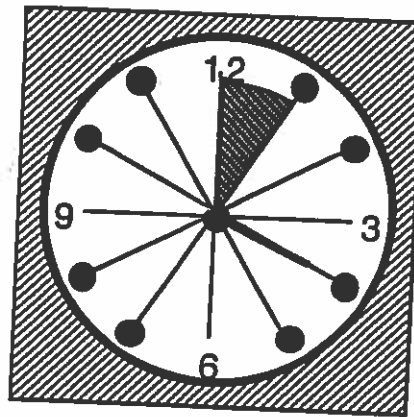
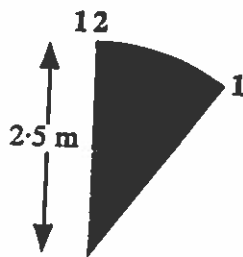
If the angle at T is 45° , calculate the area of glass to be replaced.



3. The face of a large town clock was in need of repair.

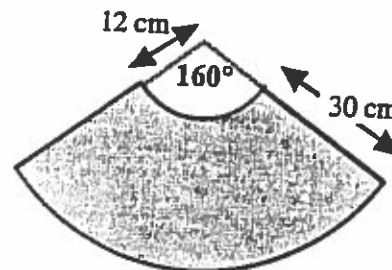
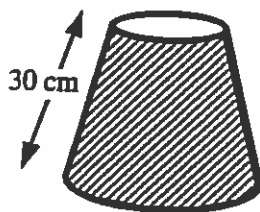
Workmen were replacing the rusted sector between the numbers 12 and 1 on the clock face.

Calculate the area of this sector.



4. A light shade is made up from the sector of a large circle with a smaller sector removed.

Calculate the area of the shade.

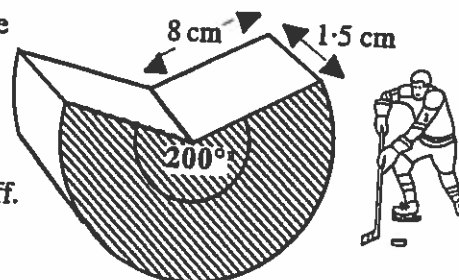


5. A cylindrical ice hockey puck of radius 8 cm and height 1.5 cm hits a goal post with such force that it splits, leaving a perfect sector as shown.

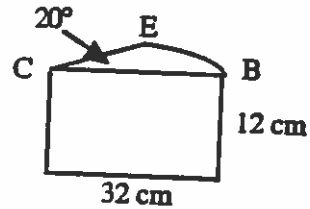
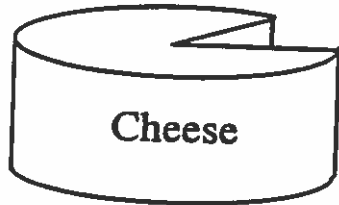
Calculate:

(a) the shaded area.

(b) the volume of the smaller part which broke off.



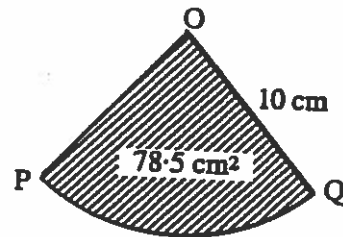
6. A wedge of cheese is cut from a large circular block of radius 32 cm and height 12 cm. For the wedge, the angle at C, the centre, is 20° .



Calculate:

- the area of the sector BCE.
 - the volume of the wedge of cheese.
7. The area of this sector is 78.5 cm^2 and the radius of the circle from which it has been cut is 10 cm.

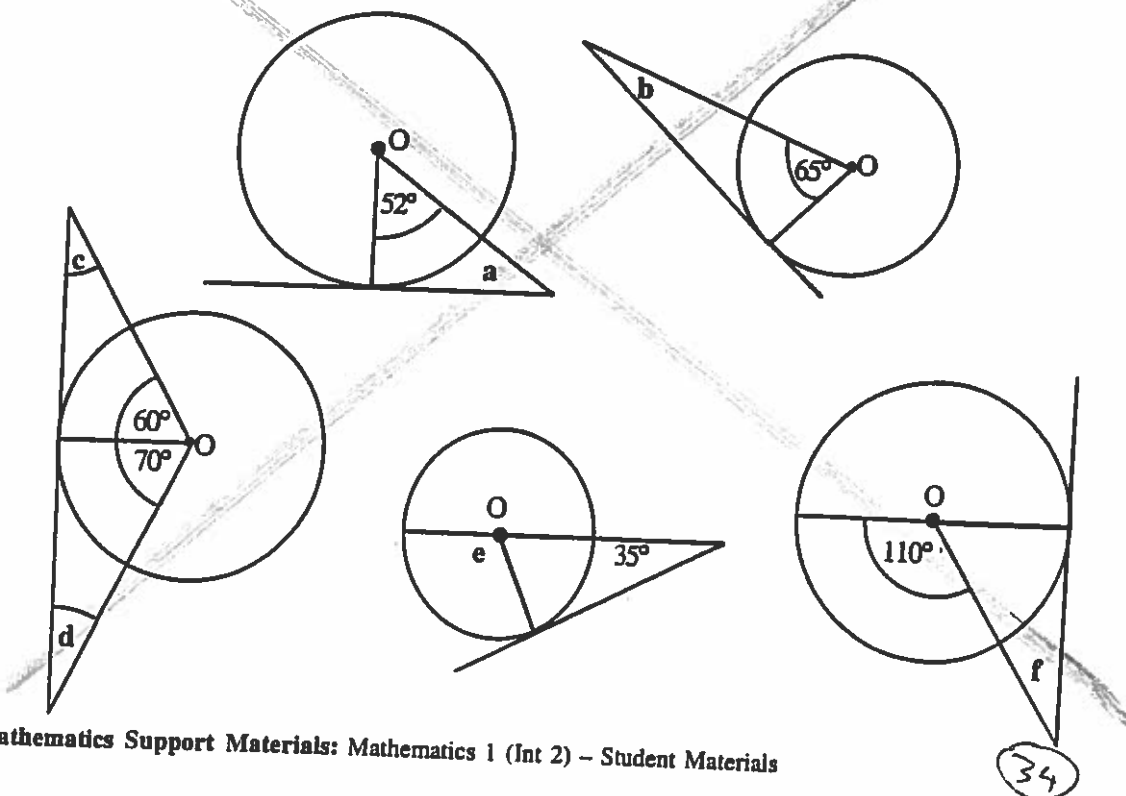
Calculate the size of angle POQ.



C. The relationship between tangent and radius

Exercise 3

1. Copy the diagrams below and fill in the sizes of the angles marked with a letter.



~~(m) $5w(2w-3)(2w+3)$ (n) $(y-2)(y-1)$ (o) $-(a-10)(a+3)$~~
~~(p) $(y-2)(y+3)$ (q) $(12-r)(2+r)$ (r) $(x-7)^2$~~
~~(s) $(2p-3)(3p-4)$ (t) $(2x+1)^2$ (u) $2(q+8)(q-9)$~~
~~(v) $(2x-y)(x+2y)$ (w) $2(a^2+1)(3a^2-2)$ (x) $(5y^2+3)(y^2-3)$~~

Properties of the Circle

Exercise 1

1. (i) 5.2 cm (ii) 9.42 cm (iii) 25.1 cm (iv) 14.0 cm (v) 47.1 cm (vi) 44.7 cm
 (vii) 42.4 cm
 2. 47.1 cm 3. 83.7 cm 4. 314 m 5. 45° 33.0 inches 6. 92°

Exercise 2

1. (a) 105 cm² (b) 177 cm² (c) 471 cm² (d) 236 cm²
 (e) 367 cm² (f) 377 cm²
 2. 2152 cm² 3. 1.64 m² 4. 2261 cm²
 5. (a) 112 cm² (b) 134 cm³ 6. (a) 179 cm² (b) 2144 cm³ 7. 90°

Exercise 3

1. $a=38, b=25, c=30, d=20, e=125, f=20, g=130, h=70, i=24, j=26, k=51$
 2. $w=117$ 3. $x=63$ 4. $v=12, w=7$
 5. $x=30, y=38.9, z=66$ 6. (a) 100mm (b) $p=48, q=42, r=42$

Exercise 4

1. $a=59, b=45, c=15, d=50, e=34, f=40, g=59$
 2. (a) 23° (b) 49° 3. (a) 12.2 (b) 12.0
 4. (a) $v=100, w=36.9$ (b) $v=10.5, w=58.2$ (c) $v=10.2, w=42.8$
 (d) $v=96.6, w=75$

Exercise 5

1. (a) 140° 20° 20° (b) 20° 20° 70° 70° (c) 10° 10° 80° 80° 90° 90° 90° 90°
 (d) 4 x 90°, 4 x 75°, 4 x 15°
 2. (a) 6 (b) 7 (c) 4.77 3. (a) 11.0 (b) 14.4
 4. $a=7.8, b=8.9, c=41.4, d=48.6$
 5. 18 cm 6. 73.9 cm 7. 59.2 cm

11. Calculate the percentage depreciation of the value of this car:
- from 1995 to 1996.
 - from 1997 to 1998.
 - from 1995 to 1999.

				
1995	1996	1997	1998	1999
£12 000	£4 800	£2 400	£1 920	£1 800

12. The value of an antique jug rose by 5% to £10 500.
Work out its previous value. (not £9 975!)

C. Significant Figures

Exercise 4

- Round the following numbers to one significant figure (1 sig. fig.).

(a) 4269	(b) 14774	(c) 17	(d) 487
(e) 18152	(f) 2085	(g) 7510	(h) 6551
(i) 42670	(j) 451	(k) 14308	(l) 24859
(m) 6890000	(n) 55847155	(o) 38749886541	(p) 25
- Round the following numbers to two significant figures (2 sig. figs.).

(a) 5187	(b) 24885	(c) 221	(d) 555
(e) 19352	(f) 2065	(g) 7650	(h) 6549
(i) 42501	(j) 448	(k) 78209	(l) 29899
(m) 6890000	(n) 55847155	(o) 38749886541	(p) 351
- Round the following numbers to three significant figures (3 sig. figs.).

(a) 8181	(b) 24882	(c) 2217	(d) 5554
(e) 19551	(f) 2077	(g) 7682	(h) 6149
(i) 42552	(j) 4499	(k) 78209	(l) 29897
(m) 6893000	(n) 55847155	(o) 38749886541	(p) 35150001
- Round each of the following decimals to:

	(i) 1 significant figure
	(ii) 2 significant figures
	(iii) 3 significant figures
(a) 8.33333	(b) 23.81558
(c) 1.53097	(d) 347.502

ANSWERS

Calculations Involving Percentages

Exercise 1

1. (a) £12.75 (b) £21 (c) £1.10 (d) 68p (e) £9
(f) £48 (g) £7.20 (h) £4.80 (i) £3.50 (j) £7.60
(k) £1980 (l) 45p (m) £70 (n) £45 (o) £49.20
(p) £3.50 (q) £3.40 (r) 50p
2. (a) £30 (b) £80 3. (i) 42 (ii) 108 4. 482.5mm
5. 112.5g 6. (a) 76 (ii) 3724 7. 13440ft 8. 143.5cm 9. 2210
10. (a) £1308 (b) £1254 (c) £1281 (d) £1236 (e) £1245
11. 80% 12. 75% 13. 30% 14. 96.4%

Exercise 2

1. (a) £1389.15 (b) £703.04 (c) £52.02 2. £56.16
3. £623.70 4. £2803.50 5. £275.73
6. (a) Mrs. D £795.68 Mrs. E £795 (b) 3% per half year better as you get interest on the interest for rest of year.
7. £2929.64 8. 8 years

Exercise 3

1. £72600 2. £63559 3. £132848 4. £76098 5. £2160
6. £9000 7. £6055.20 8. 4.2% 9. (a) £86700 (b) 2%
10. (a) 5% (b) 8% 11. (a) 60% (b) 20% (c) 85% 12. £10000

Exercise 4

1. (a) 4000 (b) 10000 (c) 20 (d) 500 (e) 20000
(f) 2000 (g) 8000 (h) 7000 (i) 40000 (j) 500
(k) 10000 (l) 20000 (m) 7000000 (n) 60000000
(o) 40000000000 (p) 30
2. (a) 5200 (b) 25000 (c) 220 (d) 560 (e) 19000
(f) 2100 (g) 7700 (h) 6500 (i) 43000 (j) 450
(k) 78000 (l) 30000 (m) 6900000 (n) 56000000
(o) 39000000000 (p) 350
3. (a) 8180 (b) 24900 (c) 2220 (d) 5550 (e) 19600
(f) 2080 (g) 7680 (h) 6150 (i) 42600 (j) 4500
(k) 78200 (l) 29900 (m) 6890000 (n) 55800000
(o) 38700000000 (p) 35200000
4. (a) 8 (b) 20 (c) 2 (d) 300
8.3 24 1.5 350
8.33 23.8 1.53 348

Exercise 5

1. (a) £2300 (b) £2000 (c) £4390
2. £950 3. £3670 4. £10000 5. £38000 6. £23700
7. (a) 47% (b) 41.7% (c) 90%
8. (a) 8% (b) 29% (c) 100%

VOLUMES OF SOLIDS

By the end of this set of exercises, you should be able to

- (a) calculate the volumes of a prism, cone and sphere

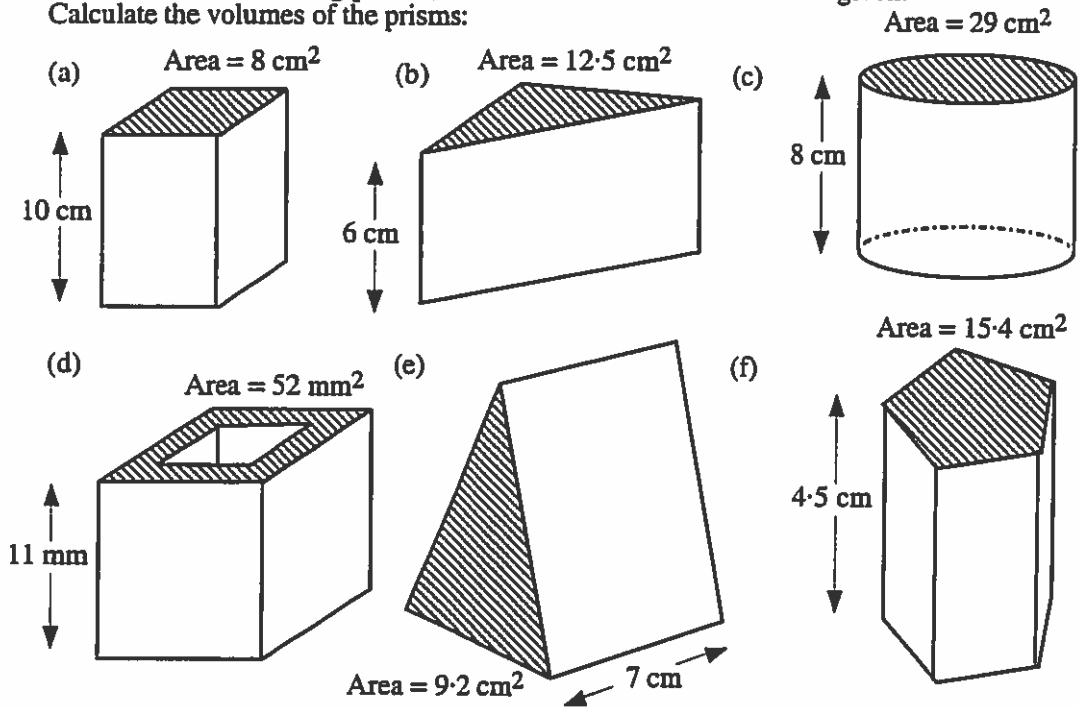
VOLUMES OF SOLIDS

A. Volume of a Prism

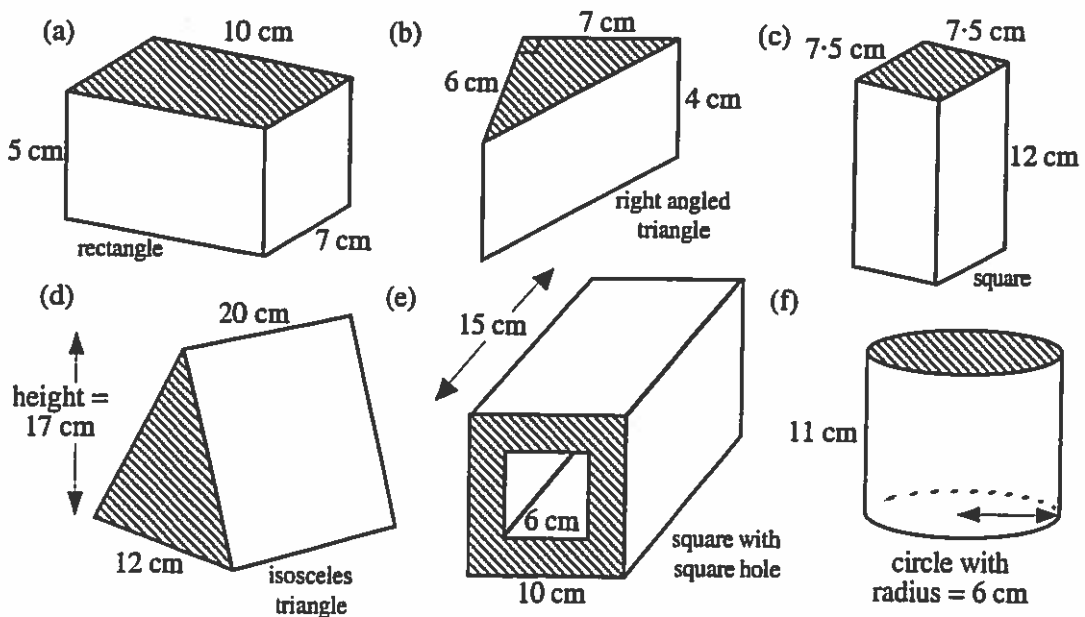
$$\text{Volume}_{\text{prism}} = \text{Area}_{\text{base}} \times \text{height}$$

Exercise 1

1. For each of the following prisms, the area of the base or end face is given. Calculate the volumes of the prisms:



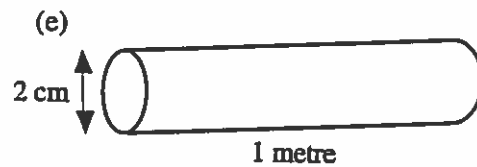
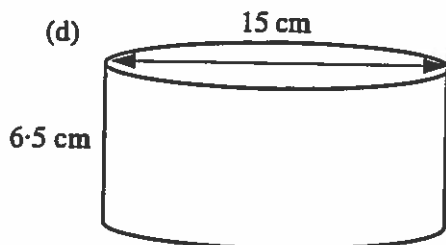
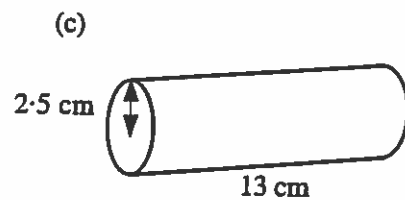
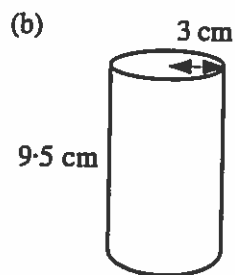
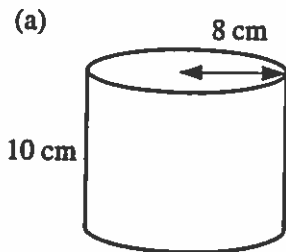
2. This time you must calculate the shaded area first, then find the volumes of the prisms.



3. The cylinder – a special prism.

Calculate the volumes of the following cylinders:

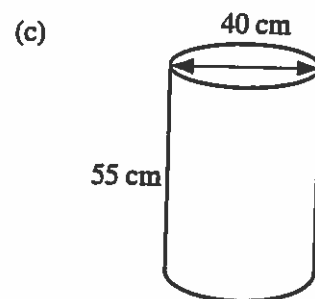
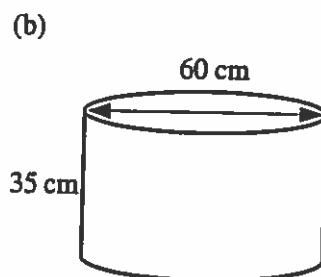
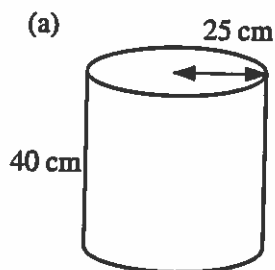
$$\text{Volume (cylinder)} = \pi r^2 h$$



4. Remember:

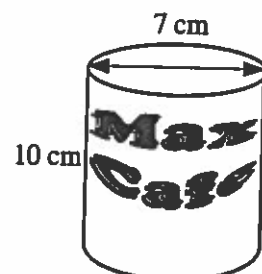
$$1 \text{ cm}^3 = 1 \text{ ml}; \quad 1000 \text{ cm}^3 = 1000 \text{ ml} = 1 \text{ litre}$$

How many litres of water will the following drums hold?



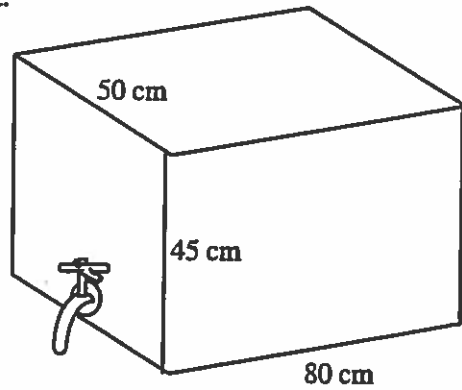
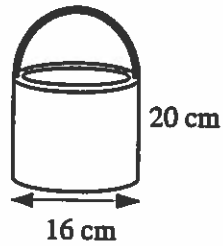
5. A cylindrical tin of Maxcafe Coffee is 10 centimetres high and has a base diameter of 7 centimetres.

What is the volume of coffee in the tin when it is full?



6. This rectangular storage tank is full of white paint.

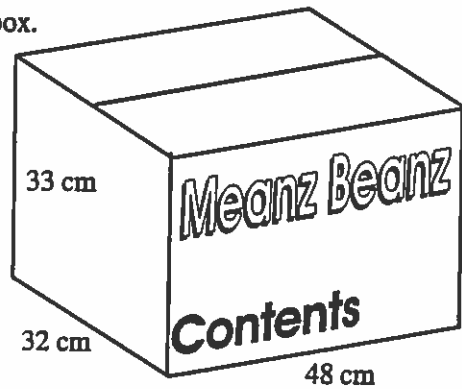
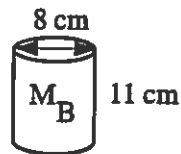
- (a) Calculate the volume of paint in the tank in cubic centimetres (cm^3).
- (b) Calculate the volume of this cylindrical paint tin.



- (c) How many times can the paint tin be completely filled from the tank?

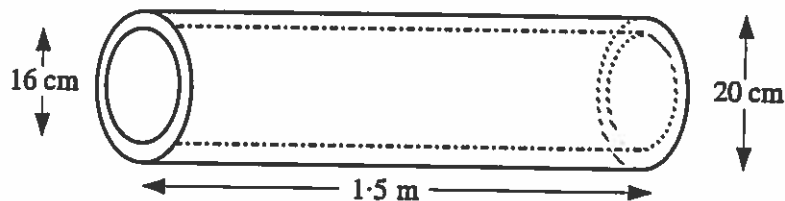
7. Meanz Beanz tins are packed into this cardboard box.

- (a) How many tins can be placed on the bottom layer?



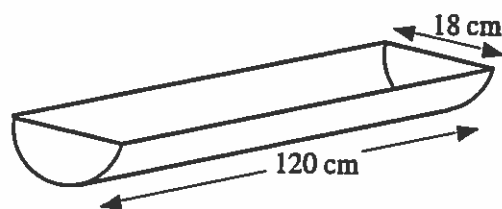
- (b) How many layers will there be?
- (c) How many tins can be packed in the box altogether?
- (d) How much air space in the box is there around all the tins?

8. This cast iron pipe has an internal diameter of 16 centimetres and an outside diameter of 20 centimetres. The pipe is 1.5 metres long.



Calculate the volume of iron needed to make the pipe.

9. How much liquid feeding will this semi-cylindrical pig-trough hold?

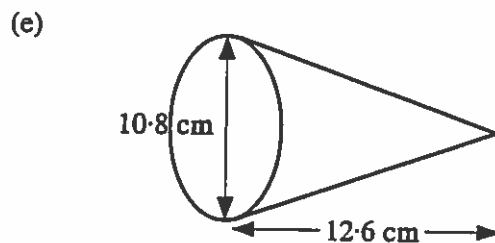
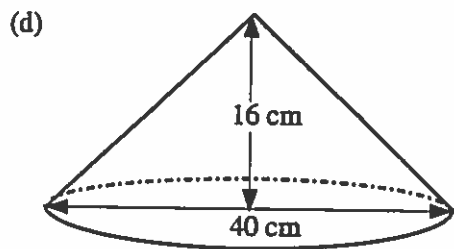
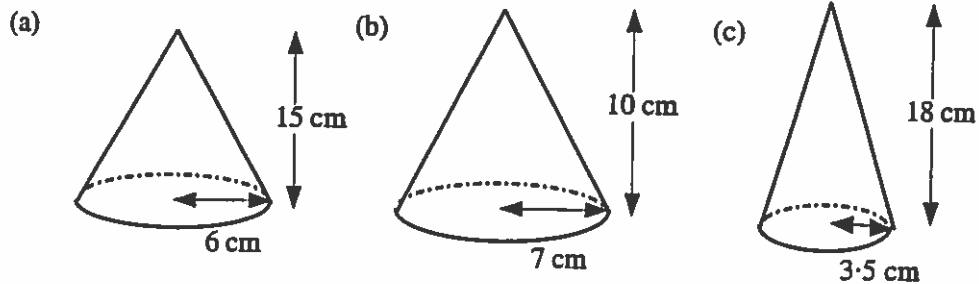


B. Volume of a Cone

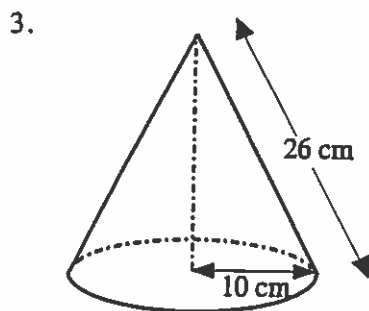
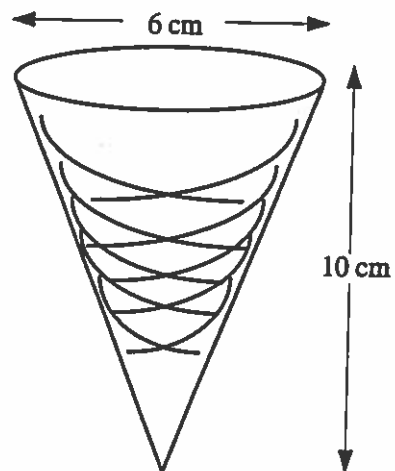
$$\text{Volume (cone)} = \frac{1}{3}\pi r^2 h$$

Exercise 2

1. Calculate the volumes of the following conical shapes:



2. The wafer of an ice-cream cone has a diameter of 6 centimetres. The cone is 10 centimetres high. Calculate the volume of the cone.

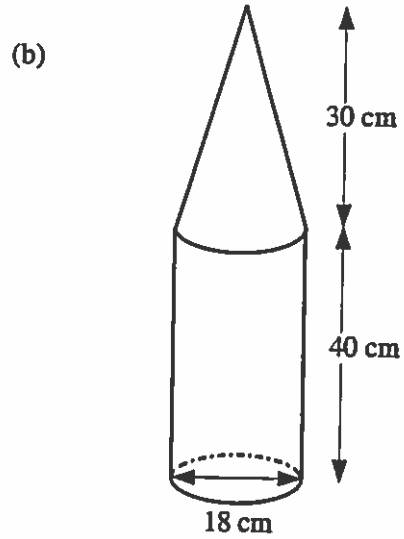
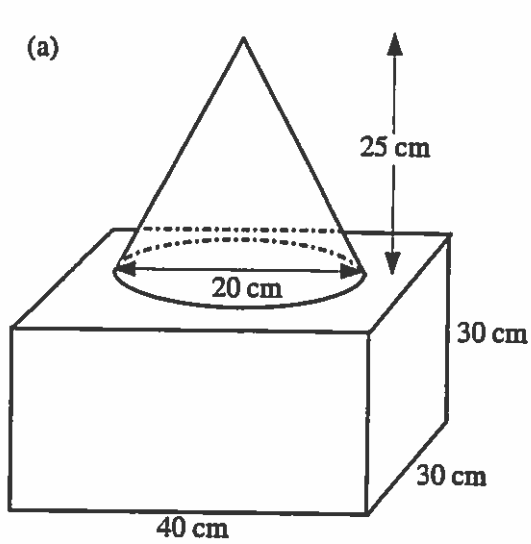


The 'sloping' height of this cone is 26 cm.

The base radius is 10 cm.

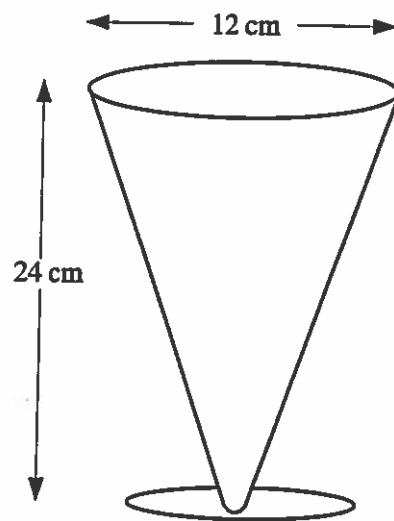
- (a) Calculate the height of the cone.
- (b) Calculate the volume of the cone.

4. Calculate the total volumes of the following shapes.



5. Water is poured into this conical flask at the rate of 50 millilitres per second.

- (a) Calculate the volume of the flask.
(b) How long will it take, to the nearest second, to fill the flask to the top?

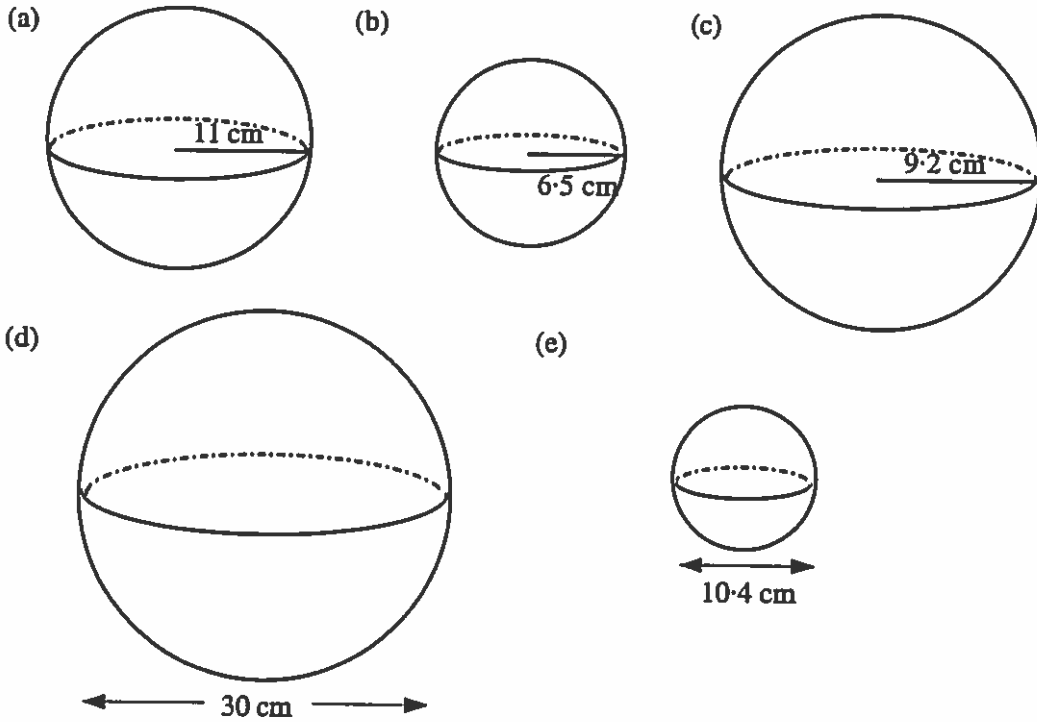


C. Volume of a Sphere

Exercise 3

$$\text{Volume}_{(\text{sphere})} = \frac{4}{3}\pi r^3$$

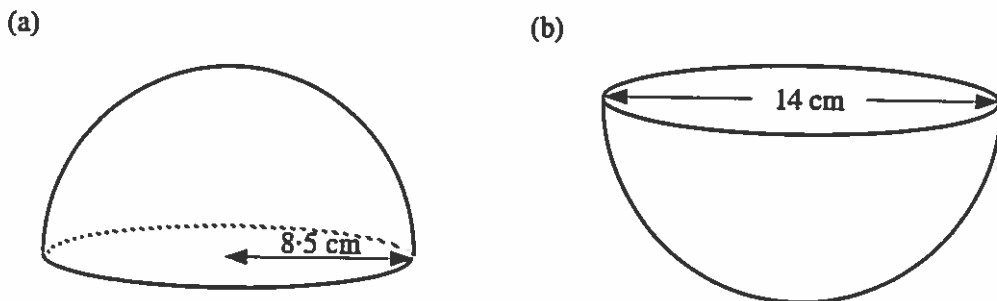
1. Calculate the volumes of the following spheres:



2. This football is fully inflated. Calculate the volume of air inside the football.

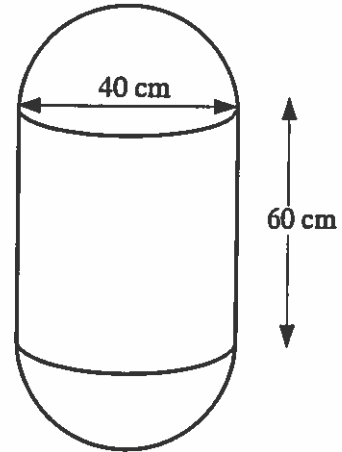


3. Calculate the volumes of these two 'hemispheres':

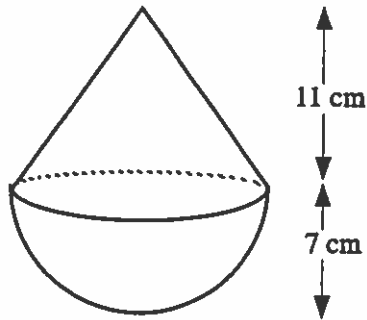


4. (a) Calculate the volume of water which can be stored in this copper hot water tank in cm^3 .
The tank consists of a cylinder with two hemispherical ends.

- (b) How many litres of water will it hold?
($1\text{cm}^3 = 1\text{ ml}$; $1000\text{ ml} = 1\text{ litre}$).



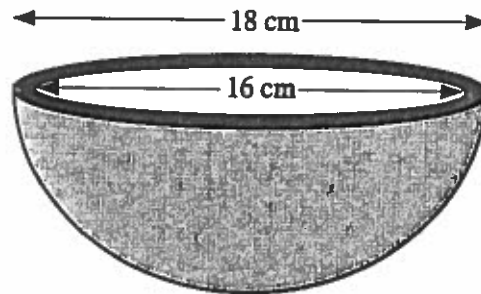
5.



Calculate the volume of this child's rocking toy which consists of a cone on top of a hemisphere.

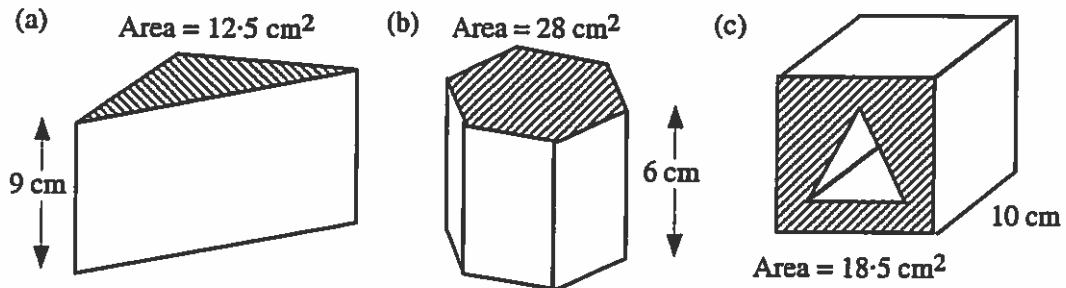
6. This decorative wooden fruit bowl is in the shape of a hollowed out hemisphere.

Calculate the volume of wood required to make it.

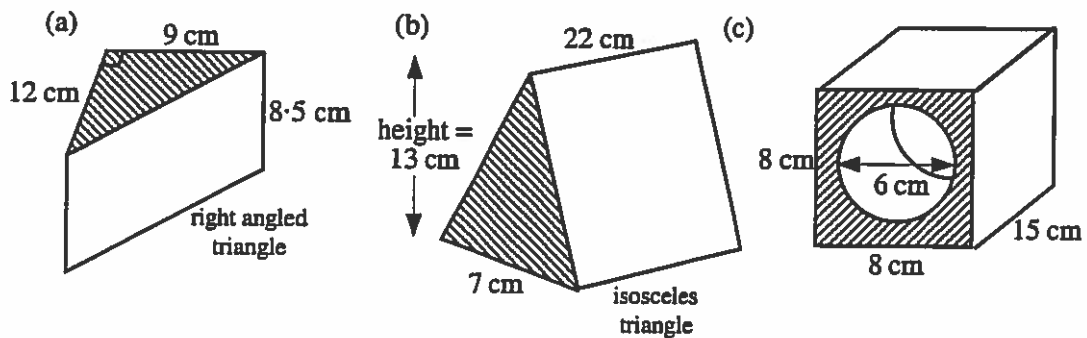


Checkup for Volumes of Solids

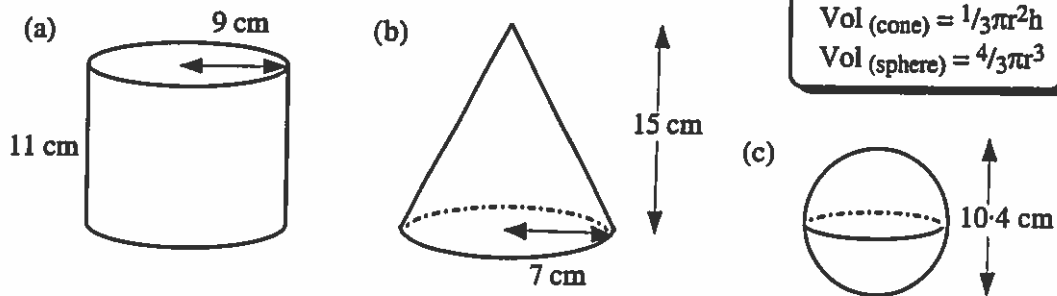
1. Calculate the volumes of the following prisms:



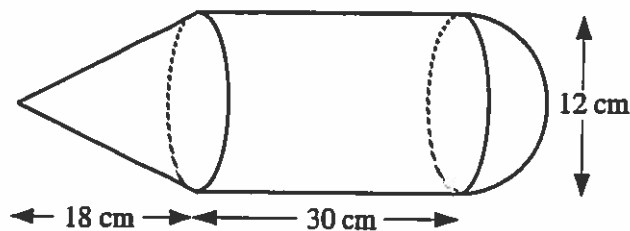
2. Calculate the shaded areas and use them to find the volume of each shape.



3. Calculate the volumes of the following shapes:



4. This shape consists of a cone, a cylinder and a hemisphere. Calculate its total volume.



Checkup for Calculations Involving Percentages

1. £33.28 2. £7400 3. £946 4. £700 5. £23800 6. £396
7. (a) £53531.25 (b) 39%
8. £13600

Volumes of Solids

Exercise 1

1. (a) 80 cm^3 (b) 75 cm^3 (c) 232 cm^3 (d) 572 cm^3
 (e) 64.4 cm^3 (f) 69.3 cm^3
2. (a) 350 cm^3 (b) 84 cm^3 (c) 675 cm^3 (d) 2040 cm^3
 (e) 960 cm^3 (f) 1243.44 cm^3
3. (a) 2009.6 cm^3 (b) 268.47 cm^3 (c) 255.125 cm^3
 (d) 1148.0625 cm^3 (e) 314 cm^3
4. (a) 78.5 litres (b) 98.91 litres (c) 69.08 litres
5. 384.65 cm^3
6. (a) 180000 cm^3 (b) 4019.2 cm^3 (c) 44
7. (a) $4 \times 6 = 24$ (b) 3 (c) 72 (d) 10897.92 cm^3
8. 16956 cm^3
9. 15260.4 cm^3

Exercise 2

1. (a) 565.2 cm^3 (b) 512.9 cm^3 (c) 230.8 cm^3 (d) 6699 cm^3 (e) 384.6 cm^3
2. 94.2 cm^3
3. (a) 24 cm (b) 2512 cm^3
4. (a) $2616.7 \text{ cm}^3 + 36000 \text{ cm}^3 = 38616.7 \text{ cm}^3$
 (b) $10173.6 \text{ cm}^3 + 2543.4 \text{ cm}^3 = 12717 \text{ cm}^3$
5. (a) 904.32 cm^3 (b) 18 seconds

Exercise 3

1. (a) 5572.5 cm^3 (b) 1149.8 cm^3 (c) 3260.1 cm^3
 (d) 14130 cm^3 (e) 588.7 cm^3
2. 7234.6 cm^3
3. (a) 1285.6 cm^3 (b) 718.0 cm^3
4. (a) $16746.66... + 16746.. + 75360 = 108853.3 \text{ cm}^3$ (b) 108.9 litres
5. $564.15.. + 718.01... = 1282.2 \text{ cm}^3$
6. 454.3 cm^3

Checkup for Volumes of Solids

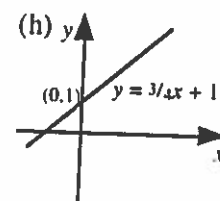
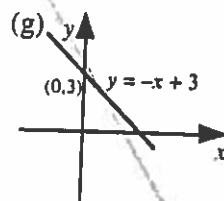
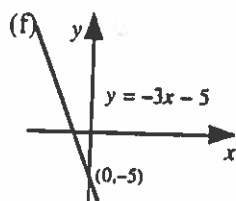
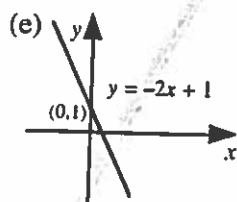
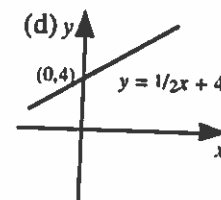
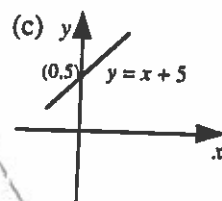
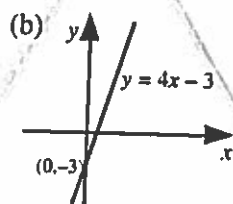
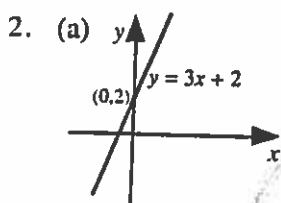
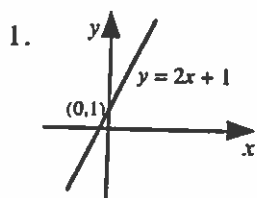
- (a) 112.5 cm^3 (b) 168 cm^3 (c) 185 cm^3
- (a) 459 cm^3 (b) 1001 cm^3 (c) 536.1 cm^3
- (a) 2797.7 cm^3 (b) 769.3 cm^3 (c) 588.7 cm^3
- $678.24 \text{ cm}^3 + 3391.2 \text{ cm}^3 + 452.16 \text{ cm}^3 = 4521.6 \text{ cm}^3$

Linear Relationships

Exercise 1

- 2, 5, $\frac{1}{2}$, $\frac{2}{3}$
- (a) 4 (b) 3 (c) $\frac{1}{2}$ (d) $\frac{2}{3}$
- (a) $\frac{1}{3}$ (b) $\frac{1}{3}$ (c) 6 (d) 3
- slows downwards if gradient is negative as you move from left to right
- 1, -4, $-\frac{1}{3}$, $-\frac{3}{4}$
- (a) -1 (b) -2 (c) -3 (d) $\frac{1}{3}$ (e) $-\frac{2}{3}$
(f) 1 (g) -3 (h) $\frac{1}{2}$ (i) $-\frac{1}{3}$
- (a) sketch showing vertical line. (b) gradient doesn't exist (error)
(c) gradient of a vertical line does not exist.

Exercise 2



- Line A - $a_6 = 2$ Line B - $a_5 = -1$ Line C - $a_1 = \frac{1}{2}$
Line D - $a_3 = -\frac{1}{2}$ Line E - $a_4 = 0$ Line F - $a_2 = -3$